Quantum description of reality is empirically (operationally) incomplete

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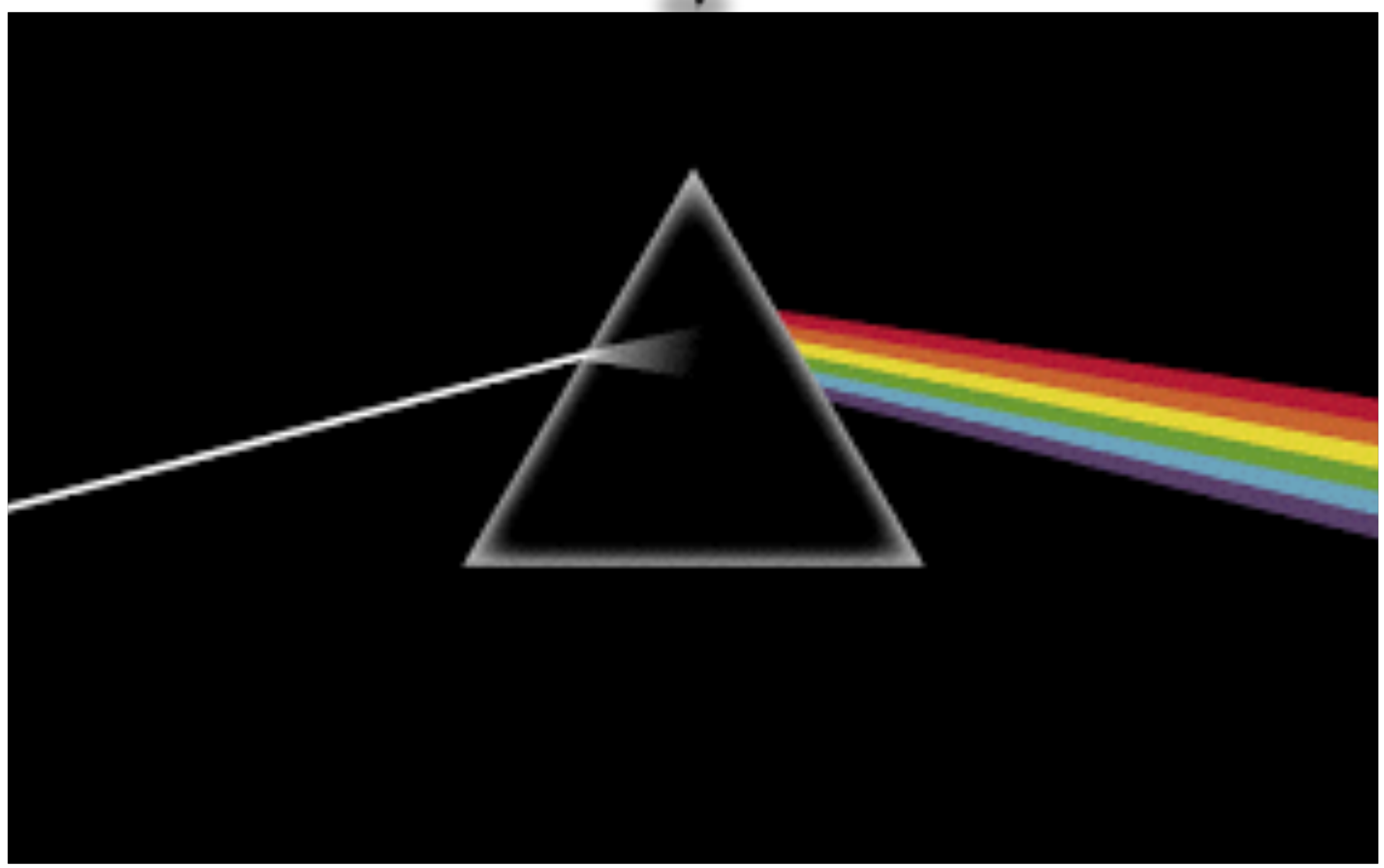


The insight: role of empirical falsifiability in realist notions of classicality

1. E*mpirically falsifiable* operational prerequisites of experimental tests

Realist notions of classicality ascribe certain operational phenomena a *not* fine-tuned realist basis.

These phenomena double as *empirically falsifiable* operational prerequisites for tests of the notions of classicality.

The insight: role of empirical falsifiability in realist notions of classicality

(Realist's) Notions of classicality	**Empirically falsifiable operational Phenomena**	**Not fine-tuned realist Basis**
Bell local causality	No signaling ($\mathcal{NS}$)	Parameter Indep. ($\mathcal{NS}_\Lambda$)
Generalized Non contextually	Operational equivalence $P_1 \equiv P_2$	Preparation noncontextuality $\mu_1(\lambda) = \mu_2(\lambda)$

The insight: role of empirical falsifiability in realist notions of classicality

2. Empirically falsifiable operational consequence

On the other hand, the realist notions of classicality yield empirically falsifiable operational consequences, typically in the form of statistical inequalities.

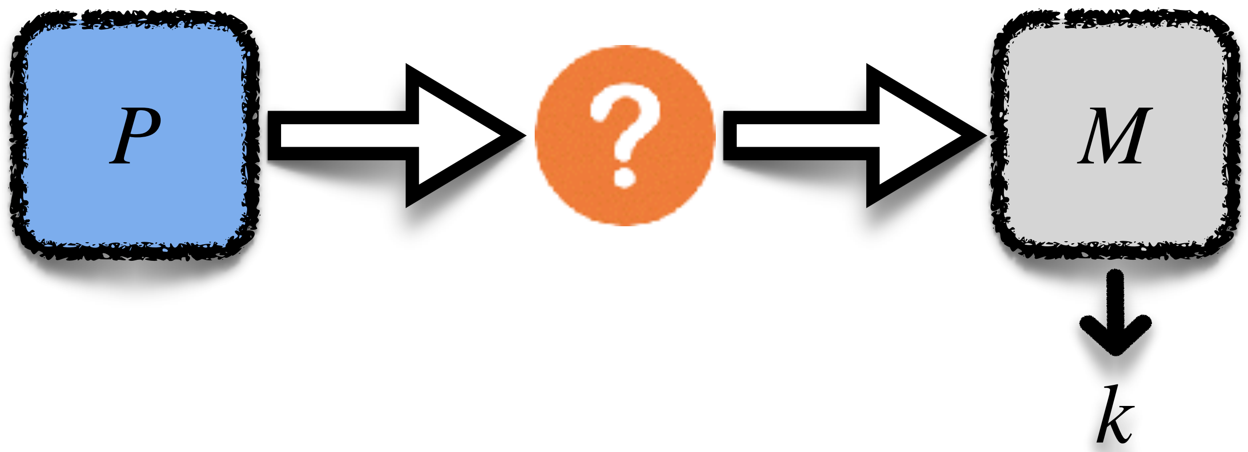
The quantum violation of these inequalities not only highlights the necessity of realist fine-tuning, discarding a large class of realist explanations, but also powers quantum advantage in a plethora of computational, communication and information processing tasks.

The insight: role of empirical falsifiability in realist notions of classicality

Empirically falsifiable phenomena feature as the operational prerequisites, as well as the operational consequences of the realist notions of classicality.

