PUSHO
a pulse-level variational optimisation algorithm for Rydberg atoms platforms

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1 – Quantum Computing with Rydberg atoms

- Qubit logical states $|0\rangle, |1\rangle$ are two electronic levels of ^{87}Rb atom.
- Atoms can be arranged in arbitrary patterns using optical traps generated by a spatial light modulator (SLM).
- Fluorescence imaging provides global measurement, only picturing atoms in $|0\rangle$.
- In Ising mode, lasers are coupled to a Rydberg transition, between a ground $|g\rangle = |0\rangle$ and a highly-excited Rydberg state $|r\rangle = |1\rangle$.

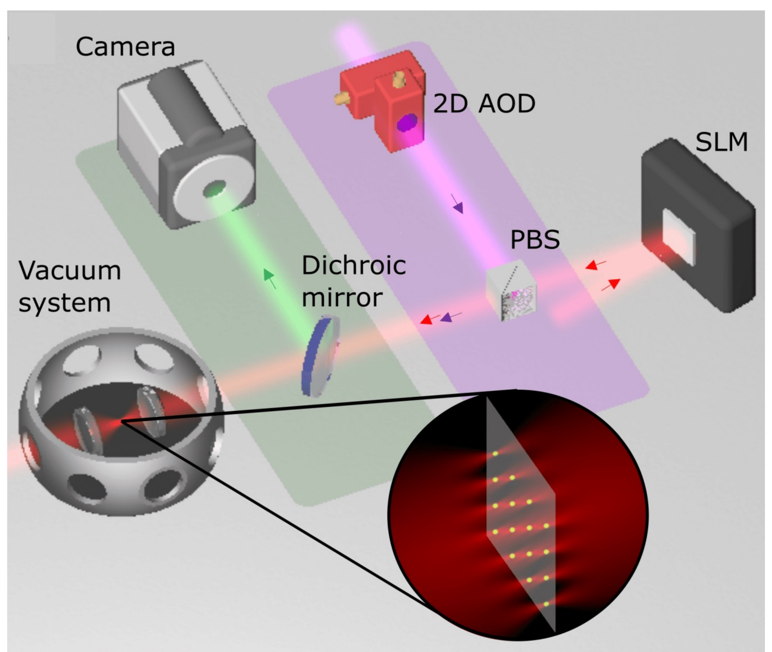


Fig. 1: Overview of the main hardware components of Pasqal QPU. [1]

- Ising Hamiltonian : $H = \frac{\hbar\Omega(t)}{2}\sum_i \hat{\sigma}_i^x - \hbar\delta(t)\sum_i \hat{n}_i + \sum_{i<j}\frac{C_6}{R_{i,j}^6}\hat{n}_i\hat{n}_j$

with $\hat{\sigma}_i^x = |r\rangle\langle g|_i + h.c.$, $\hat{n}_i = |r\rangle\langle r|_i$, $C_6 \propto n^{11}$.

- Performing optimal control on the shape of $\Omega(t)$ and $\delta(t)$ should provide a versatile method to solve various problems.

