MISCELLANEOUS: SINGLE PHOTON WAVE FUNCTION. THOMAS DURT

I. WAVE FUNCTION OF A MONOMODE SINGLE-PHOTON: N=1 FOCK STATE.

In many situations, it is appropriate to describe the single photon as a single-mode photon, in which case its physical properties are equivalent to those of a quantized one-dimensional harmonic oscillator whose Hamiltonian can be expressed in terms of the photon number operator $\hat{N}$ through $\hat{H} = \hbar\omega(\hat{N} + 1/2)$, $\omega = \sqrt{\frac{k}{m}}$ being the classical pulsation of the oscillator.

The eigenstates of $\hat{H}$ (and $\hat{N}$) are the so-called Fock states and, in this approach, a single photon is nothing else than an elementary excitation or N=1 Fock state $|N = 1\rangle$ that is to say it is an eigenstate of $\hat{N}$ for the eigenvalue $N = 1$ (and thus an eigenstate of $\hat{H}$ for the eigenvalue $3\hbar\omega/2$). Such a state can be obtained after letting act the creation (raising) operator $\hat{a}^+ = \frac{1}{\sqrt{2}}(\hat{q} - i\hat{p}_q)$ (where $q = x/\sqrt{\frac{\hbar}{m\omega}}$ and $p_q = \frac{\partial}{i\partial q}$) on the vacuum state (which is itself a N=0 Fock state $|N = 0\rangle$, of energy-eigenvalue $\hbar\omega/2$).

Note that the Hamiltonian of a one-dimensional harmonic oscillator ($\hat{H} = \hat{p}_x^2/2m + k\hat{x}^2/2$) can be expressed in terms of the creation and destructions operators (resp. $\hat{a}^+ = \frac{1}{\sqrt{2}}(\hat{q} - i\hat{p}_q)$ and $\hat{a} = \frac{1}{\sqrt{2}}(\hat{q} + i\hat{p}_q)$ through the relation $\hat{H} = \hbar\omega(\hat{a}^+\hat{a} + 1/2) = \hbar\omega(\hat{N} + 1/2) = \hbar\omega(\sum_{N=0,1,2...}^{\infty} N\,|N\rangle\langle N|\, +1/2)$.

If we consider propagation in the vacuum, very often the mode corresponds to a plane wave of given polarisation; in the case of guided light the mode is often associated to the mode used to carry the light. In any case it is worth noting that the variable q introduced above does not represent the position of the photon in real, physical space; it can be interpreted as the amplitude of the electric field.

If we wish to describe photons propagating in space and time, a multimodal description in terms of wave packets is often more appropriate as we shall see now.

II. SPATIO-TEMPORAL WAVE FUNCTION OF A MULTIMODE SINGLE-PHOTON.

To be able to address the probability to observe a photon at a given time in a given place, let us introduce a one-photon state (in Schrödinger representation)

$$|\,\psi\,(t)\rangle = \sum_{\lambda=\pm} \int \mathrm{d}^3k\, c_\lambda\,(\mathbf{k}, t = 0)\, \mathrm{e}^{-\mathrm{i}c||\mathbf{k}||t}\, |\,1_{\lambda,\mathbf{k}}\rangle \tag{1}$$

where $\hat{a}^\dagger_{(\lambda)}\,(\mathbf{k})\,|0\rangle = |\,1_{\lambda,\mathbf{k}}\rangle$ represents the quantum state of a single-mode single-photon for a (plane wave) transverse mode of wave vector $\mathbf{k}$ and polarisation λ. It is obtained after letting act the raising (creation) operator $\hat{a}^+_{\lambda,\mathbf{k}}$ associated to this mode on the vacuum state here denoted $|\,0\rangle$. The vacuum state itself is the tensor-product of the vacuum states associated to each transverse mode ($|0\rangle = \otimes\,|0_{\lambda,\mathbf{k}}\rangle, \forall\lambda, \mathbf{k}$).

