Arrival Times in Quantum and Bohmian Mechanics

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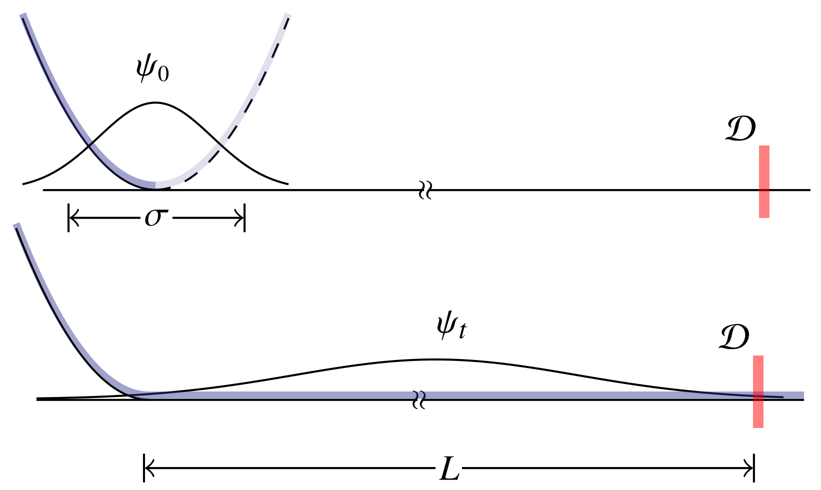
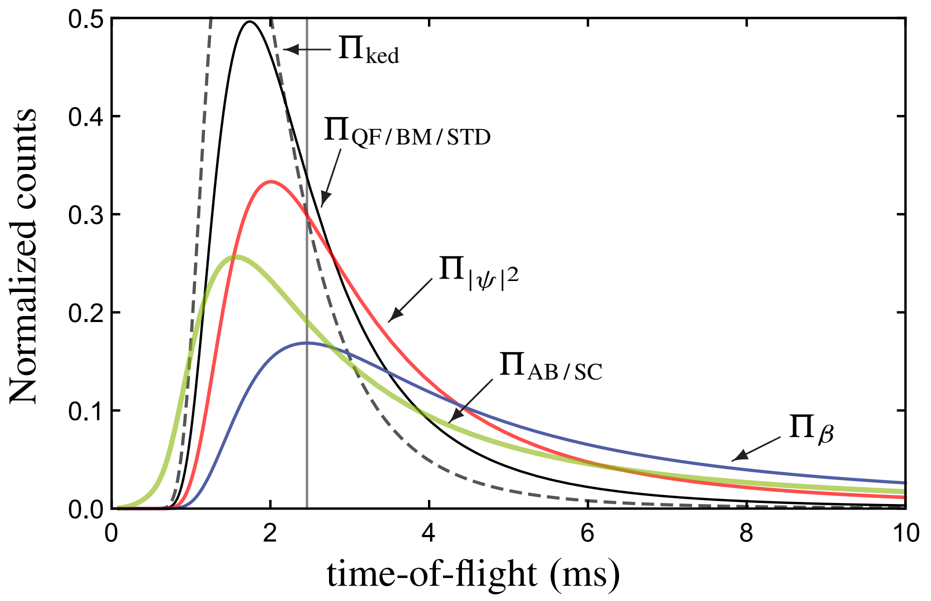
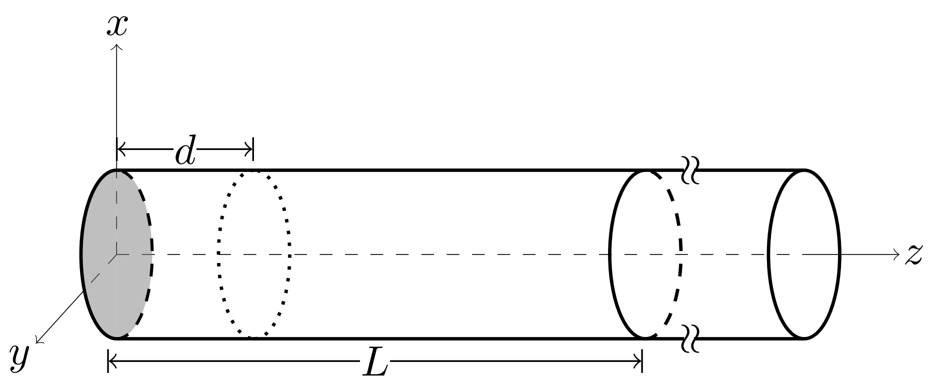
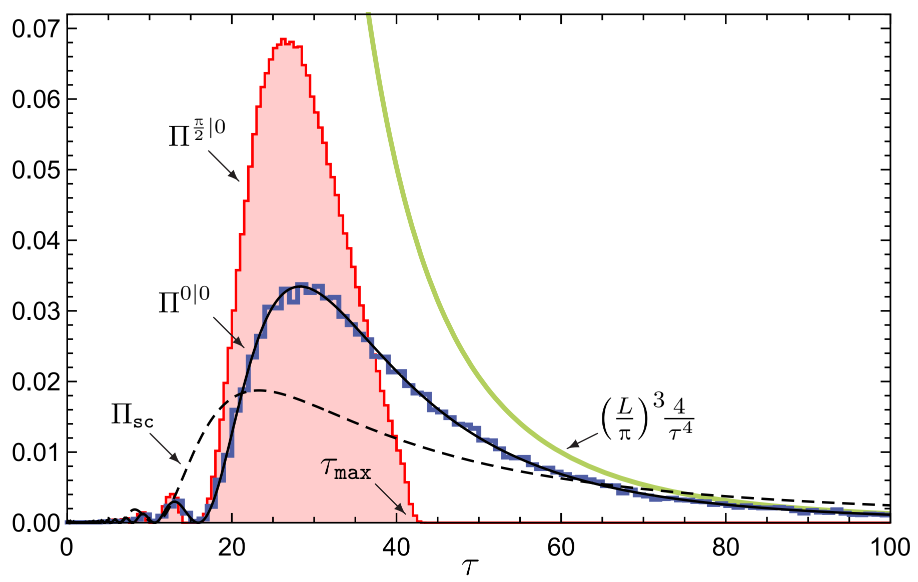
a common project with Siddhant Das and the late Detlef Dürr