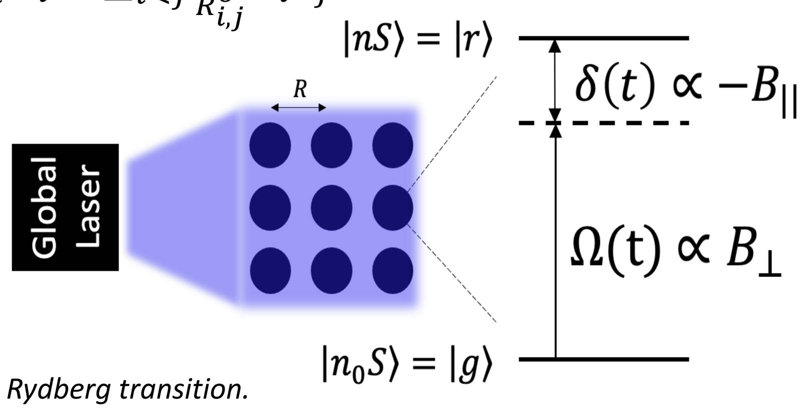
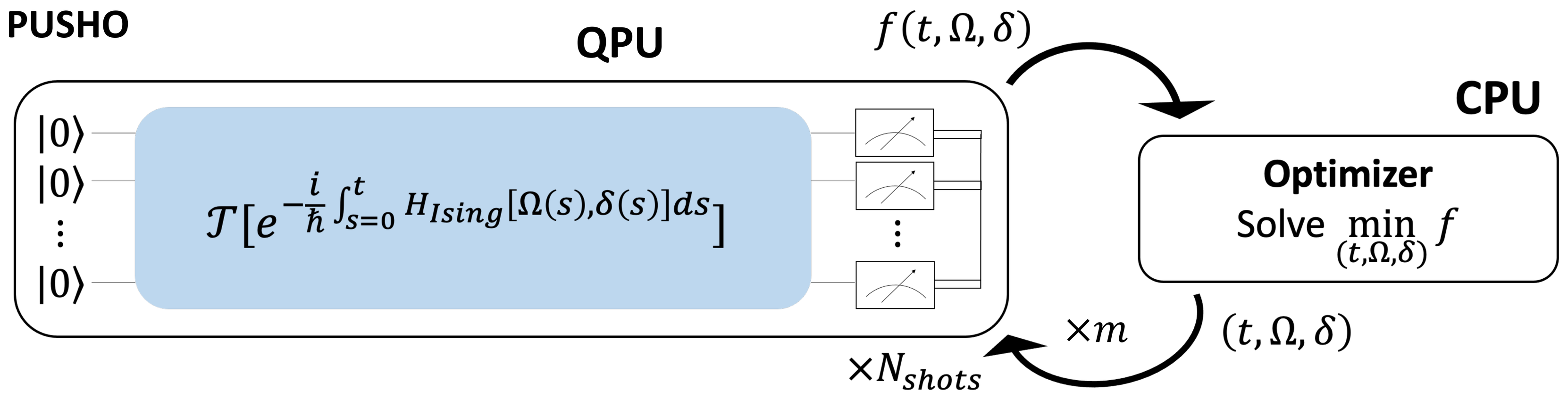


Fig. 2: Global adressing of an atom register, coupled to a Rydberg transition.

2 – Pulse Shaping Optimisation (PUSHO)

Variational Quantum Algorithms (VQA)

- Classical Processing Unit (CPU) + Quantum Processing Unit (QPU) in closed feedback loop.
- Famous QAOA [2] can approximately solve combinatorial optimisation problems which remain computationally hard to solve on CPUs alone.
- As digital approach with layers of gates, it is not well suited for current Rydberg platforms.



- *Problem related objective function (score)* f : average energy, overlap with target state, approx. ratio.
- *Classical optimizer :* Bayesian optimisation [3] works great with few iterations (small n) on low-dimensional parameter space.
- *Pulse shaping :* $\{t, \Omega(t_i), \delta(t_i)\}$ + interpolation by monotonic cubic splines.

Fig. 3 : Closed loop between CPU & QPU. The optimizer feeds the quantum device a pulse to test. Once the driving has been repeated N_{shots} times, the score evaluation is fed back to the optimizer which will decide the next point to probe.

- PUSHO = VQA closer than QAOA to Rydberg platforms, with smoother and shorter pulses.

3.1 – State Preparation

Initial States

- Atoms all initialised in $|g\rangle$ and only global addressing available.
- Some problems need to start from specific states, e.g. $|+\rangle^{\otimes N}$ or antiferromagnetic (AF) phases.

AF state on periodic chain

- AF : neighbouring atoms are in opposite states.
- Score : Neel structure factor, maximised for AF state.
- PUSHO provides adiabatic pulses, smoother than typical ramp. $S_{Néel}/S_{max} = 0.98$