The associated Glauber first order correlation function [1, 2] reads

$$\boldsymbol{\psi}^E\,(\mathbf{x}, t) = \mathrm{i}c \sum_{\lambda=\pm} \int \frac{\mathrm{d}^3k}{(2\pi)^3} \sqrt{||\mathbf{k}||}\mathrm{e}^{\mathrm{i}(\mathbf{k}\cdot\mathbf{x} - c||\mathbf{k}||t)} c_\lambda\,(\mathbf{k}, t = 0)\, \boldsymbol{\epsilon}_{(\lambda)}\,(\mathbf{k}) \tag{2}$$

where $\boldsymbol{\epsilon}_{(\lambda)}\,(\mathbf{k})$ represents the direction of polarisation of the corresponding mode.

As is well-established in the standard theory of photodetection [2], the modulus squared of $\boldsymbol{\psi}^E\,(\mathbf{x}, t)$ in the expression (2) is proportional to the probability of detecting a photon at time t in a detector located at position $\mathbf{x}$ and can thus be interpreted as a kind of single photon wave function. In analogy with the classical Poynting density, the photon wave function $\boldsymbol{\psi}^E\,(\mathbf{x}, t)$ can also be interpreted as the single photon electric field generated by the quantum dipole. It is a complex vector with 3 components. Its components transform under Lorentz transformations in the same way that Maxwell's electric field's components do transform.

III. PHOTON WAVE FUNCTION SEEN AS AN EFFECTIVE SINGLE PHOTON ELECTRIC FIELD AND QUANTUM POYNTING VECTOR.

A. Single photon wave function and Maxwell‘s equations.

The photon wave function approach emphasizes the links between Maxwell theory and quantum optics. For instance, it makes it possible to construct a local quantum Poynting vector in terms of the (complex) electric and magnetic single photon wave functions, in full analogy with Maxwell's classical optics. To show this, let us firstly consider the quantum expression of the potential vector operator:
$\hat{\mathbf{A}}(\mathbf{x},t) = \sum_{\lambda=\pm}\int \frac{\mathrm{d}^3k}{(2\pi)^3}\frac{1}{\sqrt{||\mathbf{k}||}}\left[\hat{a}_{(\lambda)}(\mathbf{k},t)\,\boldsymbol{\epsilon}_{(\lambda)}(\mathbf{k})\,\mathrm{e}^{\mathrm{i}\mathbf{k}\cdot\mathbf{x}} - \hat{a}^{\dagger}_{(\lambda)}(\mathbf{k},t)\,\boldsymbol{\epsilon}^{*}_{(\lambda)}(\mathbf{k})\,\mathrm{e}^{-\mathrm{i}\mathbf{k}\cdot\mathbf{x}}\right]$ where the creation-destruction operators associated to plane-wave modes (of wave vector $\mathbf{k}$ and polarisation $\varkappa$) $\hat{a}_{(\varkappa)}(\mathbf{k},t) = \hat{a}_{(\varkappa)}(\mathbf{k},t=0)\,\mathrm{e}^{-\mathrm{i}c||\mathbf{k}||t}$ and $\hat{a}^{\dagger}_{(\varkappa)}(\mathbf{k},t)=\hat{a}^{\dagger}_{(\varkappa)}(\mathbf{k},t=0)\,\mathrm{e}^{+\mathrm{i}c||\mathbf{k}||t}$ obey canonical commutation rules:

$$\left[\hat{a}_{(\varkappa)}(\mathbf{k},t)\,,\hat{a}^{\dagger}_{(\lambda)}(\mathbf{q},t)\right] = \delta(\mathbf{k}-\mathbf{q})\,\delta_{\varkappa\lambda}. \tag{3}$$

In analogy with Maxwell's theory, the vector operator is associated to the electric field operator (in the Coulomb gauge where the electric potential is equal to zero) through $\hat{\mathbf{E}}(\mathbf{x},t) = \frac{-\partial}{\partial t}\hat{\mathbf{A}}(\mathbf{x},t)$, so that

$$\hat{\mathbf{E}}(\mathbf{x},t) = \mathrm{i}c\sum_{\lambda=\pm}\int\frac{\mathrm{d}^3k}{(2\pi)^3}\sqrt{||\mathbf{k}||}\left[\hat{a}_{(\lambda)}(\mathbf{k},t)\,\boldsymbol{\epsilon}_{(\lambda)}(\mathbf{k})\,\mathrm{e}^{\mathrm{i}\mathbf{k}\cdot\mathbf{x}} - \hat{a}^{\dagger}_{(\lambda)}(\mathbf{k},t)\,\boldsymbol{\epsilon}^{*}_{(\lambda)}(\mathbf{k})\,\mathrm{e}^{-\mathrm{i}\mathbf{k}\cdot\mathbf{x}}\right] \tag{4}$$