Operational interpretation of quantum theory

For the operationalist the real mystery, and the source of quantum advantage lies in the statistics obtained from the experiment, and not in the details of the particular experimental implementation,



As a consequence of the experiment we obtain outcome statistics in the form of conditional probability distributions:

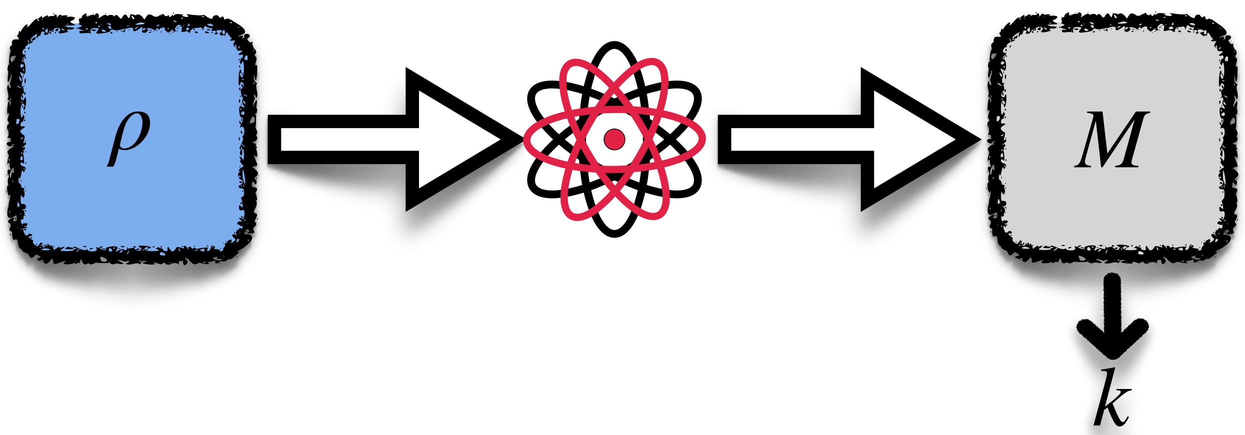
$$\{p(k \,|\, P, M)\},$$

where $P \in \mathscr{P}_{\mathcal{O}}$ is an operational preparation and $M \in \mathscr{M}_{\mathcal{O}}$ is an operational measurement

Operational interpretation of quantum theory

Quantum theory serves a two-fold purpose:

- ***Prescriptions***: It attributes a density operator $\rho \geq 0$ to each preparation such that $tr(\rho) = 1$, and a POVM element to each measurement effect $M_k \geq 0$ such that $\sum_k M_k = \mathbb{I}$:

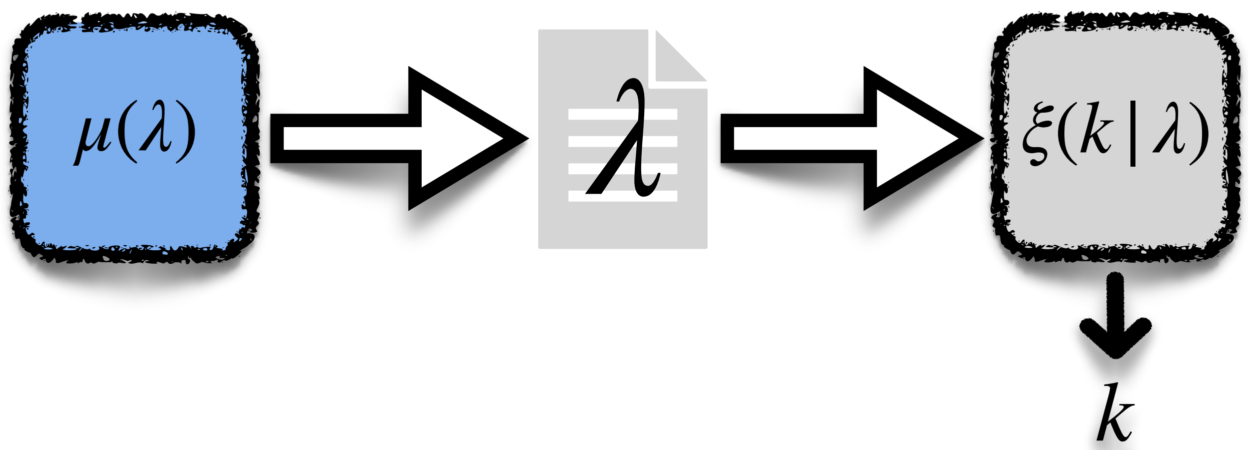


- ***Prediction***: The Born-rule yields the desired conditional probabilities:

$$\{p(k \,|\, P, M) = tr(\rho M_k)\}$$

Realist interpretation of operational theories

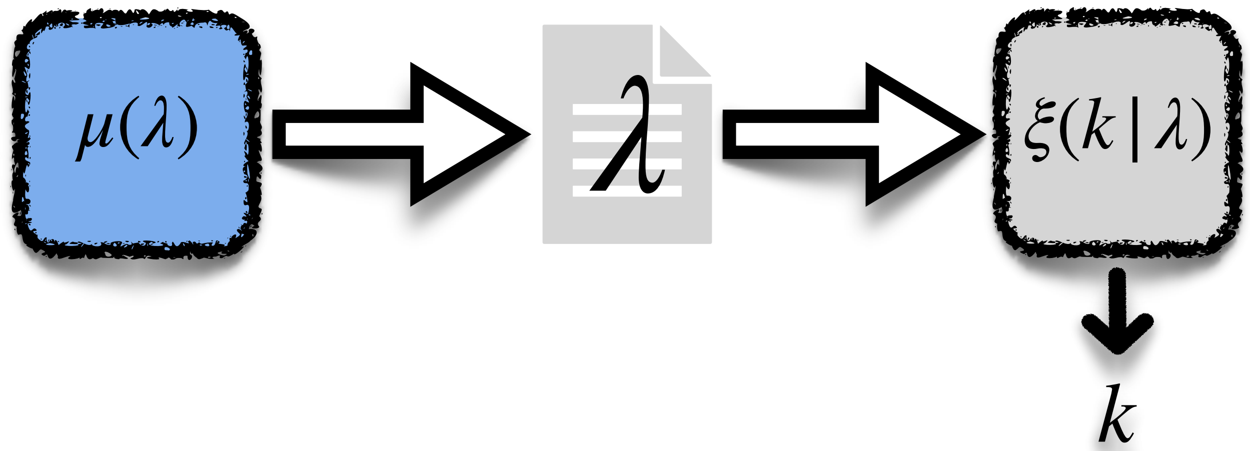
Realists attribute a hidden variable (optic-state) $\lambda \in \Lambda$ to each instance of a physical system, where Λ (antic-state space) is a measurable space



Even the realist can prescribe and predict:

- ***Prescriptions***: A realist theory attributes a probability distribution $\mu(\lambda)$ to each preparation such that $\int_\Lambda \mu(\lambda) d\lambda = 1$, and a response scheme to each measurement effect $\{\xi(k \mid \lambda)\}$ such that $\forall \lambda, k : \xi(k \mid \lambda) \geq 0$ and $\sum_k \xi(k \mid \lambda) = 1$