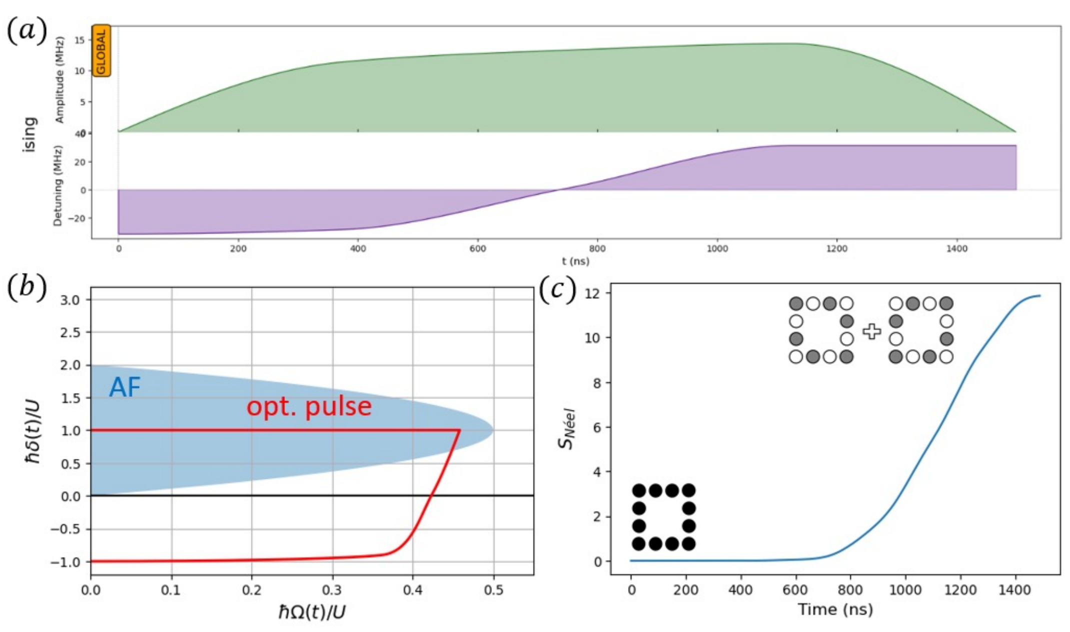


Fig. 4 : (a) Optimal pulse found after 30 iterations. (b) Optimal path in 1D phase diagram [4] (c) Increase of structure factor shows AF phase transition.

Uniform superposition

- No local gate available, global pulse required.
- PUSHO increases fidelity by 27% over a global Blackman pulse.

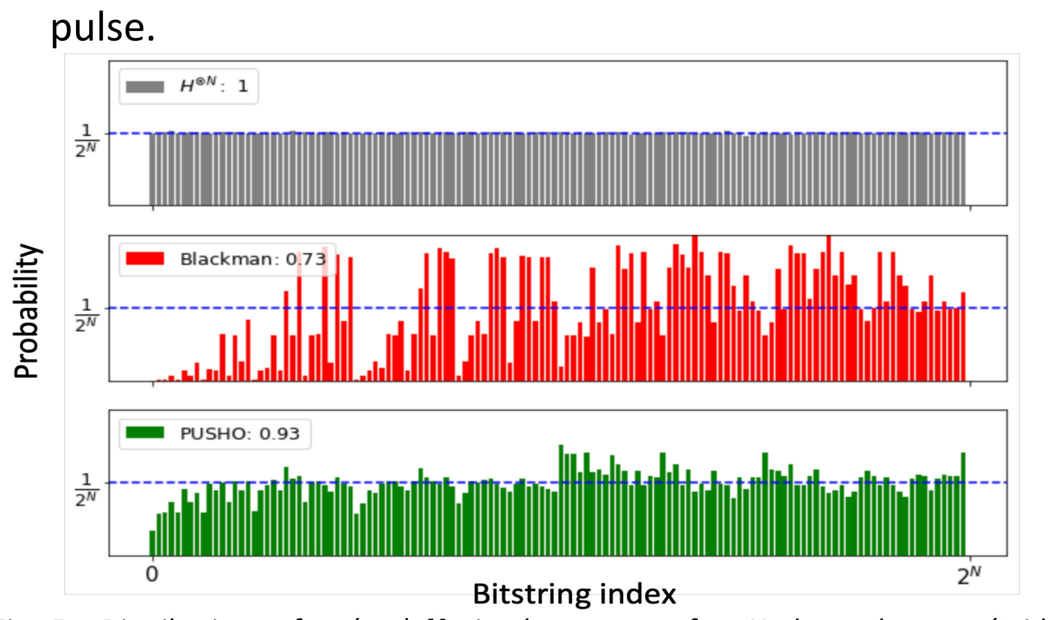


Fig. 5 : Distributions after (top) N simultaneous perfect Hadamard gates, (middle) global Blackman pulse (250 ns, 2 MHz) and (bottom) optimised pulse (160 ns, 2 MHz) for triangular lattice of 10 atoms separated by $8.5\mu m$.

3.2 – UD MIS

Maximum Independent Set

- Graph $G = (V, E)$ with V, vertices and E, edges.
- Independent Set (IS) : set of vertices in a graph, no two of which are sharing an edge.

- NP-hard problem to find an IS of maximum size (MIS).
- Even harder to find all MIS of a graph.