Similarly, the magnetic field operator obeys $\hat{\mathbf{B}}(\mathbf{x},t) = rot\hat{\mathbf{A}}(\mathbf{x},t)$ so that

$$\hat{\boldsymbol{B}}(\boldsymbol{x},t) = \mathrm{i}\sum_{\lambda=\pm}\int\frac{\mathrm{d}^3k}{(2\pi)^3}(1/\sqrt{||\mathbf{k}||})\left(\boldsymbol{k}\times\boldsymbol{\epsilon}_{(\lambda)}(\boldsymbol{k})\right)\left[\hat{a}_{(\lambda)}(\boldsymbol{k})\,\mathrm{e}^{-\mathrm{i}(c||\boldsymbol{k}||t-\boldsymbol{k}\cdot\boldsymbol{x})} - \hat{a}^{\dagger}_{(\lambda)}(\boldsymbol{k})\,\mathrm{e}^{\mathrm{i}(c||\boldsymbol{k}||t-\boldsymbol{k}\cdot\boldsymbol{x})}\right],$$

It can be shown by straightforward computation that the Glauber first order correlation function (2) can be expressed in terms of the electric field operator through Glauber's extraction rule

$$\begin{aligned}\boldsymbol{\psi}^{E}(\mathbf{x},t) &\equiv \langle 0|\,\hat{\mathbf{E}}(\mathbf{x},0)\mid\psi(t)\rangle\\ &= \sum_{\lambda=\pm}\int\mathrm{d}^3k\,\langle 0|\,\hat{\mathbf{E}}(\mathbf{x},0)\,c_{\lambda}(\mathbf{k},t=0)\,\mathrm{e}^{-\mathrm{i}c||\mathbf{k}||t}\,|\,1_{\lambda,\mathbf{k}}\rangle\end{aligned} \tag{5}$$

Remark that higher orders correlations associated to N photon wave functions can be estimated in a similar fashion; for instance the 9 components of the two-photon wave function obey

$$\boldsymbol{\psi}^{E}_{k,l}(\mathbf{x_1},\mathbf{x_2},t) \equiv \langle 0|\,\hat{\mathbf{E}}_k(\mathbf{x_1},0)\,\hat{\mathbf{E}}_l(\mathbf{x_2},0)\mid\psi(t)\rangle \tag{6}$$

Inspired by Maxwell theory, let us define, making use of the magnetic field operator, the (transverse) magnetic single-photon wave function as follows:

$$\boldsymbol{\psi}^{B}(\mathbf{x},t) \equiv \langle 0|\hat{\boldsymbol{B}}^{+}(\boldsymbol{x},t=0)\mid\psi(t)\rangle\quad . \tag{7}$$

A straightforward calculation yields

$$\boldsymbol{\psi}^{B}(\mathbf{x},t) = \mathrm{i}c\sum_{\lambda=\pm}\int\frac{\mathrm{d}^3k}{(2\pi)^3}(1/\sqrt{||\mathbf{k}||})\left(\boldsymbol{k}\times\boldsymbol{\epsilon}_{(\lambda)}(\boldsymbol{k})\right)\mathrm{e}^{\mathrm{i}(\mathbf{k}\cdot\mathbf{x}-c||\mathbf{k}||t)}c_{\lambda}(\mathbf{k},t)\,. \tag{8}$$

One can show that Maxwell's equations written explicitly for these single-photon wave functions are obeyed:

$$\nabla\times\boldsymbol{\psi}^{E}(\boldsymbol{x},t) = -\partial_t\boldsymbol{\psi}^{B}(\boldsymbol{x},t) \tag{9a}$$

$$\nabla\times\boldsymbol{\psi}^{B}(\boldsymbol{x},t) = \frac{1}{c^2}\partial_t\boldsymbol{\psi}^{E}(\boldsymbol{x},t) \tag{9b}$$