Realist interpretation of operational theories

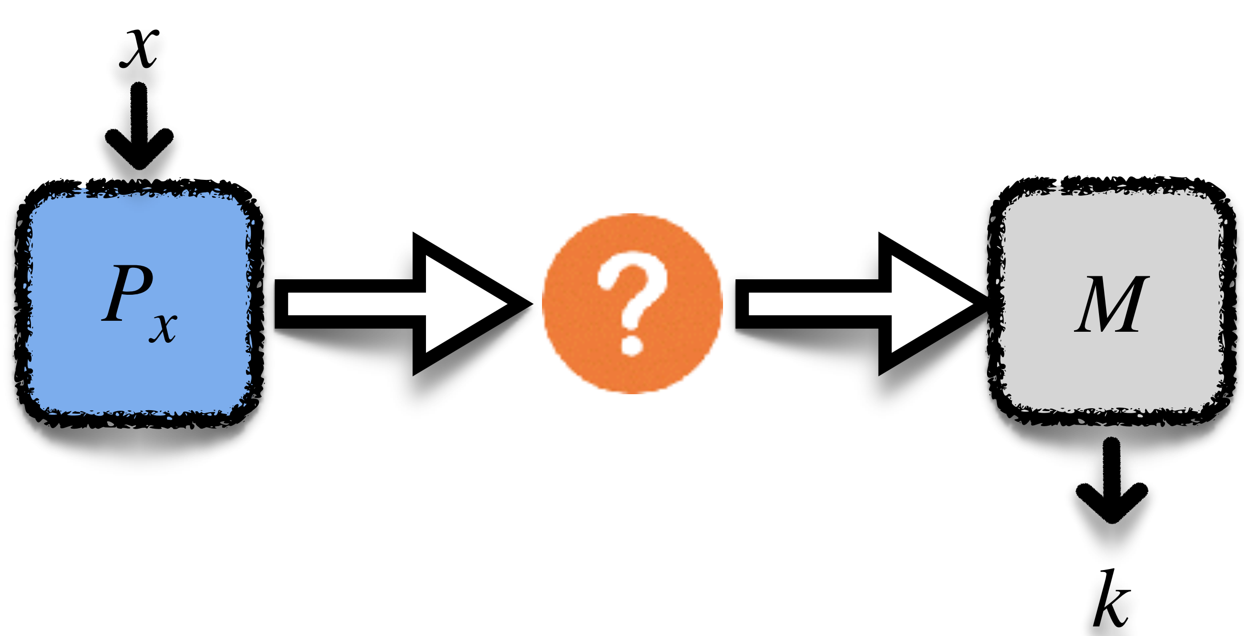


- ***Predictions***: The desired statistics are computed by averaging the response function over our ignorance of the underlying antic-state λ:

$$\{p(k\,|\,P, M) = \int_{\Lambda} d\lambda \mu(\lambda) \xi(k\,|\,\lambda)\}$$

Operational properties of preparations

Consider a set of preparations $\overrightarrow{P} \equiv \{P_x\}$,



We define a generic operational property for sets and subsets of the preparations as the maximal value of a success metric associated with a one-way communication task:

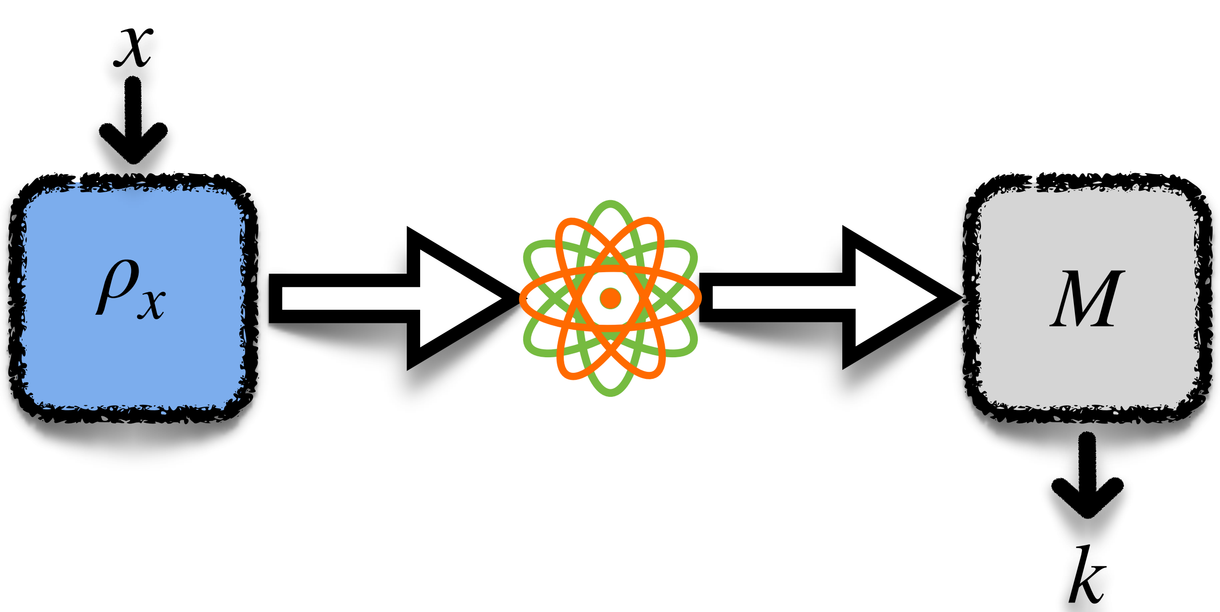
$$S^{(\mathcal{O})}(\overrightarrow{P}) = \max_{M \in \mathcal{M}_{\mathcal{O}}} \{ \sum_{x,k} c_k^x p(k \,|\, P_x, M) \}$$

Empirical falsifiability of operational properties

For any set of preparations $\overrightarrow{P} \equiv \{P_x \in \mathscr{P}_{\mathscr{O}}\}$ the properties of the form $S^{(\mathscr{O})}(\overrightarrow{P})$ constitute empirically falsifiable properties, as if one can experimentally falsify the operational theory or its prescriptions by attaining a higher value of the success metric $S(\mathbf{P})$.

Operational properties of quantum preparations

For a set of quantum preparations $\overrightarrow{\rho} = \{\rho_x\}$,

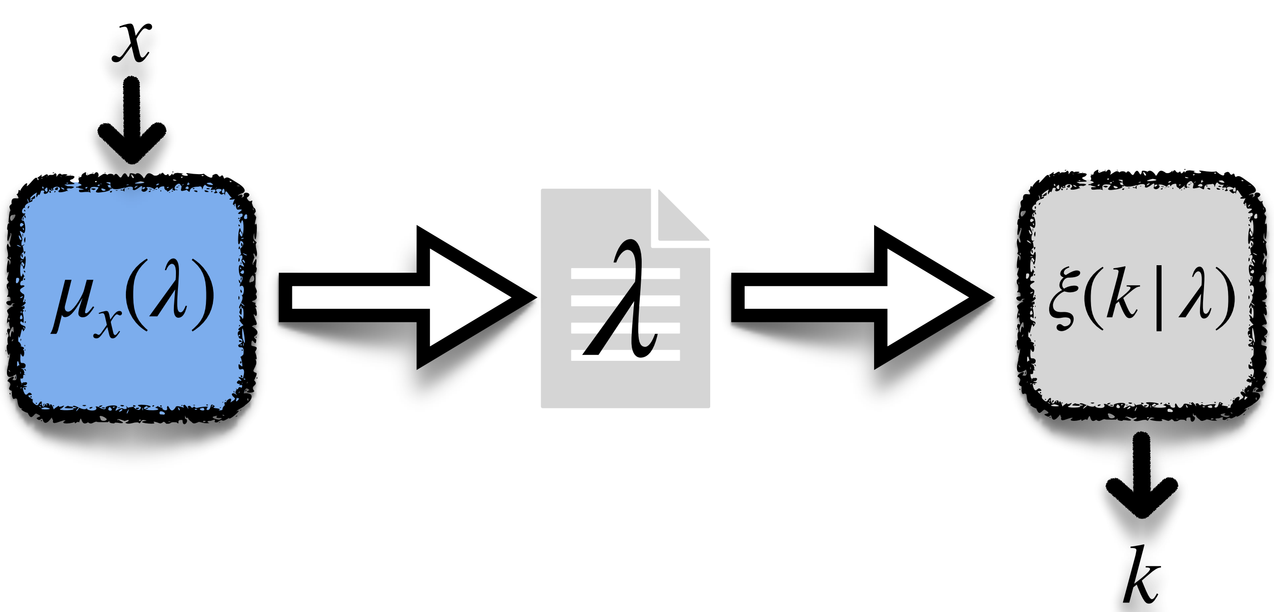


Finding the maximal value of a success metric associated with a one-way communication task constitutes a semi-definite program:

$$S^{(\mathcal{Q})}(\overrightarrow{\rho}) = \max_{M \in \mathcal{M}_{\mathcal{Q}}} \{ \sum_{x,k} c_k^x tr(\rho_x M_k) \}$$

(Not fine-tuned) realist properties

For a set of realist preparations $\overrightarrow{\mu} \equiv \{\mu_x(\lambda)\}$,



Finding the maximal value of a success metric associated with a one-way communication task constitutes a linear program:

$$S^{(\Lambda)}(\overrightarrow{\mu}) = \max_{\{\xi(k|\lambda)\}} \{ \sum_{x,k} c_k^x \int_\Lambda d\lambda \mu_x(\lambda) \xi(k\,|\,\lambda) \}$$

(Not fine-tuned) realist properties

As the set of response schemes constrained by only by positivity and completeness forms a convex polytope with deterministic response functions as extremal points, we can solve the maximization by picking the response functions that for each ontic-state λ, yield the outcome k which maximises the function $\sum_x c_k^x \mu_x(\lambda)$ such that,

$$S^{(\Lambda)}(\overrightarrow{\mu}) = \int_\Lambda d\lambda \max_k \{ \sum_x c_k^x \mu_x(\lambda) \}$$

This expression further substantiates the fact that the maximization over response schemes relieves $S^{(\Lambda)}(\overrightarrow{\mu})$ from its dependence on response schemes, deeming it an exclusive property of the set of epistemic states $\overrightarrow{\mu} \equiv \{\mu_x\}_{x=1}^n$.

