Weak measurements: fundamental aspects

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Introduction to quantum weak measurements and weak values

A few relevant references to weak measurements (some of which inspired this course)

Review and introductive papers

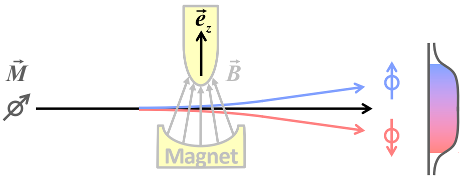
[1] *Introduction to weak measurements and weak values,*
B. Tamir and E. Cohen, Quanta **2** (2013) 7–17.
[2] *Understanding quantum weak values: Basics and applications,*
J. Dressel, M. Malik, F. M. Miatto, A. N. Jordan, and R. W. Boyd, Rev. Mod. Phys. **86** (2014) 307.
[3] *Quantum paradoxes,*
Y. Aharonov and D. Rohrlich, Wiley-VCH, 2005
[4] *Nonperturbative theory of weak pre- and post-selected measurements,*
A. G. Kofman, S. Ashhab, and F. Nori, Physics Reports 520 (2012) 43–133.
[5] *Pedagogical review of quantum measurement theory with an emphasis on weak measurements,*
B. E. Y. Svensson, Quanta **2** (2013) 18–49.
[6] *A time-symmetric formulation of quantum mechanics,*
J. Tollaksen, Y. Aharonov, and S. Popescu, Phys. Today **63** (2010) 27–32.
[7] *The Two-State Vector Formalism: An Updated Review,*
Y. Aharonov and L. Vaidman, Lect. Notes Phys. **734** (2008) 399–447.

Selected personal papers

[1] *Revealing geometric phases in modular and weak values with a quantum eraser,*
M. Cormann, M. Remy, B. Kolaric, and Y. Caudano, Phys. Rev. A **93** (2016) 042124.
[2] *Geometric description of modular and weak values in discrete quantum systems using the Majorana representation,*
M. Cormann and Y. Caudano, J. Phys. A: Math. Theor. **50** (2017) 305302.

Introduction to quantum weak measurements and weak values

Magnetic dipole deviation in a non uniform field



Energy

$E = -\vec{M} \cdot \vec{B}$

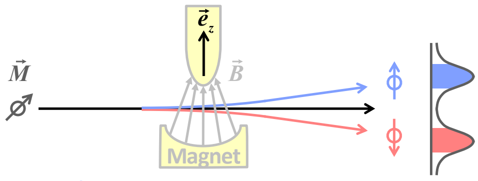
Force

$\vec{F} = -\vec{\nabla} E = \frac{d\vec{p}}{dt}$

$\approx M_z \partial B_z / \partial z \, \vec{e}_z$

Classical description

Magnetic dipole deviation in a non uniform field



Hamiltonian

$H_{int} = -\vec{M} \cdot \vec{B}$

$\vec{M} = \gamma \vec{S}$

Momentum

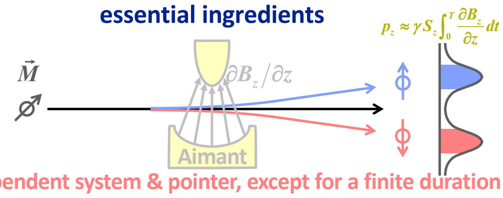
$\frac{d\vec{p}}{dt} = \frac{i}{\hbar} \left[H_{int}, \vec{p} \right]$

$p_z \approx \gamma S_z \int_0^T \partial B_z / \partial z \, dt$

Quantum description

Gyromagnetic ratio γ

This quantum measurement exhibits several essential ingredients



- Independent system & pointer, except for a finite duration
- Result linked to the observable value S_z
- Measured observable unaffected
- Short interaction (measurement) time
- Quantum process H_{int}

Ref.: *Quantum paradoxes*, Y. Aharonov and D. Rohrlich, Wiley-VCH, 2005

These properties are the basis of the von Neumann model of quantum measurement

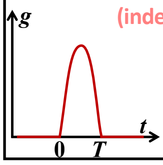
Full Hamiltonian (system S and pointer M)

$$H = H_s + H_m + H_{int}$$

Interaction Hamiltonian

$$H_{int} = g(t) A_s \otimes P_m$$

(independent observables of S and M)



Coupling strength

$$g_0 = \int_{-\infty}^{+\infty} g(t)dt = \int_0^T g(t)dt$$

The pointer evolves according to Heisenberg's equation

Canonically conjugates operators of the pointer M

$$[Q_m, P_m] = i\hbar$$

Heisenberg formalism

$$Q_m(T) - Q_m(0) = \int_0^T \frac{dQ_m}{dt} dt = \frac{i}{\hbar}\int_0^T [H, Q_m] dt$$

Additional assumptions

- Defined pointer initial state $Q_m(0)=0$
- Defined system initial state A_s

Heisenberg's equation gives the final pointer value of the measuring device

Full Hamiltonian

$$H = H_s + H_m + g(t) A_s \otimes P_m$$

Heisenberg formalism

$$Q_m(T) = \frac{i}{\hbar}\int_0^T [H, Q_m] dt$$

⬇

Final position of the pointer

$$Q_m(T) = \int_0^T \frac{i}{\hbar}[H_m, Q_m]dt + \int_0^T \frac{i}{\hbar}[g(t) A_s P_m, Q_m]dt$$

The pointer position gives the observable value

$$\lim_{T\to 0} Q_m(T) = g_0 A_s$$

⬆

$$\int_0^T \frac{i}{\hbar}[H_m, Q_m]dt + g_0 A_s$$

⬆

$$\int_0^T \frac{i}{\hbar}[H_m, Q_m]dt + \frac{i}{\hbar} A_s [P_m, Q_m] \int_0^T g(t)dt$$

⬆

Final position of the pointer

$$Q_m(T) = \int_0^T \frac{i}{\hbar}[H_m, Q_m]dt + \int_0^T \frac{i}{\hbar}[g(t) A_s P_m, Q_m]dt$$

von Neumann measurement scheme in Schrödinger's formalism

Initial and final states

$$|\psi_i\rangle = |\psi_s\rangle \otimes |\psi_m\rangle \qquad |\psi_f\rangle = U|\psi_s\rangle \otimes |\psi_m\rangle$$

Temporal evolution

$$U = e^{-\frac{i}{\hbar}\int_0^T H_{int} dt} = e^{-\frac{i}{\hbar} g_0 A_s \otimes P_m}$$

State decomposition in the A_s observable eigenstates basis

$$A_s|a_j\rangle = a_j|a_j\rangle \qquad |\psi_s\rangle = \sum_j \alpha_j |a_j\rangle$$

We consider a position measurement of the pointer state

Initial pointer state

$$|\psi_m(q_m)\rangle = \int \psi_m(q_m)|q_m\rangle dq_m$$

Gaussian state

$$\psi_m(q_m) = \frac{1}{\sqrt[4]{2\pi\sigma^2}} e^{-\frac{q_m^2}{4\sigma^2}}$$

$$|\psi_f(q_m)\rangle = e^{-\frac{i}{\hbar} g_0 A_s \otimes P_m} \sum_j \alpha_j |a_j\rangle\, |\psi_m(q_m)\rangle$$

The system and the pointer states become entangled

Action of the translation operator

$$|\psi_f(q_m)\rangle = \sum_j \alpha_j |a_j\rangle |\psi_m(q_m - g_0 a_j)\rangle$$

$$|\psi_f(q_m)\rangle = \sum_j \alpha_j |a_j\rangle e^{-\frac{i}{\hbar} g_0 a_j P_m} |\psi_m(q_m)\rangle$$

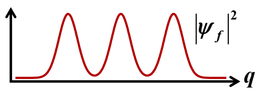
$$|\psi_f(q_m)\rangle = e^{-\frac{i}{\hbar} g_0 A_s \otimes P_m} \sum_j \alpha_j |a_j\rangle\, |\psi_m(q_m)\rangle$$

The overlap of the pointer wavepackets define two situations

Final state in position representation

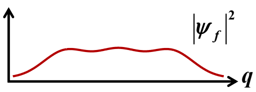
$$\langle q_m|\psi_f(q_m)\rangle = \sum_j \alpha_j\, \psi_m(q_m - g_0 a_j)\, |a_j\rangle$$

Strong coupling



$$\langle A\rangle = \sum_j a_j P(a_j)$$

Weaker coupling



$$\langle q\rangle_f - \langle q\rangle_0 \approx g_0 \langle A\rangle$$

The different wavepackets of the final global state do not interfere

Final state in position representation

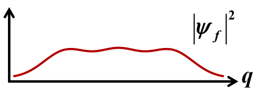
$$\langle q_m|\psi_f(q_m)\rangle = \sum_j \alpha_j\, \psi_m(q_m - g_0 a_j)\, |a_j\rangle$$

$$|\psi_f|^2 = \sum_j |\alpha_j \psi_m(q_m - g_0 a_j)|^2$$

Gaussian state

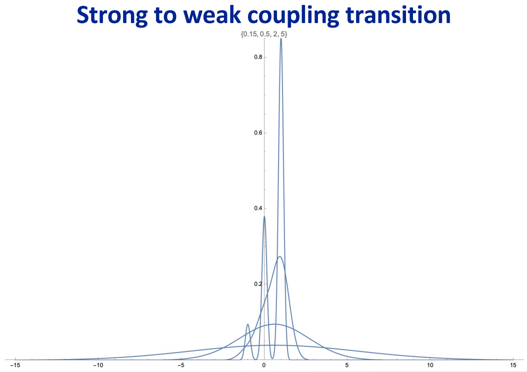
$$P(q_m) \approx \frac{1}{\sqrt{2\pi\sigma}} e^{-\frac{\left(q_m - \sum_j |\alpha_j|^2 a_j\right)^2}{2\sigma^2}}$$