Formalism

- 1 atom = 1 node of a Unit Disk (UD) graph (edge if $d_{ij} < d$).
- Rydberg blockade implements the IS constraint.
- MISs correspond to product states with as many excitations as allowed [5].
- $\hat{C} = -\sum_{i\in V}\hat{n}_i + K\sum_{(i,j)\in E}\hat{n}_i\hat{n}_j$ with $K \gg 1$.
- Minimize approximation ratio

$$r = \frac{\sum_{n=1}^{N_{shots}}\langle n|\hat{C}|n\rangle}{N_{shots}}$$

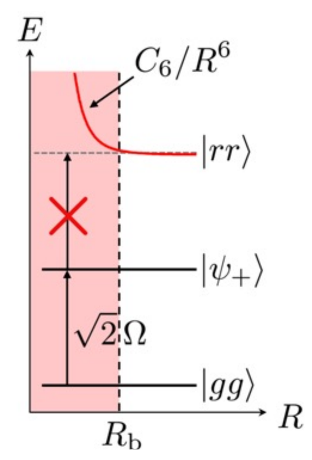


Fig. 6 : Diagram showing Rydberg blockade effect.

Results

- $\mathcal{G}$ dataset of 50 randomly generated UD graphs of 15 atoms
- Average ratio on $\mathcal{G}$ reaches 0.92 for large m.

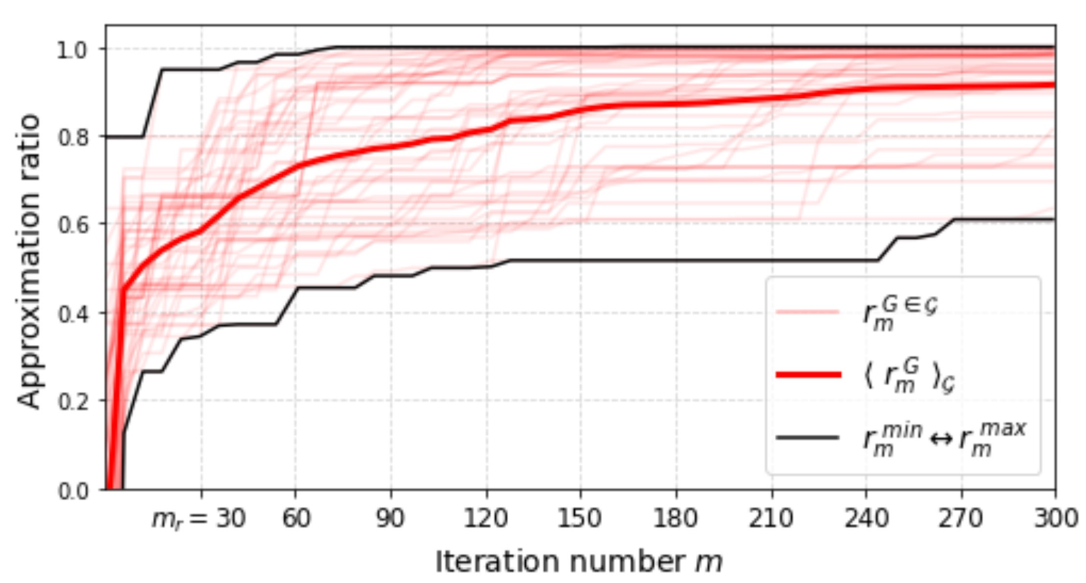


Fig. 7 : Convergence study of PUSHO on UD graph dataset $\mathcal{G}$. After $m_r = 30$ random calls to learn about the score landscape, Bayesian optimisation iteratively provides new pulses to test. Already after 150 calls, the mean ratio achieved is 0.87.

3.3 – QUBO

Quadratic Unconstrained Binary Optimisation

- $Q \in M_n(\mathbb{R})$, find $x^* \in \mathbb{B}^n$ minimising $f_Q(x) = x^T Q x$.
- Many problems can be embedded as QUBOs [6].
- Study symmetric case with $h_i = Q_{ii} \leq 0$ and $Q_{i\neq j} \geq 0$.
- Connection with Ising model but limited by connectivity.

Formalism

- $\hat{C} = -\sum_i h_i\hat{n}_i + \sum_{i\neq j} Q_{ij}\hat{n}_i\hat{n}_j$
- Optimally arrange atoms such that $C_6/R_{i>j} \sim Q_{i>j}$.
- Choose $\delta \sim h$ and $\Omega \sim \max_{i\neq j} Q_{ij}$.
- Start from uniform superposition of states (using PUSHO).
- Approx. ratio as defined in MIS part.

Results

- For 5 $Q \in M_{10}$, PUSHO outperforms QAOA with pulses of similar duration and amplitude.