$$\nabla\cdot\boldsymbol{\psi}^{E}(\boldsymbol{x},t) = 0 \tag{9c}$$

$$\nabla\cdot\boldsymbol{\psi}^{B}\boldsymbol{x},t) = 0 \tag{9d}$$

B. Quantum versus classical.

In other words, in the vacuum, the single-photon wave function propagates according to Maxwell's equation(s). This could suggest that Maxwell fields are already quantized to begin with but we do not share this interpretation: it is only in particular situations that the quantum wave function "lives" in the three dimensional, physical, space. Most often, whenever entanglement is present, a three dimensional representation does not hold anymore and must be replaced by a description in the configuration space. A good example of this feature can be found in appendix 1 where it is shown that it is impossible to explain HOM experiment [3] resorting only to Maxwell's theory. Ultimately the reason for this is that the two-photon wave function is entangled and requires to be interpreted in the 6-dimensional configuration space and not in the usual 3 dimensional space.

There are numerous situations however where Maxwell's theory alone makes it possible to interpret the observations. This is the case for instance when the source of light is either a laser source or a thermal source. Then the N photon wave function associated to light which is prepared at time $t = 0$ in the monomode (polychromatic) Fock state [4, 5] $\mid N\rangle = \frac{(\hat{a}^\dagger)^N}{\sqrt{N!}} \mid N = 0\rangle$ (with $\hat{a}^\dagger = \sum_{\lambda=\pm} \int \mathrm{d}^3k \, \tilde{c}_\lambda (\mathbf{k}) \, \boldsymbol{\epsilon}_{(\lambda)} (\mathbf{k}) \, \hat{a}^\dagger_{(\lambda)} (\mathbf{k})$) can be shown to factorize into the product of identical single-photon wave functions:

$$\boldsymbol{\psi}^E (\mathbf{x_1}, \mathbf{x_2}, ..., \mathbf{x_N}, t) = \boldsymbol{\psi}^E (\mathbf{x_1}, t) \cdot \boldsymbol{\psi}^E (\mathbf{x_2}, t) ... \boldsymbol{\psi}^E (\mathbf{x_N}, t) \tag{10}$$

where each individual (single photon) wave function obeys Maxwell equations as we have shown before.

C. Quantum expression of the single photon Poynting vector.

The local quantum Poynting vector reads $\frac{1}{2\mu_0}((\boldsymbol{\psi}^E (\mathbf{x}, t))^* \times \boldsymbol{\psi}^B (\mathbf{x}, t) + \boldsymbol{\psi}^E (\mathbf{x}, t) \times (\boldsymbol{\psi}^B (\mathbf{x}, t))^*)$.

In full analogy with classical Maxwell's equations, one can also show that the density $\frac{\epsilon_0}{2} \left[|\boldsymbol{\psi}^E (\mathbf{x}, t) |^2 + c^2 |\boldsymbol{\psi}^B (\mathbf{x}, t) |^2 \right]$ of the photonic population at a given place (x, y, z) and a given time t, obeys the conservation equation

$$\frac{\partial}{\partial t} (\frac{\epsilon_0}{2} \left[|\boldsymbol{\psi}^E (\mathbf{x}, t) |^2 + c^2 |\boldsymbol{\psi}^B (\mathbf{x}, t) |^2 \right]) + div. \frac{1}{2\mu_0} ((\boldsymbol{\psi}^E (\mathbf{x}, t))^* \times \boldsymbol{\psi}^B (\mathbf{x}, t) + \boldsymbol{\psi}^E (\mathbf{x}, t) \times (\boldsymbol{\psi}^B (\mathbf{x}, t))^*) = 0 \tag{11}$$

In analogy with classical fluid dynamics, it is natural to define a local velocity as follows $\boldsymbol{v} (\mathbf{x}, t)$:

$\boldsymbol{v} (\mathbf{x}, t) = \frac{\frac{1}{2\mu_0}((\boldsymbol{\psi}^E (\mathbf{x}, t))^* \times \boldsymbol{\psi}^B (\mathbf{x}, t) + \boldsymbol{\psi}^E (\mathbf{x}, t) \times (\boldsymbol{\psi}^B (\mathbf{x}, t))^*)}{(\frac{\epsilon_0}{2}[|\boldsymbol{\psi}^E (\mathbf{x}, t)|^2 + c^2 |\boldsymbol{\psi}^B (\mathbf{x}, t)|^2])}$, and one may consistently interpret the weak measurement of local velocities [6] in terms of single-photon Poynting velocities of this type [7, 8].