Weaker coupling



$$\langle q\rangle_f - \langle q\rangle_0 \approx g_0 \langle A\rangle$$

Strong to weak coupling transition



Introduction to quantum weak measurements and weak values

1. von Neumann measurement scheme
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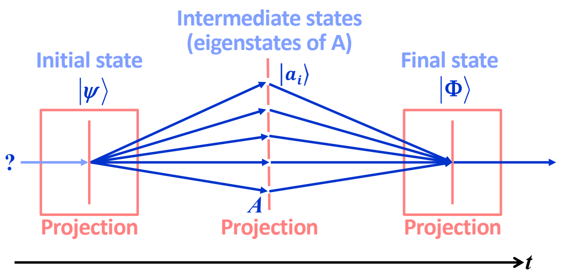
Let's consider sequential projections



We can compute the probabilities of sequences

$$|\psi\rangle \longrightarrow |a_i\rangle \longrightarrow |\Phi\rangle$$

$$P\left(|a_i\rangle \,\middle|\, |\psi\rangle\right) = |\langle a_i|\psi\rangle|^2 = \langle\psi|\hat{\pi}_i|\psi\rangle$$

$$P\left(|a_i\rangle, |\phi\rangle \,\middle|\, |\psi\rangle\right) = |\langle\phi|a_i\rangle|^2 |\langle a_i|\psi\rangle|^2 = |\langle\phi|\hat{\pi}_i|\psi\rangle|^2$$

$$P(|\phi\rangle \mid |\psi\rangle) = \sum_i P\left(|a_i\rangle, |\phi\rangle \,\middle|\, |\psi\rangle\right) = \sum_i |\langle\phi|\hat{\pi}_i|\psi\rangle|^2$$

Considering conditional probabilities for the intermediate results using Bayes' rule

$$|\psi\rangle \longrightarrow |a_i\rangle \longrightarrow |\Phi\rangle$$

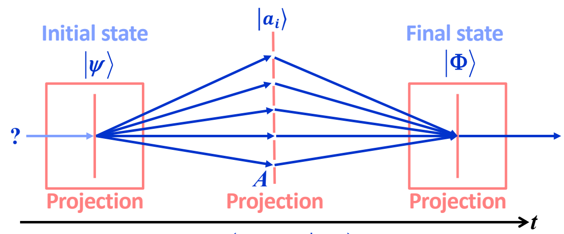
$$P\left(|a_i\rangle, |\phi\rangle \,\middle|\, |\psi\rangle\right) = |\langle\phi|a_i\rangle|^2 |\langle a_i|\psi\rangle|^2 = |\langle\phi|\hat{\pi}_i|\psi\rangle|^2$$

$$P(|\phi\rangle \mid |\psi\rangle) = \sum_i P\left(|a_i\rangle, |\phi\rangle \,\middle|\, |\psi\rangle\right) = \sum_i |\langle\phi|\hat{\pi}_i|\psi\rangle|^2$$

$$P\left(|a_i\rangle \,\middle|\, |\psi\rangle, |\phi\rangle\right) = \frac{P\left(|a_i\rangle, |\phi\rangle \,\middle|\, |\psi\rangle\right)}{P(|\phi\rangle \mid |\psi\rangle)} = \frac{|\langle\phi|\hat{\pi}_i|\psi\rangle|^2}{\sum_i |\langle\phi|\hat{\pi}_i|\psi\rangle|^2}$$

$P(a,b) = P(a|b)P(b)$

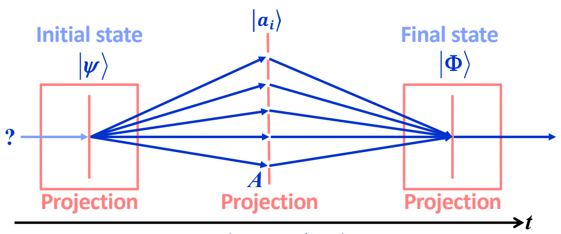
Considering conditional probabilities for the intermediate results using Bayes' rule



$$P\left(|a_i\rangle \,\middle|\, |\psi\rangle, |\phi\rangle\right) = \frac{P\left(|a_i\rangle, |\phi\rangle \,\middle|\, |\psi\rangle\right)}{P(|\phi\rangle \mid |\psi\rangle)} = \frac{|\langle\phi|\hat{\pi}_i|\psi\rangle|^2}{\sum_i |\langle\phi|\hat{\pi}_i|\psi\rangle|^2}$$

$P(a,b) = P(a|b)P(b)$

Considering conditional probabilities for the intermediate results using Bayes' rule



$$P\left(|a_i\rangle \,\middle|\, |\psi\rangle, |\phi\rangle\right) = \frac{P\left(|a_i\rangle, |\phi\rangle \,\middle|\, |\psi\rangle\right)}{P(|\phi\rangle \mid |\psi\rangle)} = \frac{|\langle\phi|\hat{\pi}_i|\psi\rangle|^2}{\sum_i |\langle\phi|\hat{\pi}_i|\psi\rangle|^2}$$

$P(a,b) = P(a|b)P(b)$

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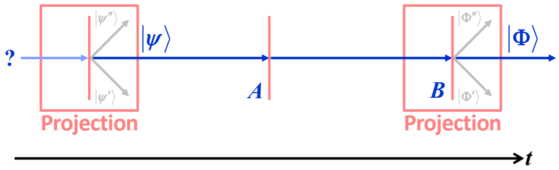
In quantum mechanics, the initial state of a sytem does not determine its final state

Initial conditions — Intermediate measurement — Final conditions



Pre-selection and post-selection of a system (PPS)

Post-selection & strong projective measurements involve contextual conditional probabilities

CI — CF

$$|\psi\rangle \longrightarrow A \longrightarrow |\Phi\rangle$$

Joint and conditional probabilities for A and B results

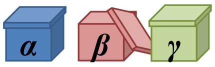
$$P_{i\Phi} = P_i P_\Phi \qquad P_{i|\Phi} = \frac{P_{i\Phi}}{\sum_j P_{j\Phi}} \qquad P_\Phi = |\langle \Phi | a_i \rangle|^2 \qquad P_i = |\langle a_i | \psi \rangle|^2$$

(Bayes – ABL rules)

The three box paradox illustrates the contextuality in a PPS projective measurement

Pre-selection

CI : $|\psi\rangle = \frac{1}{\sqrt{3}}\left[|\alpha\rangle + |\beta\rangle + |\gamma\rangle\right]$



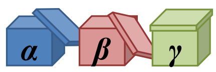
CF : $|\Phi\rangle = \frac{1}{\sqrt{3}}\left[|\alpha\rangle + |\beta\rangle - |\gamma\rangle\right]$

Post-selection

Measured projectors

$$A = \pi_\alpha = |\alpha\rangle\langle\alpha|$$
$$A = \pi_\beta = |\beta\rangle\langle\beta|$$
$$A = \pi_\gamma = |\gamma\rangle\langle\gamma|$$

The three box paradox illustrates the contextuality in a PPS projective measurement



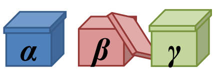
Opening two boxes

$$P_{\alpha|\Phi} = \frac{|\langle \Phi | \pi_\alpha | \psi \rangle|^2}{|\langle \Phi | \pi_\alpha | \psi \rangle|^2 + |\langle \Phi | \pi_\beta | \psi \rangle|^2 + |\langle \Phi | \pi_\gamma | \psi \rangle|^2} = 1/3$$

$$P_{\beta|\Phi} = 1/3$$

$$P_{\gamma|\Phi} = 1/3$$

The three box paradox illustrates the contextuality in a PPS projective measurement



Opening a single box

$$P_{\alpha|\Phi} = \frac{|\langle \Phi | \pi_\alpha | \psi \rangle|^2}{|\langle \Phi | \pi_\alpha | \psi \rangle|^2 + |\langle \Phi | \pi_\beta + \pi_\gamma | \psi \rangle|^2} = 1$$

$$P_{\beta|\Phi} = 1$$

$$P_{\gamma|\Phi} = 1/5$$

Counterfactuals!

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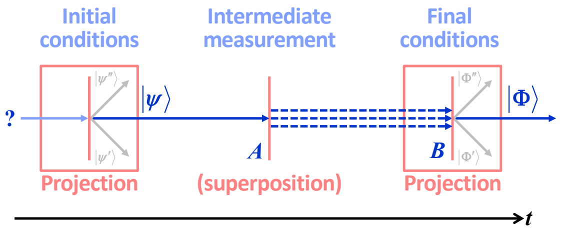
Let us consider the PPS evolution when the measurement of A preserve the coherence



Pre-selection and post-selection of a system (PPS)

We consider the momentum representation of the pointer state to describe the measurement

Initial pointer state

$$|\psi_m(p_m)\rangle = \int \psi_m(p_m)|p_m\rangle dp_m$$

Gaussian state

$$\psi_m(p_m) = \frac{\sqrt[4]{2}}{\sqrt[4]{\pi}} e^{-p_m^2\sigma^2}$$

⬇

$$|\psi_f(p_m)\rangle = e^{-\frac{i}{\hbar}g_0 A_s \otimes P_m}|\psi_s\rangle \otimes |\psi_m(p_m)\rangle$$

We project on a momentum state (pointer) and on the post-selected state (system)

Post-selection

$$\langle\phi_s|\langle p_m|\psi_f(p_m)\rangle = \psi_m(p_m)\langle\phi_s|e^{-\frac{i}{\hbar}g_0 p_m A_s}|\psi_s\rangle$$

⬆

Momentum representation

$$\langle p_m|\psi_f(p_m)\rangle = e^{-\frac{i}{\hbar}g_0 p_m A_s}\psi_m(p_m)|\psi_s\rangle$$

⬆

$$|\psi_f(p_m)\rangle = e^{-\frac{i}{\hbar}g_0 A_s \otimes P_m}|\psi_s\rangle \otimes |\psi_m(p_m)\rangle$$

A weak (PPS) measurement aims at minimizing the state perturbation

Post-selection

$$\langle\phi_s|\langle p_m|\psi_f(p_m)\rangle = \psi_m(p_m)\langle\phi_s|e^{-\frac{i}{\hbar}g_0 p_m A_s}|\psi_s\rangle$$

✚

Weak coupling

$$g_0/\hbar \ll 1$$

⬇

Linearization

$$\langle\phi_s|\langle p_m|\psi_f(p_m)\rangle \approx \psi_m(p_m)\langle\phi_s|1 - \frac{i}{\hbar}g_0 p_m A_s|\psi_s\rangle$$