Completeness of operational theories

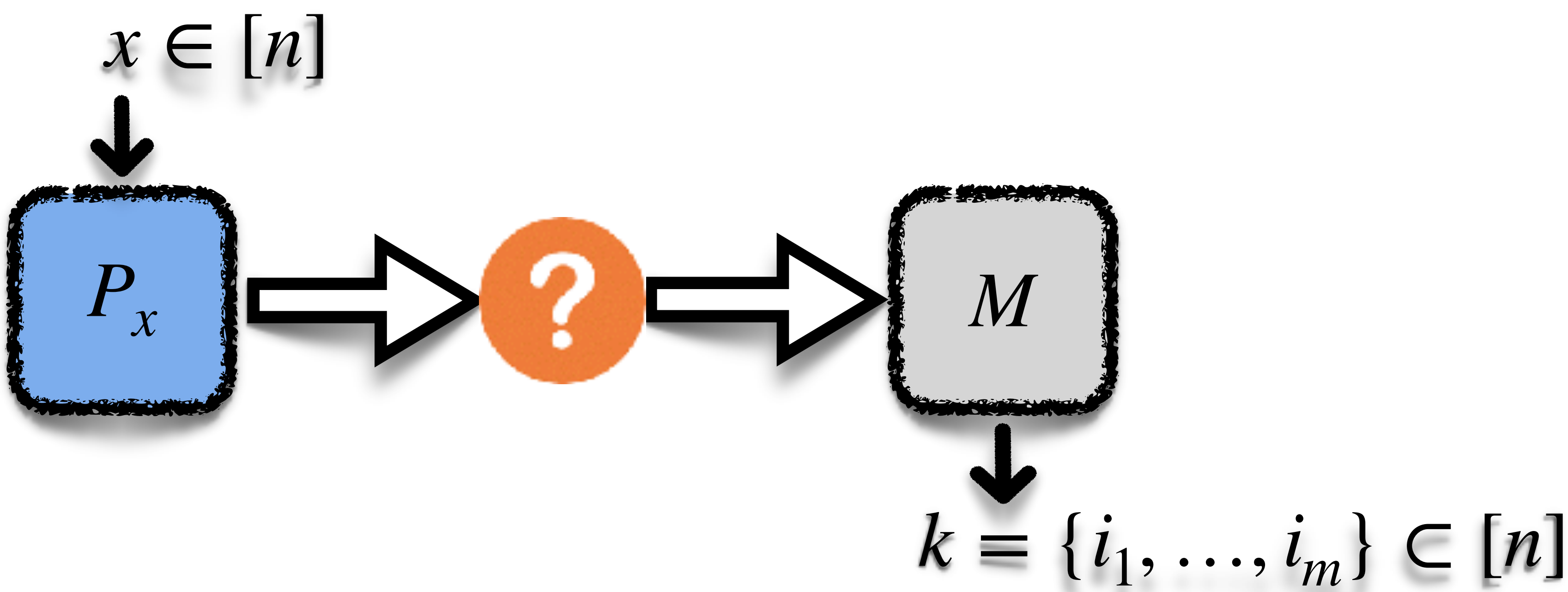
Empirically complete theories

An operational theory or a fragment thereof is said to be empirically complete if for all sets of preparations $\overrightarrow{P} \equiv \{P_x \in \mathscr{P}_{\mathscr{O}}\}$, and all associated empirically falsifiable operational properties $S^{(\mathscr{O})}(\overrightarrow{P})$, there exists underlying sets of epistemic states $\overrightarrow{u} \equiv \{\mu_x\}$ with *not* fine-tuned realist properties $S^{(\Lambda)}(\overrightarrow{\mu})$such that,

$$S^{(\Lambda)}(\overrightarrow{\mu}) = S^{(\mathscr{O})}(\overrightarrow{P})$$

- **This is a generalisation of Bounded Ontological Distinctness (BOD) introduced in** *Quantum 4, 345 (2020)*
- **Also a generalisation of the no-fine tuning principle**

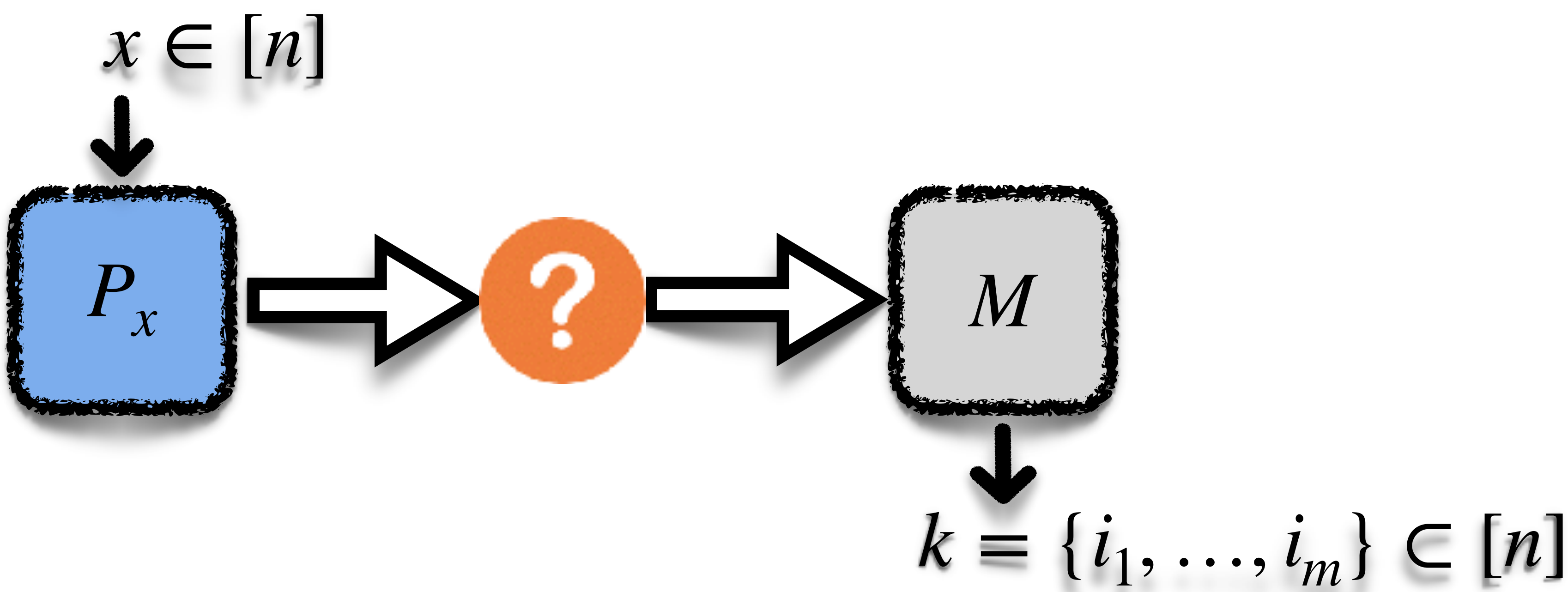
Set distinguishability



Maximum success probability of correctly guessing which non-trivial m-member subset a given preparation P_x belongs to.