IV. ACKNOWLEDGEMENTS

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V. APPENDIX 1: HOM EXPERIMENT IN TERMS OF TWO-PHOTON WAVE FUNCTIONS

In the HOM experiment [3], two single photons are sent along two orthogonal beams on a semi-transparent mirror.
Let us denote
Ψ^{HOR}_{IN} and Ψ^{VERT}_{IN}
the corresponding wave functions.

Then, the IN state, prior to the passage through the mirror is described by the symmetrized two-photon wave function
$(1/\sqrt{2})(\Psi^{HOR}_{IN}(\mathbf{x}_1)\Psi^{VERT}_{IN}(\mathbf{y}_2)+\Psi^{HOR}_{IN}(\mathbf{x}_2)\Psi^{VERT}_{IN}(\mathbf{y}_1))$

After passing through the mirror, each reflected wave gets dephased by a $\pi/2$ factor, while the transmitted waves do not undergo any dephasing.

At the output of the mirror we get thus
$(1/\sqrt{2})((1/\sqrt{2})(i\Psi^{HOR-ref}_{OUT}(\mathbf{y}_1) + \Psi^{HOR-trans}_{OUT}(\mathbf{x}_1))(1/\sqrt{2})(\Psi^{VERT-trans}_{OUT}(\mathbf{y}_2) + i\Psi^{VERT-ref}_{OUT}(\mathbf{x}_2))$

$$+(1/\sqrt{2})((1/\sqrt{2})(i\Psi_{OUT}^{HOR-ref}(\mathbf{y}_2)+\Psi_{OUT}^{HOR-trans}(\mathbf{x}_2))(1/\sqrt{2})(\Psi_{OUT}^{VERT-trans}(\mathbf{y}_1)+i\Psi_{OUT}^{VERT-ref}(\mathbf{x}_1))$$

If the two incoming photons share a same wave function, that is to say if

$$\Psi_{OUT}^{HOR-ref}(\mathbf{y})=\Psi_{OUT}^{VERT-trans}(\mathbf{y})=\Psi_{OUT}(\mathbf{y}) \text{ and } \Psi_{OUT}^{HOR-trans}(\mathbf{x})=\Psi_{OUT}^{VERT-ref}(\mathbf{x})=\Psi_{OUT}(\mathbf{x}),$$

the expression above simplifies to

$$(i/\sqrt{2})(\Psi_{OUT}^{HOR-trans}(\mathbf{x_1})\Psi_{OUT}^{VERT-ref}(\mathbf{x_2})+\Psi_{OUT}^{HOR-ref}(\mathbf{y}_1)\Psi_{OUT}^{VERT-trans}(\mathbf{y}_2))$$
$$=(i/\sqrt{2})(\Psi_{OUT}(\mathbf{x_1})\Psi_{OUT}(\mathbf{x_2})+\Psi_{OUT}(\mathbf{y_1})\Psi_{OUT}(\mathbf{y_2})),$$

which means that either both photons are located along the horizontal axis (X_{OUT}) or they are located along the vertical axis (Y_{OUT}). The absence of simultaneous clicks in the horizontal and vertical detectors at the output is thus a signature of the equivalence of the wave functions of the photons sent along the incoming horizontal and vertical axes.

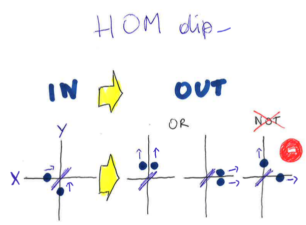

Abbildung 1: HOM dip

COMMENTS

Note that if we send two identical photons through the same input port, for instance if at the input the wave function reads

$$\Psi_{IN}^{HOR}(\mathbf{x}_1)\Psi_{IN}^{HOR}(\mathbf{x}_2),$$

no entanglement is present and at the output we get the wave function

$$(1/2)(i\Psi_{OUT}^{HOR-ref}(\mathbf{y}_1)+\Psi_{OUT}^{HOR-trans}(\mathbf{x}_1))(i\Psi_{OUT}^{HOR-ref}(\mathbf{y}_2)+\Psi_{OUT}^{HOR-trans}(\mathbf{x}_2))$$

so that the probability to find two photons in the same output port is 1/4, and in different output ports 1/2, just like in Maxwell's theory. This result also holds if we send N identical photons through the same input port (which corresponds to a N-photon multimode Fock state discussed before), the statistics of clicks is again the same as in Maxwell‘s theory.