We perform a first order approximation on the exponential

Linearization

$$\langle\phi_s|\langle p_m|\psi_f(p_m)\rangle \approx \langle\phi_s|1 - \frac{i}{\hbar}g_0 p_m A_s|\psi_s\rangle\, \psi_m(p_m)$$

⬇

$$= \left(\langle\phi_s|\psi_s\rangle - \frac{i}{\hbar}g_0\langle\phi_s|A_s|\psi_s\rangle p_m\right)\psi_m(p_m)$$

⬇

$$= \langle\phi_s|\psi_s\rangle\left(1 - \frac{i}{\hbar}g_0\frac{\langle\phi_s|A_s|\psi_s\rangle}{\langle\phi_s|\psi_s\rangle}p_m\right)\psi_m(p_m)$$

The weak value of the observable A for the pre- and post-selected states appears in the expression

Complex number, not bounded

$$\langle A_s\rangle_w = \frac{\langle\Phi_S|A_s|\psi_S\rangle}{\langle\Phi_S|\psi_S\rangle}$$

Weak value (for the PPS)

⬆

$$= \langle\phi_s|\psi_s\rangle\left(1 - \frac{i}{\hbar}g_0\frac{\langle\phi_s|A_s|\psi_s\rangle}{\langle\phi_s|\psi_s\rangle}p_m\right)\psi_m(p_m)$$

4. Weak measurement 37

The weakness allows to perform a second approximation

Non normalized pointer state after post-selection

$$\langle \phi_s | \langle p_m | \psi_f(p_m) \rangle \approx \langle \phi_s | \psi_s \rangle e^{-\frac{i}{\hbar} g_0 \langle A_s \rangle_w p_m} \psi_m(p_m)$$

$$= \langle \phi_s | \psi_s \rangle \left(1 - \frac{i}{\hbar} g_0 \langle A_s \rangle_w p_m\right) \psi_m(p_m)$$

$$= \langle \phi_s | \psi_s \rangle \left(1 - \frac{i}{\hbar} g_0 \frac{\langle \phi_s | A_s | \psi_s \rangle}{\langle \phi_s | \psi_s \rangle} p_m\right) \psi_m(p_m)$$

Weak measurements – 2021

4. Weak measurement 38

The pointer state is shifted by amounts defined by the weak value

Non normalized pointer state after post-selection

$$\langle \phi_s | \langle p_m | \psi_f(p_m) \rangle \approx \langle \phi_s | \psi_s \rangle e^{-\frac{i}{\hbar} g_0 \langle A_s \rangle_w p_m} \psi_m(p_m)$$

Average shifts of the pointer variables

$$\langle q \rangle_{PPS} - \langle q \rangle_0 = g_0 \, \mathrm{Re} \langle A \rangle_w$$

$$\langle q \rangle_{PPS} - \langle q_0 \rangle_f = g_0 \, \mathrm{Re} \langle A \rangle_w$$

Reminder: weak measurement without post-selection

$$\langle q \rangle_f - \langle q \rangle_0 = g_0 \langle A \rangle$$

Weak measurements – 2021

4. Weak measurement 39

A very large shift of the pointer is obtained at the expense of the post-selection probability

Non normalized pointer state after post-selection

$$\langle \phi_s | \langle p_m | \psi_f(p_m) \rangle \approx \langle \phi_s | \psi_s \rangle e^{-\frac{i}{\hbar} g_0 \langle A_s \rangle_w p_m} \psi_m(p_m)$$

Average shifts of the pointer variables

$$\langle q \rangle_{PPS} - \langle q \rangle_0 = g_0 \, \mathrm{Re} \langle A \rangle_w$$

$$\langle q \rangle_{PPS} - \langle q_0 \rangle_f = g_0 \, \mathrm{Re} \langle A \rangle_w$$

Large shifts if $\langle \Phi_S | \psi_S \rangle \to 0$

$$\langle A \rangle_w = \frac{\langle \Phi_S | A_s | \psi_S \rangle}{\langle \Phi_S | \psi_S \rangle} \to \infty \qquad P(\Phi | \psi) \approx |\langle \Phi_S | \psi_S \rangle|^2 \to 0$$

Weak measurements – 2021

4. Weak measurement 40

A large deflection is obtained at the expense of the post-selected intensity

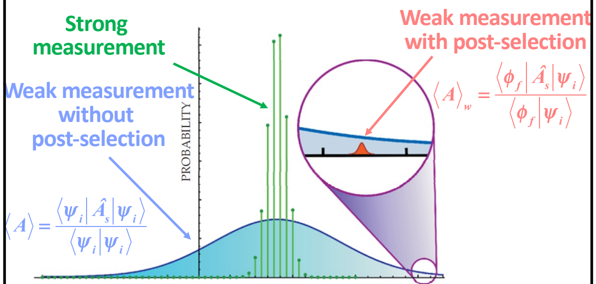


Fig. credits: J. Tollaksen, Y. Aharonov, S. Popescu. *Phys. Today*, 63 :061801, Nov 2010

Weak measurements – 2021

Talk outline 41

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Weak measurements – 2021

5. A few characteristics of weak measurements 42

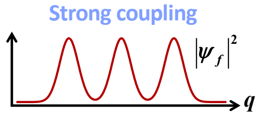
A large deflection of the pointer state is due to interferences

Pointer state after a von Neumann measurement

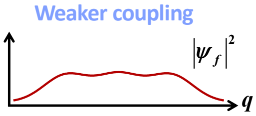
$$|\psi_f(q_m)\rangle = \sum_j \alpha_j \psi_m(q_m - g_0 a_j) |a_j\rangle$$

(after interaction but before projection, no post-selection)

Strong coupling



Weaker coupling



Weak measurements – 2021

A large deflection of the pointer state is due to interferences

Pointer state after a von Neumann measurement

$$\left|\psi_f(q_m)\right\rangle = \sum_j \alpha_j \psi_m\left(q_m - g_0 a_j\right)\left|a_j\right\rangle$$

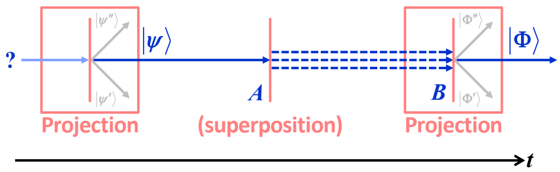
Probability distribution, no post-selection: no interference

$$\left\langle \psi_f \middle| \psi_f \right\rangle = \sum_j \left|\alpha_j\right|^2 \left|\psi_m\left(a_j\right)\right|^2$$

Probability distribution with post-selection: interferences

$$\left|\left\langle \Phi \middle| \psi_f(q_m)\right\rangle\right|^2 = \left|\sum_j \alpha_j \varphi_j \psi_m\left(a_j\right)\right|^2 \qquad \varphi_j = \left\langle \Phi \middle| a_j\right\rangle$$

A large deflection of the pointer state is due to interferences



Pre-selection and post-selection of a system (PPS)

Any observable average value can be expressed in terms of weak values

Average value

$$\langle A\rangle = \left\langle \psi_S \middle| A_s \middle| \psi_S \right\rangle$$

Eigenstate basis of post-selection

$$\langle A\rangle = \sum_i \left\langle \psi_S \middle| \Phi_{Si}\right\rangle \left\langle \Phi_{Si} \middle| A_s \middle| \psi_S \right\rangle$$

Weighted average of weak values

$$\langle A\rangle = \sum_i \left|\left\langle \psi_S \middle| \Phi_{Si}\right\rangle\right|^2 \frac{\left\langle \Phi_{Si} \middle| A_s \middle| \psi_S \right\rangle}{\left\langle \Phi_{Si} \middle| \psi_S \right\rangle} = \sum_i P_{\Phi_i|\psi} \langle A\rangle_{w,\Phi_i\psi}$$

Any observable weak value can be expressed in terms of weak values of projectors weak values

Weak value

$$\langle A\rangle_w = \frac{\left\langle \Phi_S \middle| A_s \middle| \psi_S \right\rangle}{\left\langle \Phi_S \middle| \psi_S \right\rangle} = \sum_i a_i \frac{\left\langle \Phi_S \middle| \pi_i \middle| \psi_S \right\rangle}{\left\langle \Phi_S \middle| \psi_S \right\rangle} = \sum_i a_i \langle \pi_i \rangle_w$$

(spectral decomposition of A)

Completeness relation of projectors

$$\sum_i \langle \pi_i \rangle_w = 1$$

Nonclassical, quasi-probability distribution

A projector weak value is similar to a nonclassical conditional quasi-probability distribution

Projector weak value

$$\langle \pi_i \rangle_w = \frac{\left\langle \Phi_S \middle| \pi_i \middle| \psi_S \right\rangle}{\left\langle \Phi_S \middle| \psi_S \right\rangle}$$

« Bayes theorem »

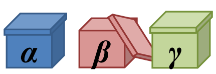
$$\langle \pi_i \rangle_w = \frac{\langle \Phi|\pi_i|\psi\rangle\langle\psi|\Phi\rangle}{\left|\langle\Phi|\psi\rangle\right|^2} = \frac{\langle\psi|\pi_\Phi \pi_i|\psi\rangle}{\left|\langle\Phi|\psi\rangle\right|^2} = \frac{\tilde{P}_{i\Phi}}{P_\Phi} = \tilde{P}_{i|\Phi}$$

Nonclassical, quasi-probability distribution

The three box paradox illustrates several key properties of weak measurements

Pre-selection

CI : $|\psi\rangle = \frac{1}{\sqrt{3}}\left[|\alpha\rangle + |\beta\rangle + |\gamma\rangle\right]$



CF : $|\Phi\rangle = \frac{1}{\sqrt{3}}\left[|\alpha\rangle + |\beta\rangle - |\gamma\rangle\right]$

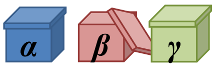
Post-selection

Measured projectors

$$A = \pi_\alpha = |\alpha\rangle\langle\alpha|$$
$$A = \pi_\beta = |\beta\rangle\langle\beta|$$
$$A = \pi_\gamma = |\gamma\rangle\langle\gamma|$$

Slide 49

The projector weak values in the 3 box paradox do not depend on how many boxes are opened