Setup and predictions for experiment on quantum arrival times in a quadrupole ion trap. A single ion is trapped and cooled to the ground state of a potential (upper left panel) in the trap and subsequently released to propagate along the axis of the trap towards a detector surface. A rough picture of the three dimensional setup is given in the lower left panel. The upper right panel shows the arrival time distribution according to a range of predictions available int he literature for a scalar particle. Most of the involved proposals have not been extended to spinor valued wave functions or electromagnetic potentials. The lower right pannel shows the arrival time distribution as directly predicted by Bohmian mechanics (summarized below) for a spin-polarized wave function $\vec{\Psi}(t,x) = \psi(t,x)\vec{\chi}$. It shows a strong dependence on the spin-state of the ion.









Elements of Bohmian Mechanics

In addition to the Pauli equation

$$i\hbar\frac{\partial\Psi(\mathbf{x},t)}{\partial t} = \frac{1}{2m}\left[\boldsymbol{\sigma}\cdot(-i\hbar\boldsymbol{\nabla} - q\mathbf{A}_{\mathrm{Paul}})\right]^2\Psi(\mathbf{x},t) + q\varphi_{\mathrm{Paul}}\Psi(\mathbf{x},t),$$

Bohmian Mechanics posits trajectories (labelled by $\mathbf{X}(t)$) for the particle to follow, which are defined by

$$\mathbf{J}(\mathbf{x},t) = \frac{\hbar}{m}\mathrm{Im}\left(\Psi^\dagger\boldsymbol{\nabla}\Psi\right) + \frac{\hbar}{2m}\boldsymbol{\nabla}\times\left(\Psi^\dagger\boldsymbol{\sigma}\Psi\right), \qquad \text{and} \qquad \dot{\mathbf{X}}(t) = \frac{\mathbf{J}(\mathbf{X}(t),t)}{|\psi_t(\mathbf{X}(t))|^2}.$$

This naturally leads to the arrival time distribution

$$\Pi_{\mathrm{BM}}(\tau) = \int_{\mathbb{R}^3} d^3\mathbf{x}\,|\psi_0(\mathbf{x})|^2\,\delta(\tau-\tau(\mathbf{x})), \qquad \text{and} \qquad \tau(\mathbf{x}) = \inf\left\{t>0\,|\,\mathbf{X}(0)=\mathbf{x}, \hat{\mathbf{z}}\cdot\mathbf{X}(t)=L\right\}.$$

The strong affect the initial spin state has on the bohmian arrival time prediction makes us hopeful that it also shows up in real experiments.

A1 *Arrival-time Distributions of Spin-1/2 Particles*, S. Das, D. Dürr, *Sci. Rep.*, 9:2242, 2019

A2 *Exotic Bohmian arrival time of spin-1/2 particles: An analytical treatment*, S. Das, D. Dürr, M. Nöth, *Phys. Rev. A*, 99:052124, 2019

A3 *Times of arrival and gauge invariance*, Proc. R. Soc. Lond. A 477.2250: 20210101, 2021