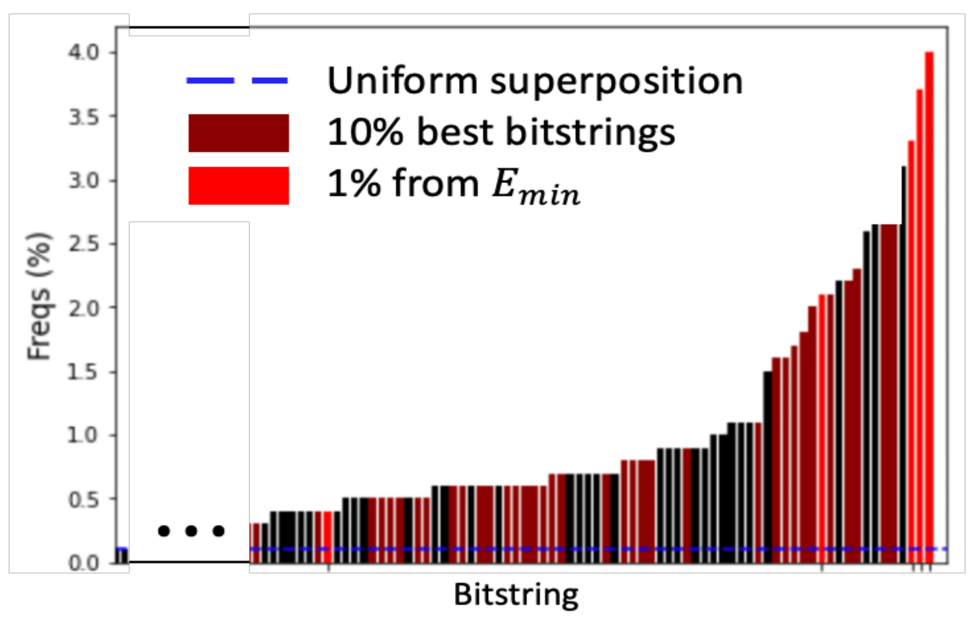


Fig. 8 : Instance of distribution obtained after an optimised pulse. Bitstrings sampled most frequently are of energy very close to the minimum one.

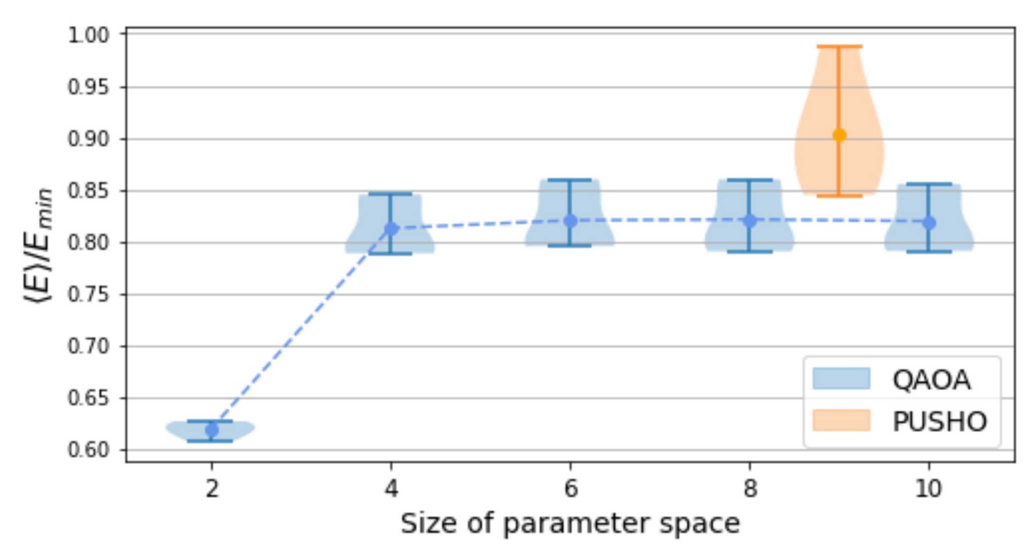


Fig. 9 : Comparison between QAOA and PUSHO. Distribution achieved with PUSHO outperforms the one QAOA converges to, even with higher-dimensional space.

4 – Conclusion & Outlooks

- PUSHO provides a straightforward but powerful alternative to QAOA, taking advantage of our ability to precisely control the shape of the driving fields.
- In all the above cases, the final score reached with PUSHO is always higher than the one found with QAOA, for similar optimization conditions.
- In the presence of noise Δ, PUSHO can also improve the average score $\langle S\rangle_\Delta$ by providing a more resilient pulse than square pulses (Master thesis)

References

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