Box weak measurement

$$\langle \pi_\alpha \rangle_w = \frac{\langle \Phi | \pi_\alpha | \psi \rangle}{\langle \Phi | \psi \rangle} = 1$$

$$\langle \pi_\beta \rangle_w = \frac{\langle \Phi | \pi_\beta | \psi \rangle}{\langle \Phi | \psi \rangle} = 1$$

$$\langle \pi_\gamma \rangle_w = \frac{\langle \Phi | \pi_\gamma | \psi \rangle}{\langle \Phi | \psi \rangle} = -1$$

Sum rule

$$\langle \pi_\alpha \rangle_w + \langle \pi_\gamma \rangle_w + \langle \pi_\gamma \rangle_w = 1$$

Consistent weak values

Slide 50

Weak value do not obey the product rule

$$|\psi_i\rangle = |\Psi_-\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow_{1z}\downarrow_{2z}\rangle - |\downarrow_{1z}\uparrow_{2z}\rangle\right)$$

$$|\phi_f\rangle = |\uparrow_{1x}\uparrow_{2y}\rangle$$

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$

$$\langle \sigma_{1y} \rangle_w = \frac{\langle \phi_f | \sigma_{1y} | \psi_i \rangle}{\langle \phi_f | \psi_i \rangle} = -1$$

$$\langle \sigma_{2x} \rangle_w = \frac{\langle \phi_f | \sigma_{2x} | \psi_i \rangle}{\langle \phi_f | \psi_i \rangle} = -1$$

$$\langle \sigma_{1y} \otimes \sigma_{2x} \rangle_w = \frac{\langle \phi_f | \sigma_{1y} \otimes \sigma_{2x} | \psi_i \rangle}{\langle \phi_f | \psi_i \rangle} = -1 \neq \langle \sigma_{1y} \rangle_w \langle \sigma_{2x} \rangle_w$$

Slide 51

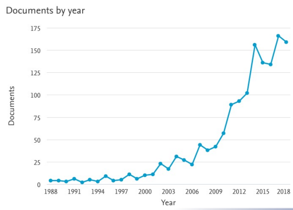
Introduction to quantum weak measurements and weak values

1. von Neumann measurement scheme
2. Aharonov – Bergman – Leibowitz rule (ABL)
3. Measurements with post-selection
4. Weak measurements (with post-selection)
5. A few characteristics of weak measurements
6. Making sense of anomalous weak values

Slide 52

Yakir Aharonov's most cited papers

Document title	Authors	Year	Source	Cited by
Significance of electromagnetic potentials in the quantum theory Open Access	Aharonov, Y., Bohm, D.	1959	Physical Review 115(3), pp. 485-491	4249
Phase change during a cyclic quantum evolution	Aharonov, Y., Anandan, J.	1987	Physical Review Letters 58(16), pp. 1593-1596	1506
How the result of a measurement of a component of the spin of a spin-1/2 particle can turn out to be 100	Aharonov, Y., Albert, D.Z., Vaidman, L.	1988	Physical Review Letters 60(14), pp. 1351-1354	1173

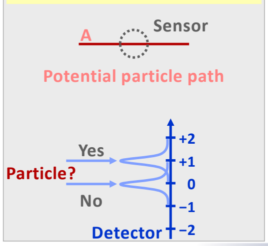


Weak measurement and weak value papers

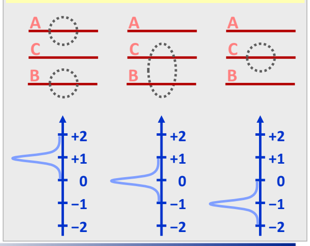
Slide 53

Anomalous weak values provoke sensor responses that never occur with strong measurements

Sensor calibration in strong measurements



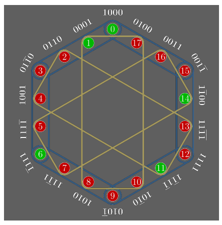
Sensor observations in weak measurements

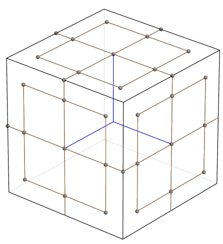


Slide 54

Anomalous weak values are proof of contextuality

$$\langle A_S \rangle_w = \sum_i a_i \langle \pi_i \rangle_w$$





Ref.: Matthew Pusey, Phys. Rev. Lett. **113** (2014) 200401; Phys. Rev. A **100** (2019) 0402116

Left Fig. credit: Z.-P. Xu et al., Phys. Rev. Lett **124** (2020) 230401

The imaginary part of the weak value gives the 1st order correction to the post-selection probability

State evolution & post-selection probability

$$P_\epsilon = |\langle f|e^{-i\epsilon H}|i\rangle|^2$$

$$P_\epsilon \approx |\langle f|1 - i\epsilon H|i\rangle|^2 = |\langle f|i\rangle|^2\ |1 - i\epsilon\langle H\rangle_w|^2$$

$$P_\epsilon \approx P_0(1 + 2\epsilon\ \mathrm{Im}\langle H\rangle_w)$$

Reference: J. Dressel et al., Rev. Mod. Phys. **86** (2014) 307

The argument of weak values is related to a geometric phase

Qubit projector

$$\langle\pi_r\rangle_w = \frac{\langle\phi_f|\pi_r|\psi_i\rangle}{\langle\phi_f|\psi_i\rangle}$$

Polar representation of weak value

$$\mathrm{Arg}\langle\pi_r\rangle_w = \mathrm{Arg}\frac{\langle\psi_i|\phi_f\rangle\langle\phi_s|\pi_r|\psi_s\rangle}{|\langle\phi_f|\psi_i\rangle|^2}$$

$$= \mathrm{Arg}\langle\psi_i|\phi_f\rangle\langle\phi_f|r\rangle\langle r|\psi_i\rangle$$

Gm phase (Berry, Pancharatnam – Bargmann invariant)

See e.g.: M. Cormann et al., Phys. Rev. A 93 (2016) 042124
M. Cormann and Y. Caudano, J. Phys. A: Math. Theor. 50 (2017) 305302

The polar representation of qubit weak values can be described using the Bloch sphere

Qubit projector

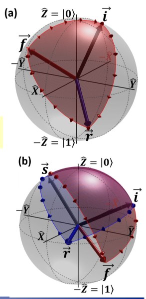
$$|\Pi_{r,w}| = \sqrt{\frac{1}{2}\frac{(1+\vec{f}\cdot\vec{r})(1+\vec{r}\cdot\vec{i})}{(1+\vec{f}\cdot\vec{i})}} \qquad \arg\Pi_{r,w} = -\frac{1}{2}\Omega_{irf}$$

Geometric phase (Pancharatnam)

Pauli observable

$$|\sigma_{r,w}| = \sqrt{\frac{1}{2}\frac{(1+\vec{f}\cdot\vec{s})}{(1+\vec{f}\cdot\vec{i})}} \qquad \arg\sigma_{r,w} = -\frac{1}{2}\Omega_{irsf}$$

$$\vec{s} = 2(\vec{r}\cdot\vec{i})\vec{r} - \vec{i}$$



Introduction to quantum weak measurements and weak values

Weak measurements: practical aspects

Yves Caudano

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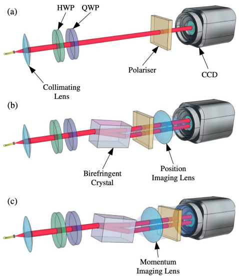
esphin
naxys Namur Institute for Complex Systems
NISM Namur Institute of Structured Matter
UNIVERSITÉ DE NAMUR

Introduction to quantum weak measurements and weak values

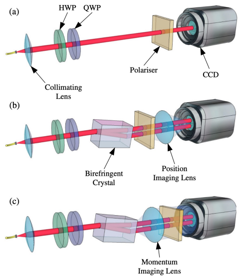
The pointer deflection in weak measurement depends on the weak value

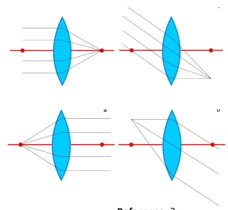
$$\langle A\rangle_w = \frac{\langle\phi_f|\hat{A}_s|\psi_i\rangle}{\langle\phi_f|\psi_i\rangle}$$



Reference: J. Dressel et al., Rev. Mod. Phys. **86** (2014) 307

The pointer deflection in weak measurement depends on the weak value

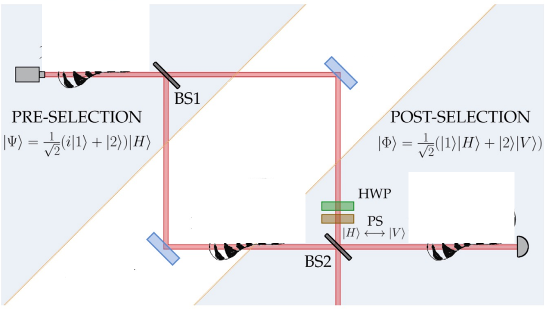




Reference: ?

