Classical analogs of unitary quantum evolutions

Vincent Hardel

M2 supervisor: P.A. HERVIEUX

PhD supervisor: G. MANFREDI

Université de Strasbourg and IPCMS

Faculté de **physique et ingénierie**
Université de Strasbourg

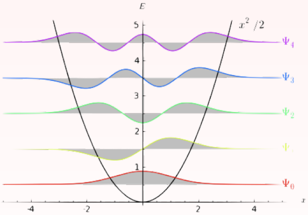
QUANTUM SCIENCE AND NANOMATERIALS

Motivations and Objectives

Simulate Quantum Mechanics using Classical experiments

Why : Probing **quantum** scales using **classical** physics

Quantum



Classical

$\longrightarrow$ ***Need to find adapted formulation of QM***

Outline :

1 - **Pilot wave formulation** (de Broglie - Bohm)

2 - **Stochastics formulation** (Nelson)

3 - **Numerical simulation**

4 - **Experimental classical analogs**

1 - ***Pilot wave formulation***
(de Broglie - Bohm)

de Broglie - Bohm formulation of QM

Particles : well defined **continuous trajectories**

Wavefunction : not enough

$\longrightarrow$ need to specify the **position**

Particles are *guided* by wavefunction ψ

$\longrightarrow$ **evolution drifted by** ψ

Form of the wavefunction

$$\psi = R\mathrm{e}^{\frac{iS}{\hbar}}$$

$$\mathrm{d}\boldsymbol{x} = \frac{\boldsymbol{\nabla} S}{m}\mathrm{d}t$$

de Broglie guiding equation

Wave function **drift evolution** of position : $\boldsymbol{\nabla} S$

Time interval $\mathrm{d}t \longrightarrow$ position evolution $\mathrm{d}\boldsymbol{x}$

2 - *Stochastic formulation* (Nelson)

Nelson : **universal Brownian motion** (*quantum fluctuations*)

$\longrightarrow$ *Equation of motion* :

$$\underbrace{\mathrm{d}x(t) = b(x(t),t)\mathrm{d}t}_{\text{deterministic}} + \underbrace{\mathrm{d}w(t)}_{\text{stochastic}}$$

w : **Wiener process**

Diffusion coefficient ν related to $w(t)$:

$$\nu = \frac{\hbar}{2m}$$

Recall : $\mathrm{d}x = b\mathrm{d}t + \mathrm{d}w$ $\psi = R \exp\left\{ i \frac{S}{\hbar} \right\}$

Include **wavefunction** in **position processes** :

$$b = \frac{\hbar}{m}\left\{ \Re\left(\frac{\nabla\psi}{\psi}\right) + \Im\left(\frac{\nabla\psi}{\psi}\right) \right\} = \frac{\nabla S}{m} + \frac{\hbar}{m}\frac{\nabla R}{R}$$

Drift velocity : $v = \frac{\nabla S}{m}$

Osmotic velocity : $u = \frac{\hbar}{m}\frac{\nabla R}{R}$

de Broglie - Bohm

Nelson

3 - *Numerical simulations*

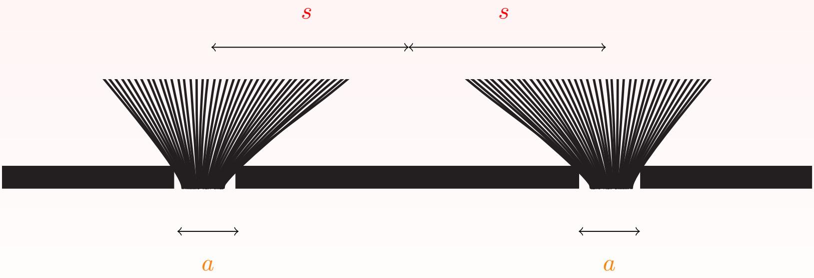
Double slit : Interferences

(Key experiment of Quantum Mechanics)

Free particles for double slit :

$$\psi(x, t=0) \sim \exp\left\{-\frac{(x-s)^2}{2a^2}\right\} + \exp\left\{-\frac{(x+s)^2}{2a^2}\right\}$$

Separation between slit   Width of the slit



Numerical verification : **Nelson** $\Longleftrightarrow$ **Schrodinger** $(\hbar = m = 1)$

$$\psi(x, t=0) \sim \exp\left\{-\frac{(x-s)^2}{2a^2}\right\} + \exp\left\{-\frac{(x+s)^2}{2a^2}\right\}$$