$$\mathcal{D}^{(\mathcal{O})}_{n,m}(\overrightarrow{P}) = \frac{1}{n} \max_{M} \sum_{i_1<\ldots<i_m} \sum_{x\in\{i_1,\ldots,i_m\}} \{p(k = \{i_1, \ldots, i_m\} \mid P_x, M)\}$$

Set distinguishability



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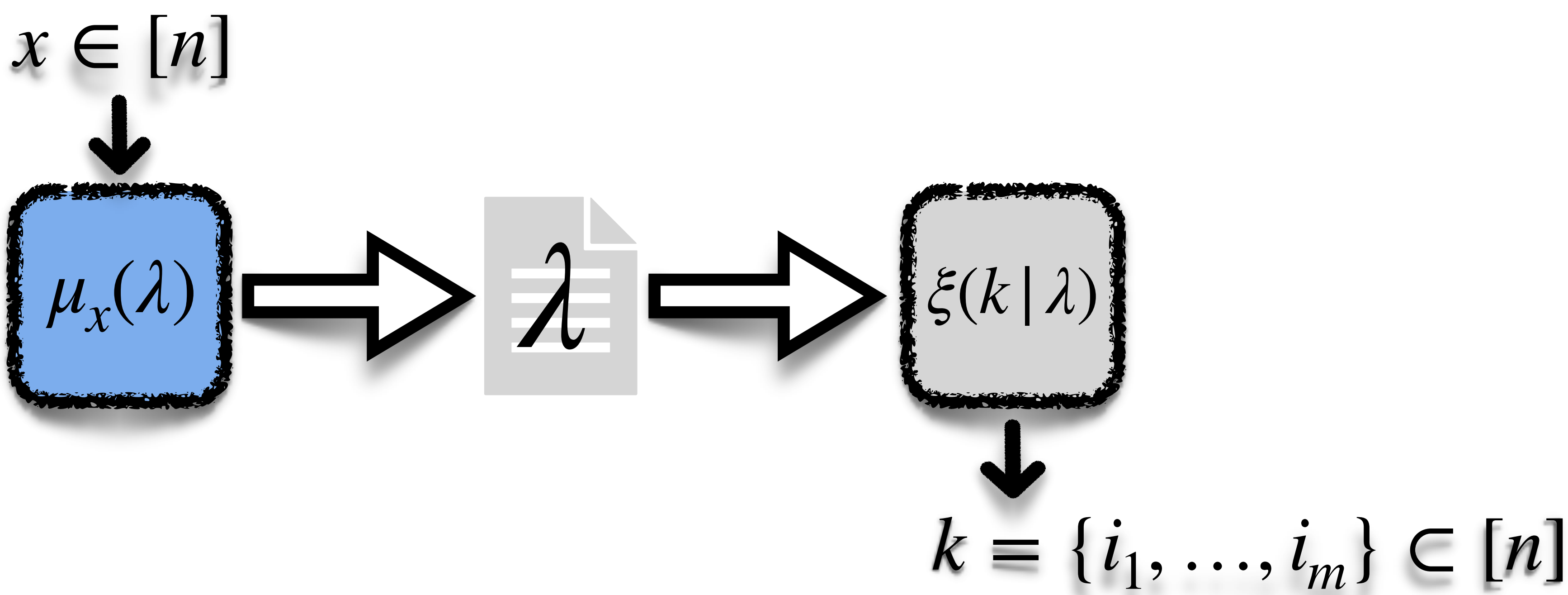
Anti-Distinctness of three epistemic states



$$\not{D}^{(\Lambda)}_{n,m} = \frac{1}{n} \int_\Lambda d\lambda \max_{i_1 < \ldots < i_m \in [n]} \sum_{x \in \{i_1, \ldots, i_m\}} \{\mu_x(\lambda)\}$$

Average set distinguishability

The average maximum success probability of correctly guessing which non-trivial m-member subset a given preparation P_x belongs to,

$$\bar{\not{D}}_n^{(\mathcal{O})}(\overrightarrow{P}) = \frac{1}{n-1}\sum_{m=1}^{n-1} \not{D}_{n,m}^{(\mathcal{O})}(\overrightarrow{P}),$$

Average pairwise distinguishability

The average of maximum success probability of perfectly distinguishing distinct pairs of preparations $\{P_i, P_j \in \{P_x\}\}$,

$$\bar{D}_n^{(\mathcal{O})}(\overrightarrow{P}) = \frac{1}{\binom{n}{2}} \sum_{i<j} \not{D}_{2,1}^{(\mathcal{O})}(\{P_i, P_j\}),$$

where,

$$\not{D}_{2,1}^{(\mathcal{O})}(\{P_i, P_j\}) = \frac{1}{2} \max_{M \in \mathscr{M}_{\mathcal{O}}} \{ \sum_{x \in \{i,j\}} p(k = x \,|\, P_x, M)\}$$

Theorem: The Equalities

Theorem: For any *empirically complete* theory, for any given set of n preparations $\overrightarrow{P} \equiv \{P_x\}_{x=1}^{n}$, the average set distinguishability is exactly equal to average pair-wise distinguishability, i.e.,

$$\bar{D}_n^{(\mathcal{O})}(\overrightarrow{P}) = \bar{\not{D}}_n^{(\mathcal{O})}(\overrightarrow{P})\,.$$

Proof: From “a first course in probabilities”

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For any three real number $\{\mu_x \in \mathbb{R}\}_{x=1}^{3}$, the following identity holds:

$$\sum_{i<j\in[3]} \max_{x\in\{i,j\}} \{\mu_x\} = \max_x\{\mu_x\} + \max_{i<j\in[3]} \{\mu_i + \mu_j\}$$

Proof: Consider the associated ordered list $\{a, b, c \in \mathbb{R}\}$

such that $a \geq b \geq c$, then,

$$\max_x\{\mu_x\} + \max_{i<j\in[3]} \{\mu_i + \mu_j\} = 2a + b$$

$$\sum_{i<j\in[3]} \max_{x\in\{i,j\}} \{\mu_x\} = 2a + b$$

Proof: From “a first course in probabilities”

ISBN-13: 978-0-13-603313-4

For any three real measures $\overrightarrow{\mu} \equiv \{\mu_x(\lambda)\}_{x=1}^{3}$, as $\forall x \in [3], \forall \lambda \in \Lambda : \mu_x(\lambda) \in \mathbb{R}$, the following identity holds:

$$\sum_{i<j\in[3]} \max_{x\in\{i,j\}} \{\mu_x(\lambda)\} = \max_x \{\mu_x(\lambda)\} + \max_{i<j\in[3]} \{\mu_i(\lambda) + \mu_j(\lambda)\}$$