Reference: J. Dressel et al., Rev. Mod. Phys. **86** (2014) 307

Example of pre- and post-selection



Reference: Quentin Duprey, Ph.D. thesis, U. Cergy-Pontoise, 2019

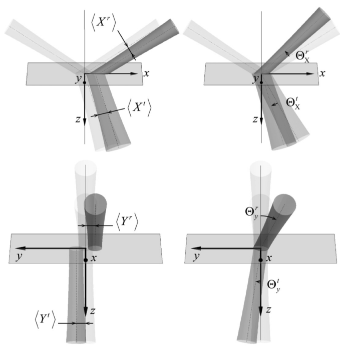
Introduction to quantum weak measurements and weak values

Weak measurements show deviations from geometrical optics for optical beam propagation

Goos–Hänchen and Imbert–Fedorov beam shifts: an overview

To cite this article: K Y Bliokh and A Aiello 2013 *J. Opt.* **15** 014001

Goos–Hänchen & Imbert-Fedorov beam shifts & deviations



Goos–Hänchen & Imbert-Fedorov beam shifts & deviations

Weak measurements show deviations from geometrical optics for optical beam propagation

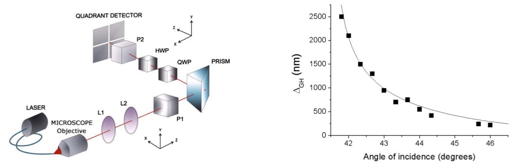
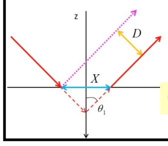
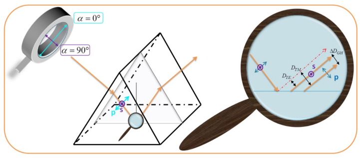


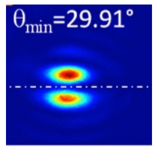
Fig. credits: G. Jayaswal, G. Mistura, and M. Merano, Opt. Lett. 38, 1232 – 1234 (2013)

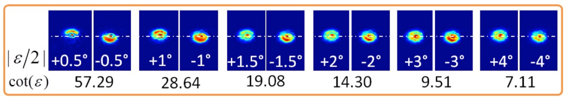


Goos – Hänchen beam shift

Weak measurements can evidence optical beam shifts through weak value amplification





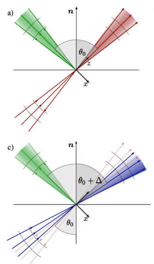


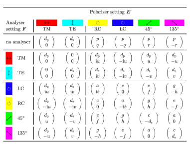
$$A_w = \frac{\langle \phi_f | \hat{A}_s | \psi_i \rangle}{\langle \phi_f | \psi_i \rangle}$$

Goos – Hänchen effect

Refs: J. Bouhy, master's thesis (2017), University of Namur

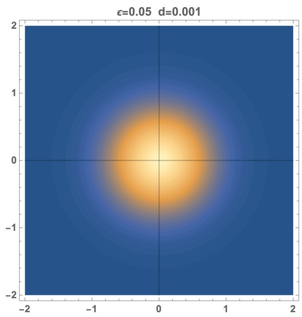
Weak measurements can evidence optical beam shifts through weak value amplification

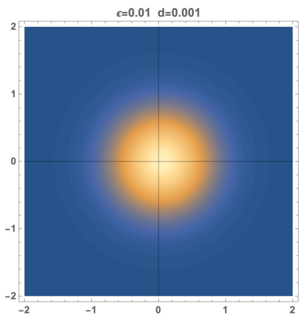


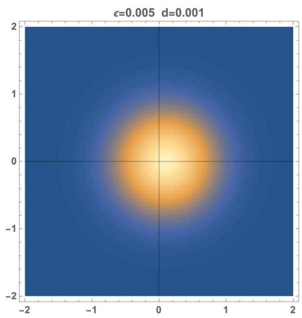


Generalized shifts and weak values for polarization components of reflected light beams

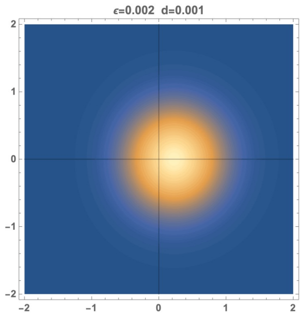
To cite this article: J B Götte and M R Dennis 2012 *New J. Phys.* **14** 073016



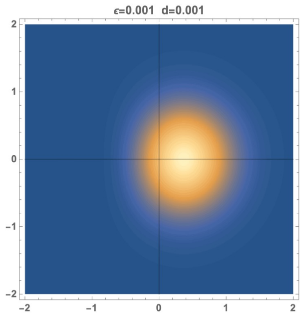




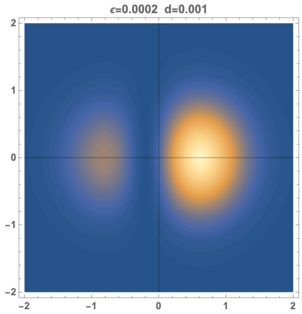
8. Exploiting amplification
74



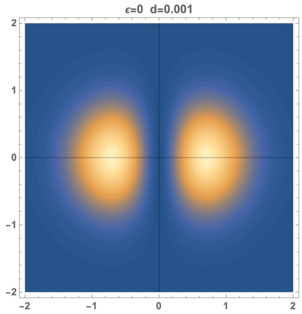
8. Exploiting amplification
75



8. Exploiting amplification
76

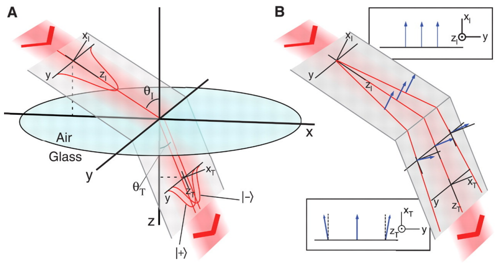


8. Exploiting amplification
77



8. Exploiting amplification
78

Spin Hall effect of light: transverse shifts of refracted beams according to circular polarizations

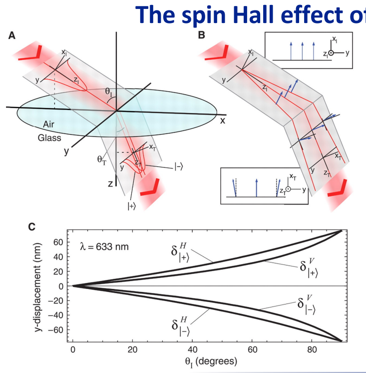


Spin-orbit coupling & geometric phase

Reference: *Observation of the Spin Hall Effect of Light via Weak Measurements,* O. Hosten and P. Kwiat, Science **319**, (2009) 787-790

8. Exploiting amplification
79

The spin Hall effect of light



Weak interaction

$$\int dk_y \Phi(k_y) \exp(-ik_y\hat{\sigma}_3\delta)|k_y\rangle|s\rangle$$
$$= \int dy \Psi(y - s\delta)|\breve{y}\rangle|s\rangle$$

Reference: *Observation of the Spin Hall Effect of Light via Weak Measurements,* O. Hosten and P. Kwiat, Science **319**, (2009) 787-790

The spin Hall effect of light

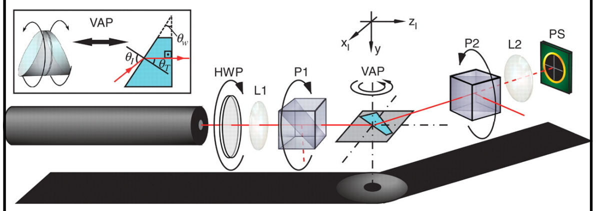
$$|\psi_1\rangle = |H\rangle = \frac{1}{\sqrt{2}}(|+\rangle + |-\rangle),$$

$$|\psi_2\rangle = |V \pm \Delta\rangle = -i\exp(\mp i\Delta)|+\rangle + i\exp(\pm i\Delta)|-\rangle$$

$$(\hat{\sigma}_3)_w = \mp i\cot\Delta \approx \mp i/\Delta.$$

Reference: *Observation of the Spin Hall Effect of Light via Weak Measurements,* O. Hosten and P. Kwiat, Science **319**, (2009) 787-790

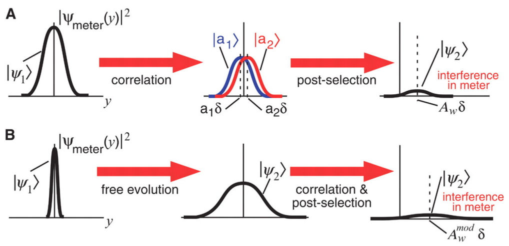
The spin Hall effect of light



$$\langle y\rangle = \frac{2\, z_{eff}\langle k_y^2\rangle}{k_I}|A_w|\delta$$

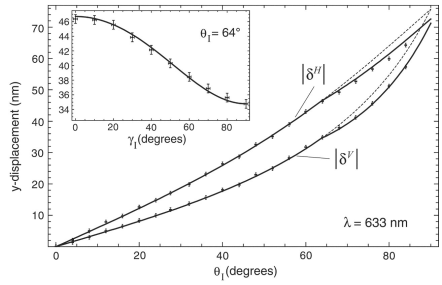
Reference: *Observation of the Spin Hall Effect of Light via Weak Measurements,* O. Hosten and P. Kwiat, Science **319**, (2009) 787-790

The spin Hall effect of light



Reference: *Observation of the Spin Hall Effect of Light via Weak Measurements,* O. Hosten and P. Kwiat, Science **319**, (2009) 787-790

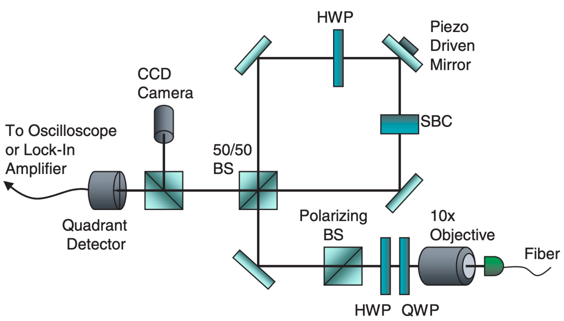
The spin Hall effect of light



1 Angstrom sensitivity, 10 000 enhancement

Reference: *Observation of the Spin Hall Effect of Light via Weak Measurements,* O. Hosten and P. Kwiat, Science **319**, (2009) 787-790

Ultrasensitive beam deflection measurement

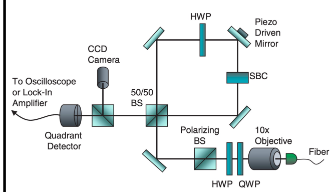


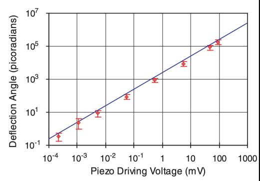
Reference: *Ultrasensitive Beam Deflection Measurement via Interferometric Weak Value Amplification* P. Ben Dixon et al., Phys. Rev. Lett. **102** (2009) 173601

Ultrasensitive beam deflection measurement

$$|\psi_i\rangle = (ie^{i\phi/2}|\circlearrowright\rangle + e^{-i\phi/2}|\circlearrowleft\rangle)/\sqrt{2}.$$

$$|\psi_f\rangle = (|\circlearrowright\rangle + i|\circlearrowleft\rangle)/\sqrt{2}$$