Procedure : compute $\psi(x,t)$ to obtain $b(x,t)$ and obtaining N trajectories

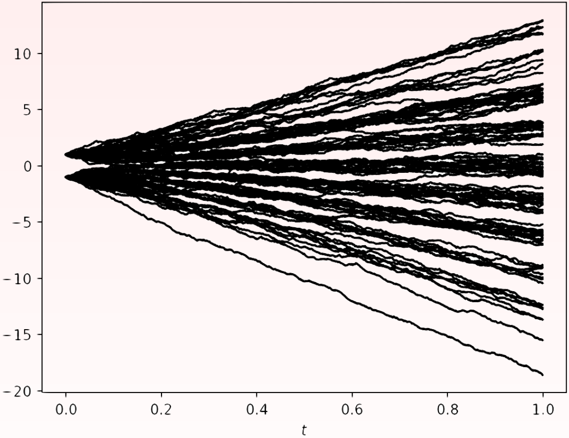
Time dependent Schrodinger equation :

$$i\frac{\partial\psi}{\partial t} = -\frac{1}{2}\frac{\partial^2\psi}{\partial x^2} \qquad \Longrightarrow \qquad b(x,t) = \Re\left(\frac{\nabla\psi}{\psi}\right) + \Im\left(\frac{\nabla\psi}{\psi}\right)$$

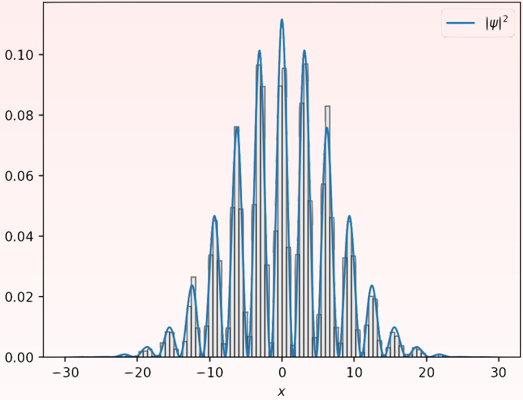
Nelson equation :

$$\mathrm{d}x = b\mathrm{d}t + \mathrm{d}w$$

Trajectories :



Parameters : $N = 10000,\ s = 1,\ a = 0.01,\ t = 1,\ \delta t = 0.001$



Histogram :

x at time $t_f = 1$

$\longrightarrow$ Quantum Mechanics

$$|\psi(x,t)|^2 \sim \mathrm{e}^{\frac{a(x-s)^2}{a^2+t^2}} + \mathrm{e}^{\frac{a(x+s)^2}{a^2+t^2}} + \mathrm{e}^{\frac{a(x^2+s^2)}{a^2+t^2}} \cos\left(\frac{2tsx}{a^2+t^2}\right)$$

4 - *Experimental classical analogs*

$\longrightarrow$ **Experimentally** : *classical analog*

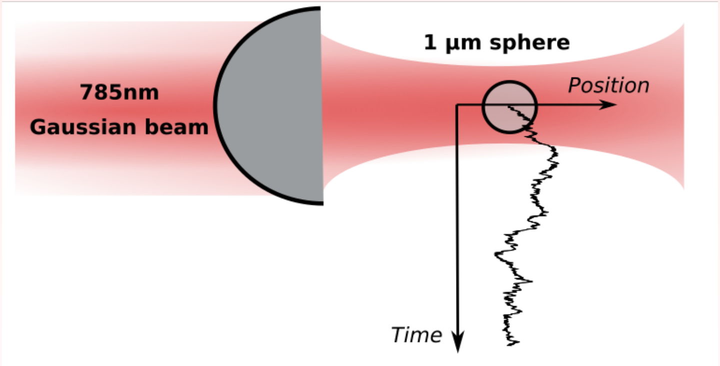
$$\mathrm{d}x(t) = b(x(t), t)\mathrm{d}t + \mathrm{d}w(t)$$

Overdamped brownian motion

b : analog to a *applied force*

w : analog to *thermal fluctuation*

Possible to simulate Nelson's dynamics with classical experiments



Optical gaussian beam : $E \sim \mathrm{e}^{-\kappa x^2} \longrightarrow \langle F \rangle \sim ||E^2|| \sim -\kappa x$

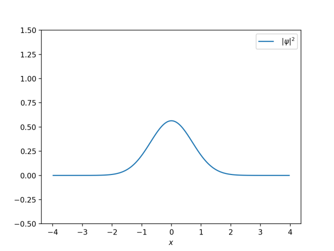
Possible experimentally : **linear force !**

Width κ changes in time : **out of equilirium** physics

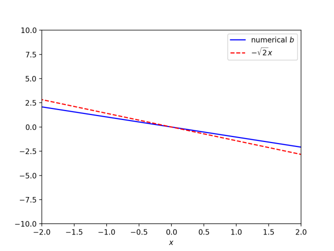
Nelson analog : **Quantum harmonic oscillator**

Wavefunction : gaussian but not stationary solution of Schrodinger

$\longrightarrow$ **width oscillates in time**



$|\psi(x,t)|^2$



$b(x,t)$

Conclusion

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Nelson adapted to **simulate** QM with classical experiments

⟶ *Quantum stochasticity analog to thermal fluctuation*

⟶ **Equivalent** to standard QM (*Numerical verification*)

Only closed systems

Go further (PhD) : Open quantum system

⟶ **Decoherence**, effect of **temperature**

⟶ More **complex** than harmonic potential (duffing, anhamonic, ...)

Nelson formalism more adapted ?

Thank you