The HOM dip experiment appears to constitute a very useful test in the context of quantum cryptography where the single photons used for encoding a quantum key must be indistinguishable. Indeed, if for instance the signal is encoded along the single photon polarisation, the spatial wave functions of the photons must be the same. If this was not the case, a malicious spy could extract some extra-information about the key by carrying out tomographic measures of the spatial wave functions.

VI. APPENDIX 2: PILOT WAVE INTERPRETATION FOR (SINGLE) PHOTONS: STILL UNSOLVED PROBLEMS CONCERNING SINGLE PHOTON "TRAJECTORIES".

Each time we can associate a conservation equation of the type $\frac{\partial P}{\partial t}+\boldsymbol{\nabla}\cdot\boldsymbol{j}=0$ to a given probability, the mean particle current

$$\boldsymbol{j}(\boldsymbol{x},t)=P(\boldsymbol{x},t)\dot{\boldsymbol{x}} \tag{12}$$

can be used to define the Bohm-de Broglie particle trajectories $\boldsymbol{x}$ via the velocity $\dot{\boldsymbol{x}}$ [11]. When one takes this idea on to special relativity, a problem arises regarding the extraction of photon trajectories. Despite the existence of a continuity equation for the electromagnetic energy,

$$\frac{\partial w}{\partial t}+\boldsymbol{\nabla}\cdot\boldsymbol{S}=0 \qquad \text{where} \quad w=\frac{\epsilon_0}{2}\left[\boldsymbol{E}^2+c^2\boldsymbol{B}^2\right] \quad \text{and} \quad \boldsymbol{S}=\frac{1}{\mu_0}\boldsymbol{E}\times\boldsymbol{B}, \tag{13}$$

that goes by the name of *Poynting's Theorem*, we cannot define a four-current j^μ that could account for the photon trajectories. One can rewrite (13) into

$$\partial_\mu T^{\mu 0}=0 \tag{14}$$

using the zeroth row (or column) of the stress-energy tensor $T^{\mu\nu}$ consisting of

$$\begin{cases} T^{00} &= \frac{\epsilon_0}{2}\left[\boldsymbol{E}^2+c^2\boldsymbol{B}^2\right], \\ T^{0j} &= \frac{1}{\mu_0}\left(\boldsymbol{E}\times\boldsymbol{B}\right)_j, \\ T^{ij} &= -\epsilon_0\left[E_iE_j+c^2B_iB_j-\frac{\delta_{ij}}{2}(\boldsymbol{E}^2+c^2\boldsymbol{B}^2)\right], \end{cases} \tag{15}$$

but since $T^{\mu 0}$ transforms like

$$T^{\mu 0}\rightarrow\left(T^{\mu 0}\right)'=\Lambda^\mu_\rho\,\Lambda^0_\sigma\,T^{\rho\sigma} \tag{16a}$$

under Lorentz transformations (with transformation matrix Λ^μ_ν) it is an unsuitable candidate for j^μ that should transform like

$$j^\mu\rightarrow(j^\mu)'=\Lambda^\mu_\nu\,j^\nu. \tag{16b}$$

Of course, this is a classical argument but it remains valid after quantisation.

We can indeed use (11) to build the quantised stress-energy tensor analogous to (15) by substituting $\boldsymbol{E}$ for $\boldsymbol{\psi}^{(E)}$ and $\boldsymbol{B}$ for $\boldsymbol{\psi}^{(B)}$. Problems occur however because the density and the current do not transform as a relativistic 4-vector.

For instance, the Poynting vector could be estimated to be equal to 0 for a certain observer, and also for an observer moving at constant velocity relatively to the first observer, which makes it impossible to interpret the Poynting vector in terms of "realistic" photon velocities as is exxplained in [12]:

> "... If we go on to situations in which the field does not correspond to a plane wave, we will not even get the same trajectories in different frames. For example, consider an electrostatic field E in the x-direction. The Poynting vector is zero, and this would therefore correspond to particles at rest. Let us consider a Lorentz transformation in the x-direction. Under this transformation E remains in this direction and there is no field H. Therefore the Poynting vector is still zero in the new frame. This would mean that the particle would have to be at rest in both frames which is clearly impossible. The reason for this result is, of course, that the energy momentum is a tensor and so cannot be regarded as describing a flux of particles..."