Reference: *Ultrasensitive Beam Deflection Measurement via Interferometric Weak Value Amplification* P. Ben Dixon et al., Phys. Rev. Lett. **102** (2009) 173601

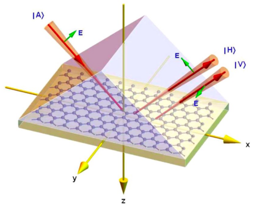
APPLIED PHYSICS LETTERS 110, 031105 (2017)

Observation of the Goos-Hänchen shift in graphene via weak measurements

Shizhen Chen, Chengquan Mi, Liang Cai, Mengxia Liu, Hailu Luo,[a] and Shuangchun Wen
Laboratory for Spin Photonics, School of Physics and Electronics, Hunan University, Changsha 410082, China

(Received 3 November 2016; accepted 5 January 2017; published online 18 January 2017)

We report the observation of the Goos-Hänchen effect in graphene via a weak value amplification scheme. We demonstrate that the amplified Goos-Hänchen shift in weak measurements is sensitive to the variation of graphene layers. Combining the Goos-Hänchen effect with weak measurements may provide important applications in characterizing the parameters of graphene. *Published by AIP Publishing.* [http://dx.doi.org/10.1063/1.4974212]



APPLIED PHYSICS LETTERS 110, 031105 (2017)

Observation of the Goos-Hänchen shift in graphene via weak measurements

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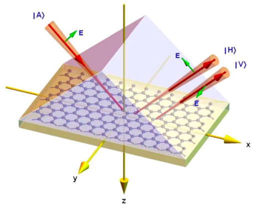

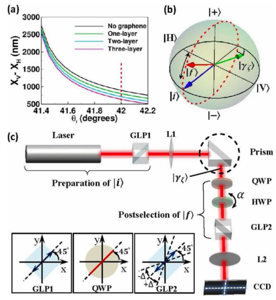

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Article

Measurement of Chiral Molecular Parameters Based on a Combination of Surface Plasmon Resonance and Weak Value Amplification

Liping Xu, Lan Luo, Hao Wu, Zhengchun Luo, Zhiyou Zhang, Haofei Shi, Tianying Chang,* Peng Wu,* Chunlei Du, and Hong-Liang Cui

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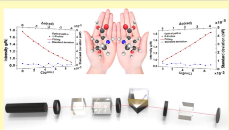
ABSTRACT: A novel combination of surface plasmon resonance (SPR) and weak value amplification (WVA) is employed to measure the optical rotation angle and refractive index of chiral enantiomers such as sugars and amino acids. An extremely low optical rotation change (2.73×10^{-4} rad) is readily measurable, with a resolution of 6.75×10^{-7} rad, 1 order of magnitude higher than that obtained using weak value amplification with intensity modulation, and a refractive index change of 1.13×10^{-6} RIU is also detected, with a resolution of 1.99×10^{-9} RIU, a nearly 1-order-of-magnitude increase in sensitivity over weak measurement based on a Mach–Zehnder interferometer. The optical activity and refractive index changes of chiral molecules are determined in real time by measurements of the output light intensity variation, whereby the absolute configuration of the chiral molecule is identified through the relation between intensity and molecular orientation. The SPR–WVA combination sensing scheme fills the gap of capability for detecting the optical activity of a molecular solution, which has not been possible with conventional SPR alone.



KEYWORDS: *chiral molecules, surface plasmon resonance, weak value amplification, optical rotation angle, refractive index change*

LETTERS
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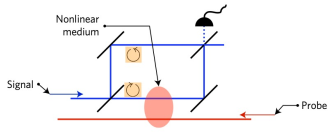
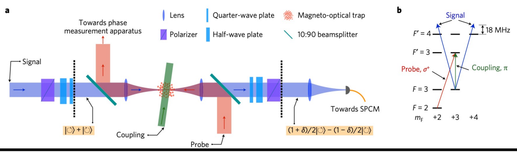
Weak-value amplification of the nonlinear effect of a single photon

Matin Hallaji[1]*, Amir Feizpour[1], Greg Dmochowski[1], Josiah Sinclair[1] and Aephraim M. Steinberg[1,2]



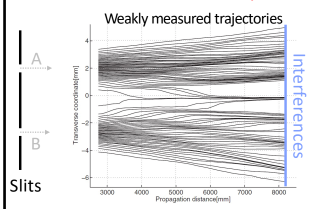


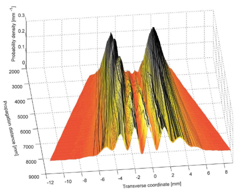
Introduction to quantum weak measurements and weak values

1. von Neumann measurement scheme
2. Aharonov – Bergman – Leibowitz rule (ABL)
3. Measurements with post-selection
4. Weak measurements (with post-selection)
5. A few characteristics of weak measurements
6. Making sense of anomalous weak values
7. Conceptual experimental set-up
8. Exploiting amplification
9. Probing trajectories
10. Exploiting complex numbers
11. Additional examples

Weak measurements can probe very fundamental questions about quantum mechanics

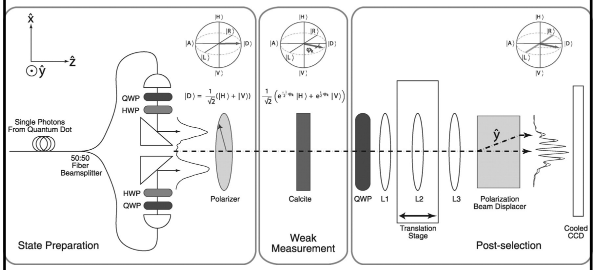
« Observing the average trajectories of single photons in a two-slit Interferometer »
Sacha Kocsis et. al., Science 332 (2011) 1170 – 1173





Bohmian interpretation of quantum mechanics!

Average trajectories of photons

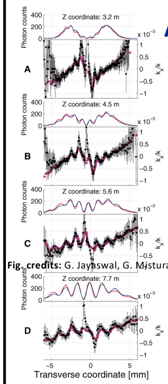


Linear phaseshift

$$\mathrm{Re}\frac{\langle x|\hat{p}|\psi_i\rangle}{\langle x|\psi_i\rangle} = \partial_x\Phi(x) = p_B(x)$$

Reference: *Observing the Average Trajectories of Single Photons in a Two-Slit Interferometer*, Sacha Kocsis et. al., Science **332** (2011) 1170 – 1173

Average trajectories of photons



$$|\Psi'\rangle = e^{-i\hat{H}_I t/\hbar}|\Psi\rangle,$$
$$= e^{-ig\hat{k}_x\hat{S}_1 t/\hbar}|D\rangle|\psi\rangle.$$

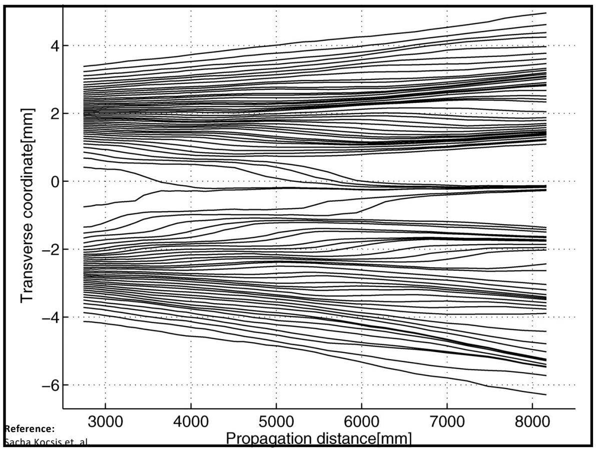
$$\cong \frac{\psi(x_f)}{\sqrt{2}}\left(e^{-\frac{igt}{2}\langle\hat{k}_x\rangle_w}|H\rangle + e^{\frac{igt}{2}\langle\hat{k}_x\rangle_w}|V\rangle\right)$$

$$\varphi(k_x) = \zeta\frac{k_x}{|\mathbf{k}|} + \varphi_0$$

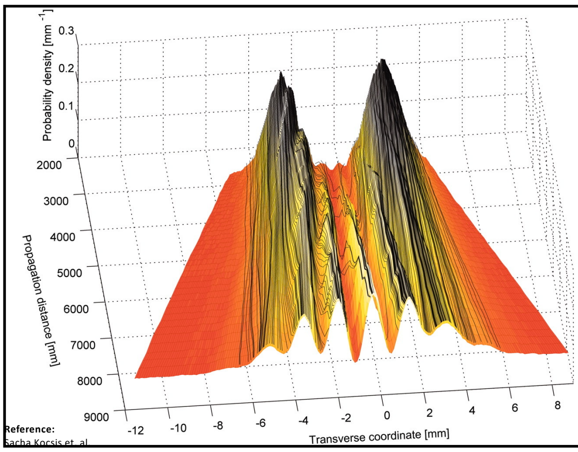
Fig. credits: G. Jayaswal, G. Mistura, and M. Merano, Opt. Lett. 38, 1232 – 1234 (2013)

$$\left\langle\hat{S}_3\right\rangle = |\langle R|\langle x_f|\Psi'\rangle|^2 - |\langle L|\langle x_f|\Psi'\rangle|^2,$$

Reference: *Observing the Average Trajectories of Single Photons in a Two-Slit Interferometer*, Sacha Kocsis et. al., Science **332** (2011) 1170 – 1173



Reference: Sacha Kocsis et. al.



Reference: Sacha Kocsis et. al.