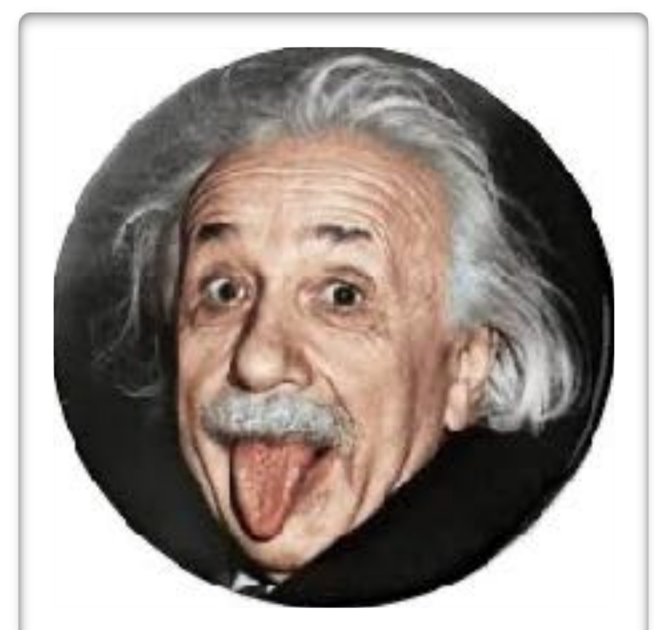
Summing of over λ, we obtain for any three general (**possibly negative**) realist preparations $\{\mu_x(\lambda) \in \mathbb{R}\}_{x=1}^{3}$,

$$\frac{1}{6}\int_\Lambda d\lambda \sum_{i<j\in[3]} \max_{x\in\{i,j\}} \{\mu_x(\lambda)\} = \frac{1}{6}\int_\Lambda d\lambda \max_x \{\mu_x(\lambda)\} + \frac{1}{6}\int_\Lambda d\lambda \max_{i<j\in[3]} \{\mu_i(\lambda) + \mu_j(\lambda)\}$$

$$\bar{D}_3^{(\Lambda)}(\overrightarrow{\mu}) = \sum_{i<j} \not{D}_{2,1}^{(\Lambda)}(\{\mu_i, \mu_j\}) = \frac{1}{2}(\not{D}_{3,2}^{(\Lambda)}(\overrightarrow{\mu}) + \not{D}_{3,1}^{(\Lambda)}(\overrightarrow{\mu})) = \bar{\not{D}}_3^{(\Lambda)}(\overrightarrow{\mu})$$

Theorem: Quantum-Mechanical Description of Reality is Empirically Incomplete

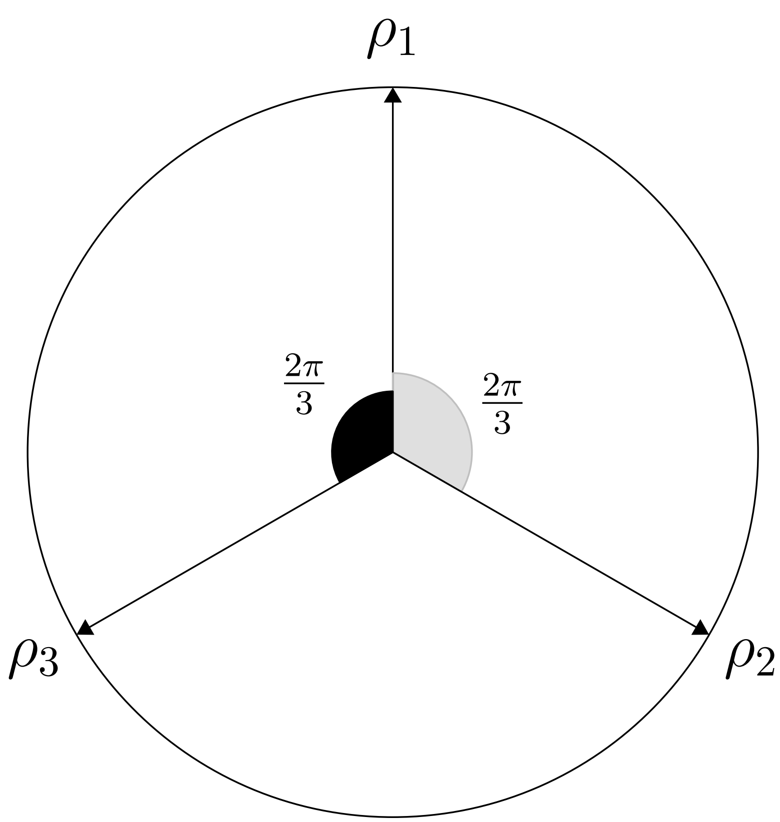
Theorem: Quantum theory prescribes sets of preparations for which there exists no realist interpretation such that $S_{\mathcal{Q}} = S_{\Lambda}$ for all operational properties S



Proof: Incompleteness of Quantum Description of Reality

Theorem: Quantum theory prescribes sets of preparations for which there exists no realist interpretation such that $S_{\mathbb{Q}} = S_{\Lambda}$ for all operational properties S

Consider the following set of three qubit states:



$\overrightarrow{\rho}_{\triangle} \equiv \{\rho_x\}$

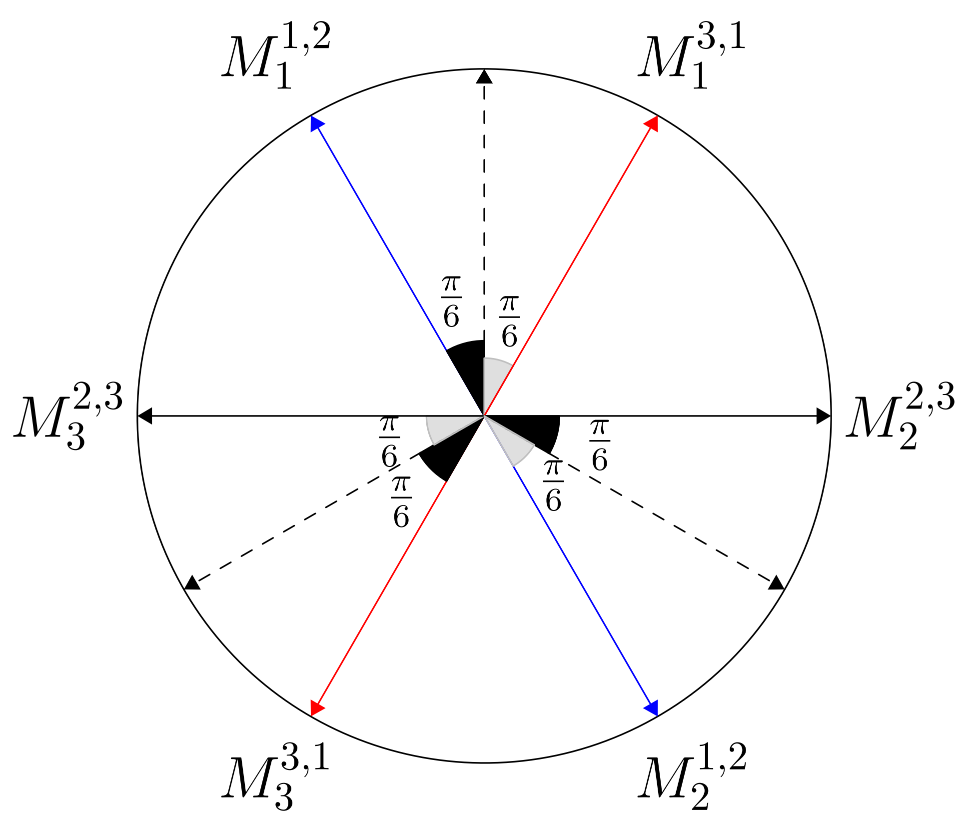
Mercedes-Benz

tool

Proof: Incompleteness of Quantum Description of Reality

The total pair-wise distinguishability of these states is,

$$\bar{D}_3^{(\mathcal{Q})}(\overrightarrow{\rho}_\triangle) = \frac{1}{2}\left(1 + \frac{\sqrt{3}}{2}\right) \approx 0.933$$