One could try other definitions of photon wave functions, like for instance the wave function obtained by "extracting" the effective potential vector instead of exctracting the effective electric field via

$$\begin{aligned}\boldsymbol{\psi}^A(\mathbf{x},t) &\equiv \langle 0|\,\hat{\mathbf{A}}(\mathbf{x},0)\mid\psi(t)\rangle \\ &= \sum_{\lambda=\pm}\int \mathrm{d}^3k\,\langle 0|\,\hat{\mathbf{A}}(\mathbf{x},0)\,c_\lambda(\mathbf{k},t=0)\,\mathrm{e}^{-\mathrm{i}c||\mathbf{k}||t}\,|\,1_{\lambda,\mathbf{k}}\rangle\end{aligned} \tag{17}$$

It is even possible [10] to add a zero component to this 3-vector (an effective electric potential) which has the merit to deliver a Lorentz covariant 4-vector, but one can show [10] that this 4-vector obeys D'Alembert equation for which the associated conserved density is not definite-positive. The same problem appears for instance in the case of massive bosons described by the Klein-Gordon equation. We must thus face the following dilemma [11] if we wish to associate velocities to the photon (and to bosons in general [13]):

-either the trajectories are not Lorentz covariant

-or they are not associated to a positive density.

In both cases no satisfactory realistic interpretation *à la* de Broglie-Bohm holds [9].

This difficulty maybe explains why many authors are reluctant to associate a wave function to the photon [14]. Another explanation of this *désamour* could be the Newton-Wigner theorem which makes it possible to associate a position operator to zero mass particles of spin 0 and 1/2, but not to spin 1 particles as the photon.

We consider however the N photon wave function as a useful tool, making it possible to develop a first quantization description of light, and in particular to provide a spatio-temporal description of the photon distribution, be it in configuration space whenever entanglement is present.

[1] Glauber, R. Coherent and incoherent states of the radiation field. *Physical Review* **131**, 2766 (1963).

[2] Scully, M. O. & Zubairy, M. S. *Quantum Optics* (Cambridge University Press, 1997).

[3] Hong, C. K., Ou, Z. Y. & Mandel, L. Measurement of subpicosecond time intervals between two photons by interference. *Phys. Rev. Lett.* **59** (1987).

[4] Titulaer, U. & Glauber, R. Density operators for coherent fields. *Phys. Rev.* **145** (1966).

[5] Smith, B. & Raymer, M. Photon wave functions, wave-packet quantization of light, and coherence theory. *New. J. Phys.* **9** (2007).

[6] Kocsis, S. *et al.* Observing the average trajectories of single photons in a two-slit interferometer. *Science* **332** (2011).

[7] Flack, R. & Hiley, B. Weak measurement and its experimental realisation. *J. Phys.: Conf. Ser.* **504** (2016).

[8] Bliokh, K., Bekshaev, A., Kofman, A. G. & Nori, F. Photon trajectories, anomalous velocities and weak measurements: a classical interpretation. *New J. Phys.* **15** (2013).

[9] Goessens, I. Decoherence and the quantum to classical transition: Do we need quantum jumps? master thesis v.u.b.-institut fresnel 2013.

[10] Debierre, V. *La fonction d'onde du photon en principe et en pratique*. Ph.D. thesis, Ecole Centrale Marseille (2015).

[11] Holland, P. R. *The Quantum Theory of Motion: An Account of the de Broglie-Bohm Causal Interpretation of Quantum Mechanics* (Cambridge University Press, 1993).

[12] Bohm, D., Hiley, B. J. & Kaloyerou, P. N. A causal interpretation of quantum fields. *Physics Reports* **6**, 349–375 (1987).

[13] Dirac equation is well-behaved regarding de Broglie-Bohm trajectories because the conserved density and the associated current form a relativistic 4-vector, while the conserved density is definite-positive.

[14] Kiessling, M.-K.-H. & A.-S.Tahvildar-Zadeh. On the quantum-mechanics of a single photon. *J. Math. Phys.* **59** (2018).