1256–1261 | PNAS | January 25, 2011 | vol. 108 | no. 4

Violation of the Leggett–Garg inequality with weak measurements of photons

M. E. Goggin[a,b,1], M. P. Almeida[a], M. Barbieri[a,c], B. P. Lanyon[a], J. L. O'Brien[d], A. G. White[a], and G. J. Pryde[e]

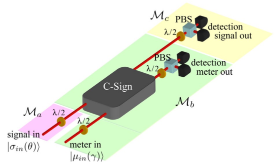


Fig. 1. Conceptual representation of the experiment. A "signal" photon is prepared with an arbitrary linearly polarized state using a half waveplate (λ/2), $\mathscr{M}_a$. A weak measurement of the polarization is then made ($\mathscr{M}_b$) by interacting the signal photon with a "meter" photon via a C-Sign gate, which operates via a measurement-induced nonlinearity (14, 25–29). Conditional on detecting a single photon in each of the two output modes (a coincidence measurement) the gate ideally performs the operation $|H\rangle\langle H|\otimes\sigma_{\mathcal{I}}+|V\rangle\langle V|\otimes\sigma_Z$ on the two-qubit polarization Hilbert space $\mathscr{H}_{\rm signal}\otimes\mathscr{H}_{\rm meter}$, where $\sigma_{\mathcal{I}}$ (σ_Z) is the 1-qubit identity (Pauli $\mathcal{Z}$) operator and $|H\rangle=|0\rangle$, $|V\rangle=|1\rangle$. See the main text for a full description of $\mathscr{M}_a$, $\mathscr{M}_b$, and $\mathscr{M}_c$ and *Materials and Methods* for more experimental details.

ARTICLES
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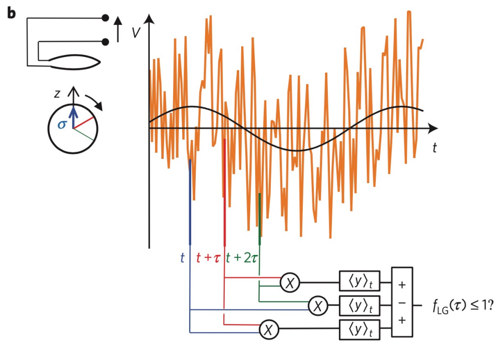
nature physics

Experimental violation of a Bell's inequality in time with weak measurement

Agustin Palacios-Laloy[1], François Mallet[1], François Nguyen[1], Patrice Bertet[1]★, Denis Vion[1], Daniel Esteve[1] and Alexander N. Korotkov[2]

The violation of Bell inequalities with two entangled and spatially separated quantum two-level systems (TLSs) is often considered as the most prominent demonstration that nature does not obey local realism. Under different but related assumptions of macrorealism—which macroscopic systems plausibly fulfil—Leggett and Garg derived a similar inequality for a single degree of freedom undergoing coherent oscillations and being measured at successive times. Here, we test such a 'Bell's inequality in time', which should be violated by a quantum TLS. Our TLS is a superconducting quantum circuit in which Rabi oscillations are continuously driven while it is continuously and weakly measured. The time correlations present at the detector output agree with quantum-mechanical predictions and violate the Leggett-Garg inequality by five standard deviations.



Introduction to quantum weak measurements and weak values

Direct measurement of the quantum wavefunction

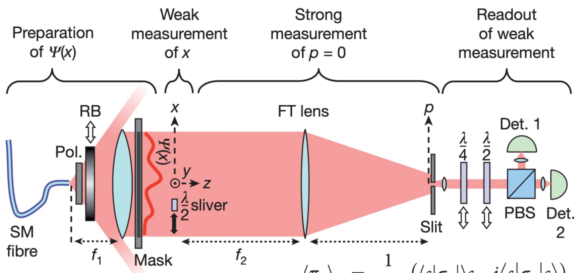
$$\langle \pi_x \rangle_W = \frac{\langle p|x\rangle \langle x|\Psi\rangle}{\langle p|\Psi\rangle}$$

$$= \frac{e^{ipx/\hbar}\Psi(x)}{\Phi(p)}$$

$\boldsymbol{p = 0}$ $\qquad \langle \pi_x \rangle_W = k\Psi(x)$

Reference: *Direct measurement of the quantum wavefunction,* J. Ludeen et. al., Nature **474** (2011) 188 – 191

Direct measurement of the quantum wavefunction



$$\langle \pi_x \rangle_W = \frac{1}{\sin\alpha}\left(\langle s|\sigma_x|\rangle s - i\langle s|\sigma_y|s\rangle\right)$$

Reference: *Direct measurement of the quantum wavefunction,* J. Ludeen et. al., Nature **474** (2011) 188 – 191

nature photonics ARTICLES
PUBLISHED ONLINE: 3 MARCH 2013 | DOI: 10.1038/NPHOTON.2013.24

Full characterization of polarization states of light via direct measurement

Jeff Z. Salvail[1]*, Megan Agnew[1], Allan S. Johnson[1], Eliot Bolduc[1], Jonathan Leach[1] and Robert W. Boyd[1,2]

Ascertaining the physical state of a system is vital in order to understand and predict its behaviour. However, due to their fragile nature, the direct observation of quantum states has, until recently, been elusive. Historically, determination of the quantum state has been performed indirectly through the use of tomography. We report on two experiments showing that an alternative approach can be used to determine the polarization quantum state in a simple, fast and general manner. The first experiment entails the direct measurement of the probability amplitudes describing pure polarization states of light, the first such measurement on a two-level system. The second experiment entails the direct measurement of the Dirac distribution (a phase-space quasi-probability distribution informationally equivalent to the density matrix), demonstrating that the direct measurement procedure is applicable to general (that is, potentially mixed) quantum states. Our work has applications to measurements in foundational quantum mechanics, quantum information and quantum metrology.

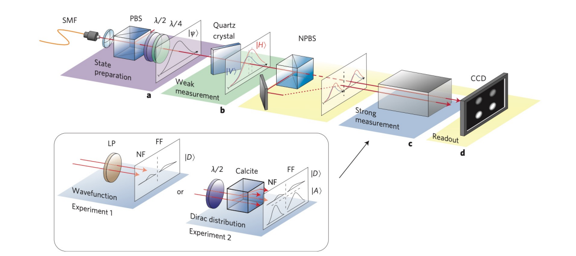


Figure 1 | Schematic representation of the experiment. a, The output of the single-mode fibre (SMF) is a near-Gaussian transverse mode of light. A polarizing beamsplitter (PBS) and waveplate(s) create a known pure polarization state. **b**, A quartz crystal at an oblique angle performs the weak measurement by introducing a small (compared to the beam waist) lateral displacement between horizontal and vertical polarization components. **c**, A strong measurement in a basis (diagonal/antidiagonal) mutually unbiased from the weak measurement is used to complete the direct measurement. Inset: To measure the wavefunction, a linear polarizer oriented to transmit diagonally polarized light performs the strong measurement and post-selection. To measure the Dirac distribution, a $\lambda/2$ wave-plate and calcite beam displacer carry out the strong measurement. **d**, A 50:50 non-polarizing beamsplitter (NPBS) splits the light into two sub-ensembles. These are imaged in the near-field (dotted line, NF) and far-field (dashed line, FF) of the quartz crystal onto non-overlapping regions of the CCD camera.

nature COMMUNICATIONS

ARTICLE

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Direct measurement of a 27-dimensional orbital-angular-momentum state vector

Mehul Malik[1,2], Mohammad Mirhosseini[1], Martin P.J. Lavery[3], Jonathan Leach[4,5], Miles J. Padgett[3] & Robert W. Boyd[1,5]

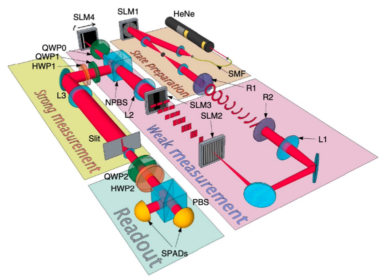
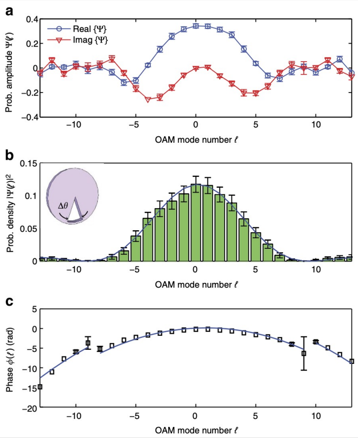


Figure 1 | Experimental setup for direct measurement of a high-dimensional state vector. State preparation: a quantum state in an arbitrary superposition of OAM modes is prepared by impressing phase information with SLM1 on spatially filtered (SMF) photons from an attenuated HeNe laser. Weak measurement: a particular OAM mode is weakly projected by rotating its polarization. In order to do so, the OAM modes are first transformed into finite-sized momentum modes by two refractive optical elements made out of Poly methyl methacrylate (R1 and R2). Then, a Fourier transform lens (L1) and a fan-out hologram implemented on SLM2 are used to generate three adjacent copies of each momentum mode. The phase between these copies is corrected by SLM3. Another lens (L2) converts these larger momentum modes into well-separated position modes at its focus. Finally, a QWP0 used in double pass with SLM4 is used to rotate the polarization of the OAM mode to be weakly projected. Another quarter-wave plate and a half-wave plate (QWP1 and HWP1) are used to remove any ellipticity introduced by transmission and reflection through the non-polarizing beam splitter (NPBS). Strong measurement: a strong measurement of angular position is performed by Fourier transforming with a lens (L3) and post-selecting state $\theta = \theta_0$ with a 10-µm slit. Readout: the OAM weak value $\langle \pi_\ell \rangle_w$ is obtained by measuring the change in the photon polarization in the linear and circular polarization bases. QWP2, HWP2, a polarizing beam splitter (PBS) and two single-photon avalanche detectors (SPADs) are used for this purpose.



PHYSICAL REVIEW LETTERS **123,** 150402 (2019)

Editors' Suggestion Featured in Physics

Direct Measurement of a Nonlocal Entangled Quantum State

Wei-Wei Pan,[1,2] Xiao-Ye Xu,[1,2] Yaron Kedem,[3,*] Qin-Qin Wang,[1,2] Zhe Chen,[1,2] Munsif Jan,[1,2] Kai Sun,[1,2] Jin-Shi Xu,[1,2] Yong-Jian Han,[1,2] Chuan-Feng Li,[1,2,†] and Guang-Can Guo[1,2]

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Entanglement and the wave function description are two of the core concepts that make quantum mechanics such a unique theory. A method to directly measure the wave function, using weak values, was demonstrated by Lundeen *et al.* [Nature **474**, 188 (2011)]. However, it is not applicable to a scenario of two disjoint systems, where nonlocal entanglement can be a crucial element, since that requires obtaining weak values of nonlocal observables. Here, for the first time, we propose a method to directly measure a nonlocal wave function of a bipartite system, using modular values. The method is experimentally implemented for a photon pair in a hyperentangled state, i.e., entangled both in polarization and momentum degrees of freedom.