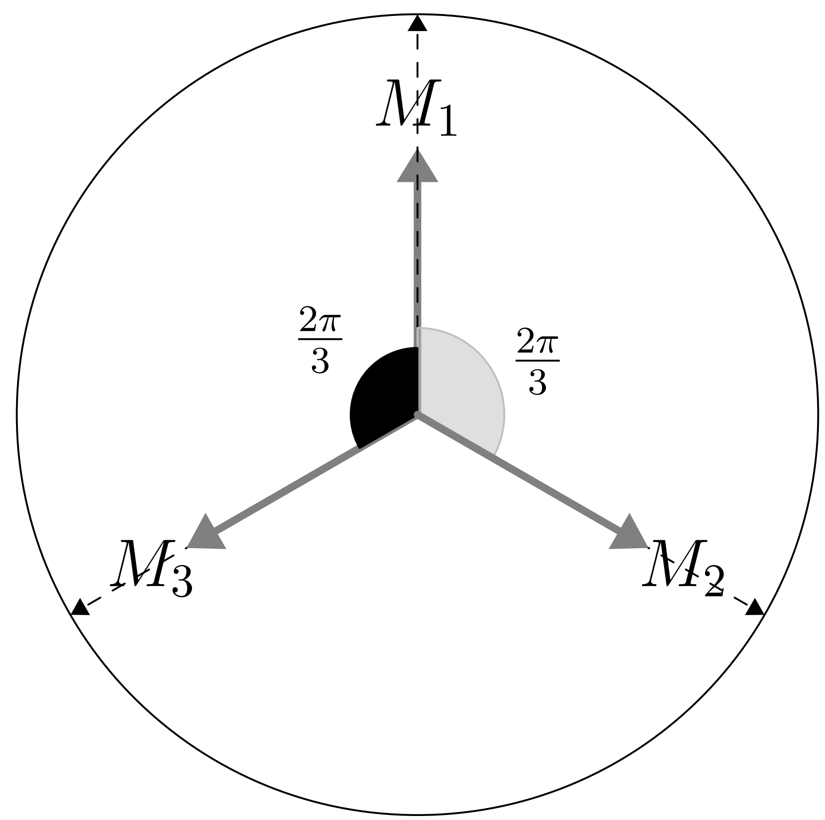


$\{M_k^{1,2}\}, \{M_k^{2,3}\}, \{M_k^{3,1}\}$

Proof: Incompleteness of Quantum Description of Reality

The distinguishability of these states is,

$$\cancel{D}_{3,1}^{(\mathcal{Q})}(\overrightarrow{\rho}_{\triangle}) = \frac{1}{3}\max_{M}\{\sum_{x} tr(\rho_x M_{k=x})\} = \frac{2}{3}$$

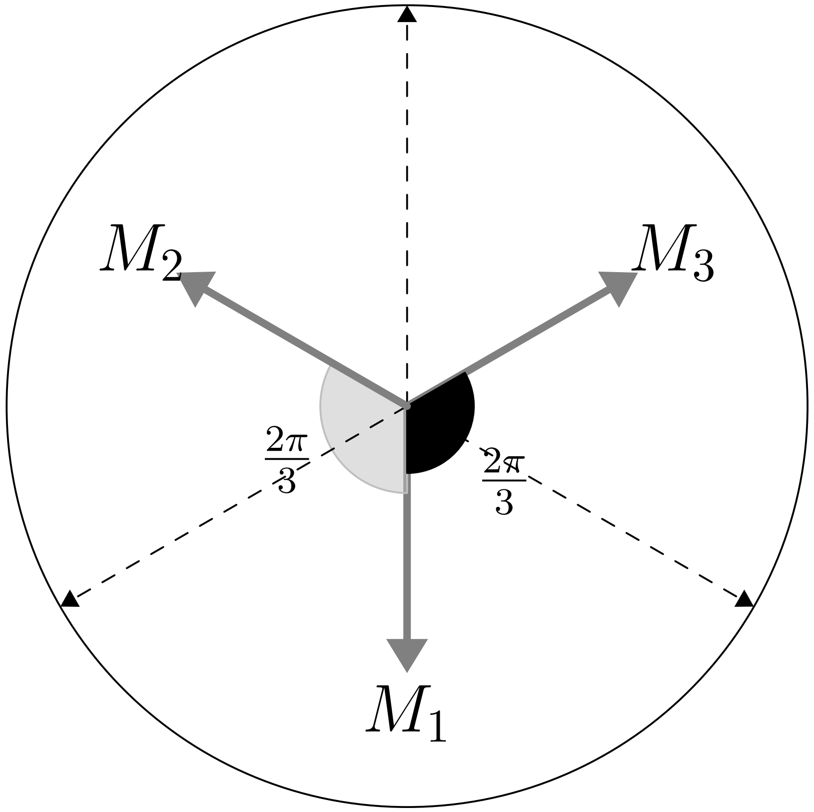


$\{M_k\}$

Proof: Incompleteness of Quantum Description of Reality

These states are completely anti-distinguishable, i.e.,

$$\not{D}^{(\mathcal{Q})}_{3,2}(\overrightarrow{\rho}_{\triangle}) = 1$$



$\{M_k\}$

Proof: Incompleteness of Quantum Description of Reality

Lemma: For any ***complete*** operational theory, and any three given preparations $\{P_x\}_{x=1}^{3}$,

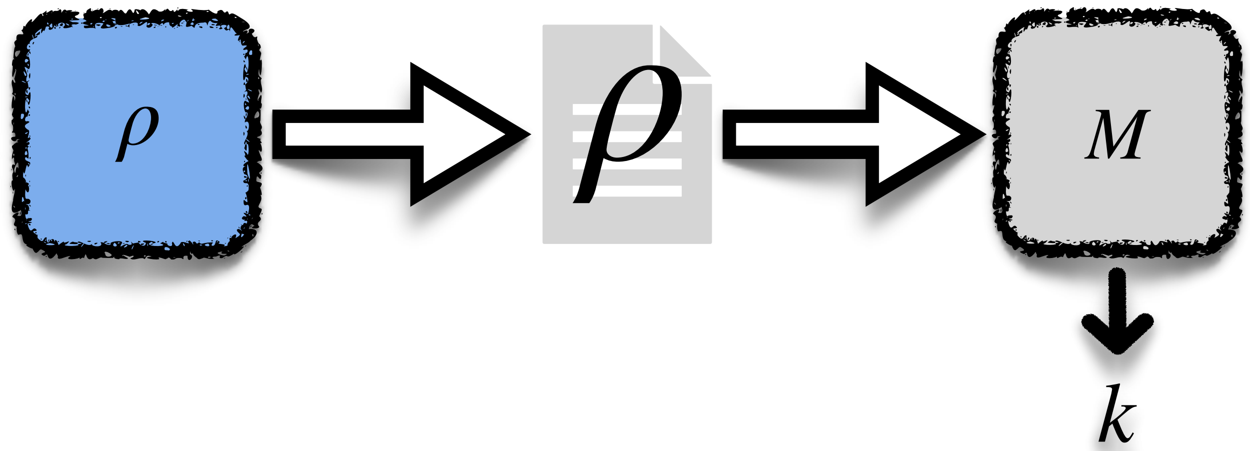
$$\overline{D}_{\mathcal{O}} = D_{\mathcal{O}} + \cancel{D}_{\mathcal{O}}$$

But we have,

$$\Delta_3^{(\mathcal{Q})}(\overrightarrow{\rho}_{\triangle}) = \frac{3\sqrt{3}-4}{12} \approx 0.0997$$

(A measure of quantum theory's incompleteness with respect to a given set of quantum preparations)

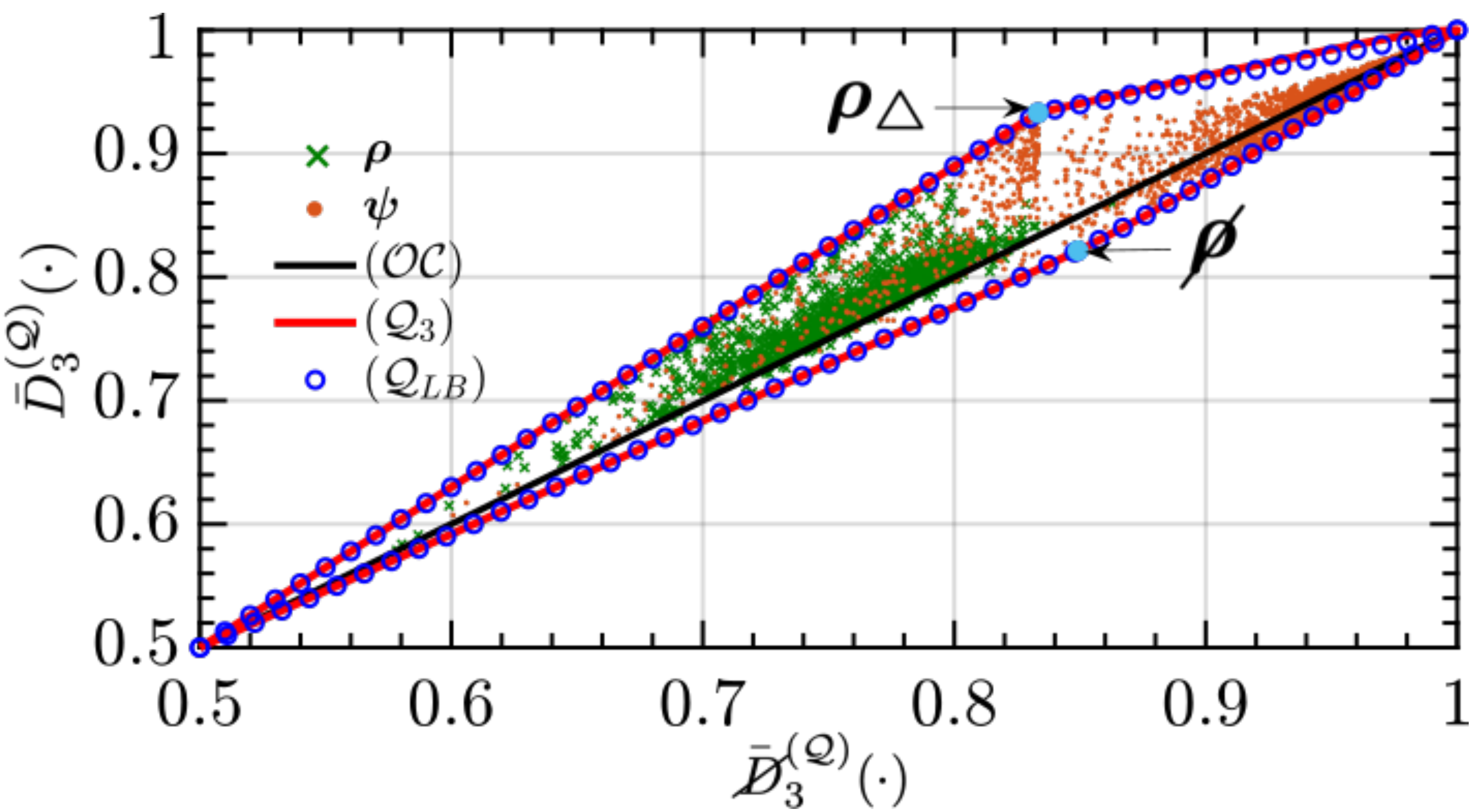
Implication: Discarding even last only escape back to classicality



- Consider a non outcome-independent, non-convex classical (realist) model of quantum theory, such a ρ-ontic model is ``local”, non-contextual (Leibnitzian). It is how our machines store quantum stuff
- However, **any** realist interpretation of quantum theory must be **incomplete**, i.e., violate BOD, as we made no assumptions about convexity, or composite systems we have,

No way back to classicality

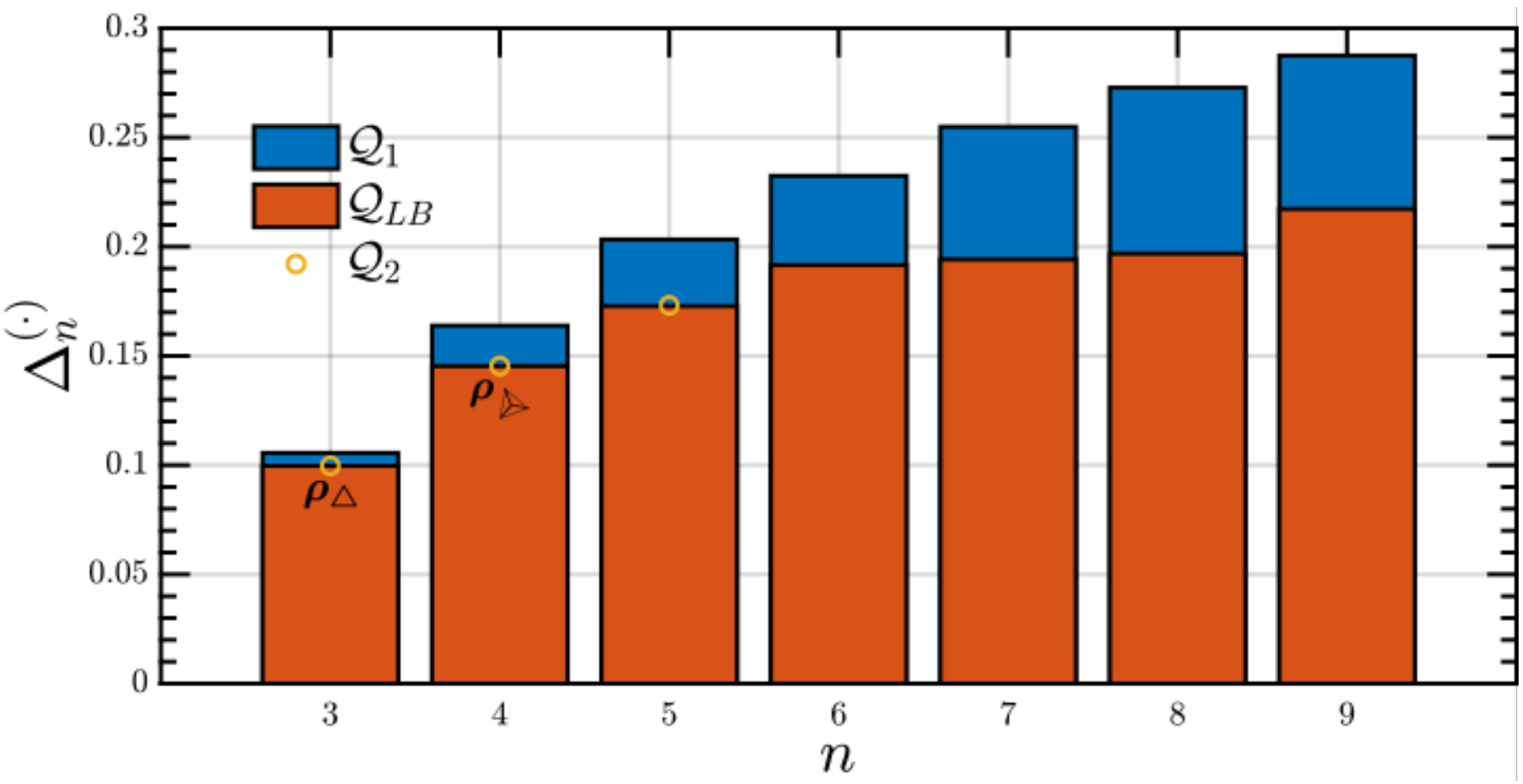
Characterising quantum deviation from the three preparation equality



All (Haar uniformly) randomly sampled triplets of pure states and density operators deviate from the equalities

Any deviation from the equality implies quantum advantage in suitable constrained communication task

Increasing quantum deviation



Summary

We introduced a novel notion of classicality,

- Defined in this way, empirical completeness underlies other well-known notions of classicality.

  As the distinguishability of a set of preparations forms an empirically falsifiable operational property, empirical completeness directly implies (symmetric) maximal ψ-epistemicity, bounded ontological distinctness of preparations, and preparation noncontextuality. While these implications follow directly from the definition of these notions, equipped with quantum theory dependent assumptions, empirical completeness can also be shown to imply generalized noncotextuality, Kochen-Specker noncontextuality and Bell local-causality.

- Other notions of classicality have zero measure operational pre-requisites and stochastic inequality as empirically falsifiable operational consequences

  However, empirical completeness has robust operational pre-requisite and zero measure empirically falsifiable operational consequences, which makes for easy and abundant quantum violation

Thank you

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Marcin (co-author)

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