DOI: 10.1103/PhysRevLett.123.150402

Reference: *Direct measurement of the quantum wavefunction,* J. Lundeen et. al., Nature **474** (2011) 188 – 191

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nature COMMUNICATIONS

ARTICLE

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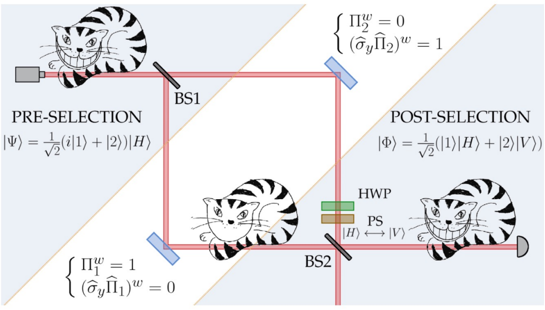
Observation of a quantum Cheshire Cat in a matter-wave interferometer experiment

Tobias Denkmayr[1], Hermann Geppert[1], Stephan Sponar[1], Hartmut Lemmel[1,2], Alexandre Matzkin[3], Jeff Tollaksen[4] & Yuji Hasegawa[1]

Example of pre- and post-selection



Reference: Quentin Duprey, Ph.D. thesis, U. Cergy-Pontoise, 2019

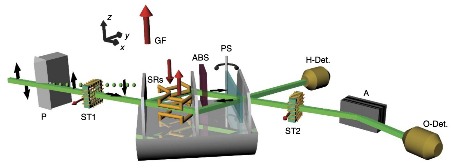


Figure 2 | Illustration of the experimental setup. The neutron beam is polarized by passing through magnetic birefringent prisms (P). To prevent depolarization, a magnetic guide field (GF) is applied around the whole setup. A spin turner (ST1) rotates the neutron spin by $\pi/2$. Preselection of the system's wavefunction $|\psi_i\rangle$ is completed by two spin rotators (SRs) inside the neutron interferometer. These SRs are also used to perform the weak measurement of $\langle\hat{\sigma}_z\hat{\Pi}_I\rangle_w$ and $\langle\hat{\sigma}_z\hat{\Pi}_{II}\rangle_w$. The absorbers (ABS) are inserted in the beam paths when $\langle\hat{\Pi}_I\rangle_w$ and $\langle\hat{\Pi}_{II}\rangle_w$ are determined. The phase shifter (PS) makes it possible to tune the relative phase χ between the beams in path *I* and path *II*. The two outgoing beams of the interferometer are monitored by the H and O detector in reflected and forward directions, respectively. Only the neutrons reaching the O detector are affected by postselection using a spin turner (ST2) and a spin analyzer (A).

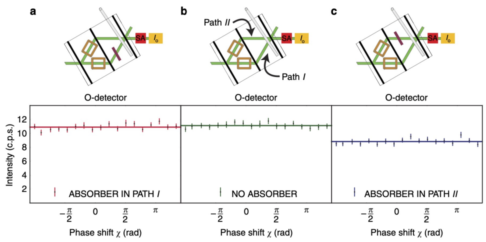


Figure 3 | Measurement of $\langle\hat{\Pi}_I\rangle_w$ and $\langle\hat{\Pi}_{II}\rangle_w$ using an absorber with transmissivity $T = 0.79(1)$. The intensity is plotted as a function of the relative phase χ. The solid lines represent least-square fits to the data and the error bars represent one s.d. (**a**) An absorber in path *I*; no significant loss in intensity is recorded. (**b**) A reference measurement without any absorber. (**c**) An absorber in path *II*: the intensity decreases. These results suggest that for the successfully postselected ensemble, the neutrons go through path *II*.

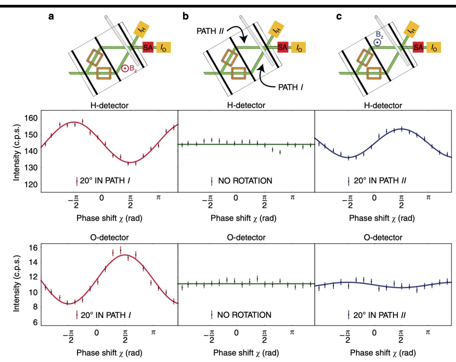


Figure 4 | Measurement of $\langle\hat{\sigma}_z\hat{\Pi}_I\rangle_w$ and $\langle\hat{\sigma}_z\hat{\Pi}_{II}\rangle_w$ applying small additional magnetic fields. The intensity of the O beam (with the spin analysis) and the H beam (without the spin analysis) is plotted as a function of the relative phase χ. The solid lines represent least-square fits to the data and the error bars represent one s.d. (**a**) A magnetic field in path *I*; interference fringes appear both at the postselected O detector and the H detector. (**b**) A reference measurement without any additional magnetic fields. Since the spin states inside the interferometer are orthogonal, interference fringes appear neither in the O, nor the H detector. (**c**) A magnetic field in path *II*; interference fringes with minimal contrast can be seen at the spin postselected O detector, whereas a clear sinusoidal intensity modulation is visible at the H detector without spin analysis. The measurements suggest that for the successfully postselected ensemble (only the O detector) the neutrons' spin component travels along path *I*.

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Experimental exchange of grins between quantum Cheshire cats

Zheng-Hao Liu [1,2,4], Wei-Wei Pan[1,2,4], Xiao-Ye Xu[1,2✉], Mu Yang[1,2], Jie Zhou[3], Ze-Yu Luo[1,2], Kai Sun[1,2], Jing-Ling Chen[3✉], Jin-Shi Xu [1,2✉], Chuan-Feng Li [1,2✉] & Guang-Can Guo[1,2]

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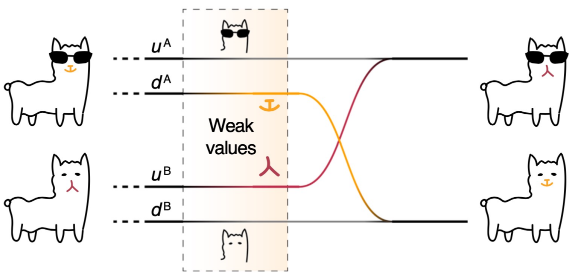


NATURE COMMUNICATIONS | https://doi.org/10.1038/s41467-020-16761-0 ARTICLE

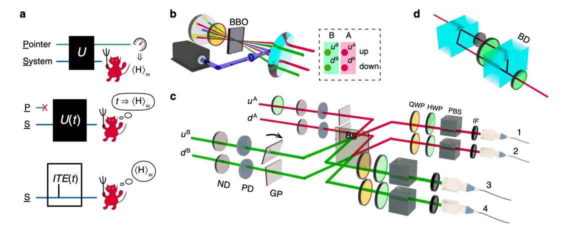
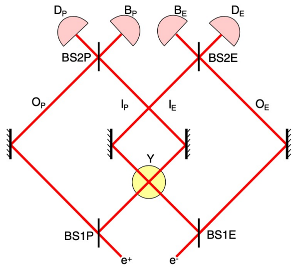


Fig. 2 Design of experiments. a Concept of weak value extraction. Top: rigorous weak measurement. The system and pointer are entangled by coupling evolution $\mathcal{U} = \exp(-i\mathcal{H}t)$ with the interaction Hamiltonian being $\mathcal{H} = O_s \otimes P$. The demon cast judicious choices of post-selection to obtain the weak value $\langle O_s \rangle_w$. Mid: interpretation of weak value without pointer. The system's evolution $\mathcal{U}_s$ due to a perturbation is characterised by weak value in the weak interaction regime[23]. Bottom: weak value extraction based on perturbation. A linear relationship between the post-selection probability and the interaction time of ITE can be established, whose incline gives the real part of the weak value $\langle O \rangle_w$. **b** Photon source. Biphoton (central wavelength 813.4 nm), four-qubit hyperentangled state is generated via type-I spontaneous parametric down-conversion (SPDC) process by pumping a β-barium borate (BBO) crystal twice in a confocal structure[24,25]. CM concave mirror, QWP quarter-wave plate. **c** The main setup to exchange grins. Neutral density filter (ND) and polarisation-sensitive density filter (PD) implement the perturbation on the path observable Π and the conditional spin observable $\sigma_z \otimes \Pi$, respectively. A polarisation independent BS, followed by four polarisation analysers, each composed of a set of quarter- or half-wave plates (QWP, HWP) and a PBS, implements post-selection. Four 0.2-mm-thick glass plates (GPs) are inserted in the four arms with one of them rotatable to compensate the biphoton phase. The photons are filtered by 3-nm-bandwidth interference filters (IF) with a central wavelength of 813.4 nm before collected by four single-mode fibres and guided to single-photon detectors. **d** A PD is constructed from two polarisation beam displacers (BDs) and an ND filter inserted in one arm.

PRL 102, 020404 (2009) PHYSICAL REVIEW LETTERS week ending 16 JANUARY 2009

Experimental Joint Weak Measurement on a Photon Pair as a Probe of Hardy's Paradox

J. S. Lundeen and A. M. Steinberg



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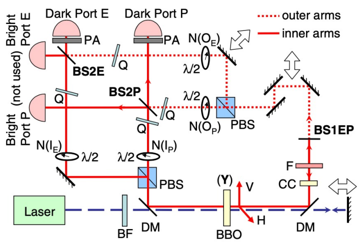

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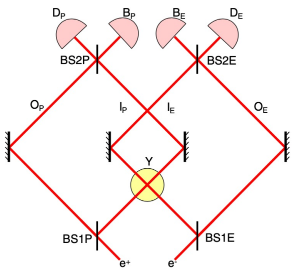
J. S. Lundeen and A. M. Steinberg



$N^{-}{}_{Ow}=1, N^{+}{}_{Ow}=1,$

$N^{-}{}_{NOw}=0, N^{+}{}_{NOw}=0,$

$N^{+,-}{}_{O,Ow}=0,$

$N^{+,-}{}_{O,NOw}=1, N^{+,-}{}_{NO,Ow}=1,$

$N^{+,-}{}_{NO,NOw}=-1.$

Weak measurements: fundamental and practical aspects

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