

Thematic school
*From the first to the second
quantum revolution*

Applications of
non-locality

Djeylan Aktas

Senior PDRA at Quantum Engineering & Technologies Labs

20 sept 2021, Peyresq

Photonics is all around us

Photonics is all around us

Photonics is about light:

Photonics is all around us

Laser sources

Light Peak



Photonics is about light:

- Generation

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Distribution

Fiber optics & comp.



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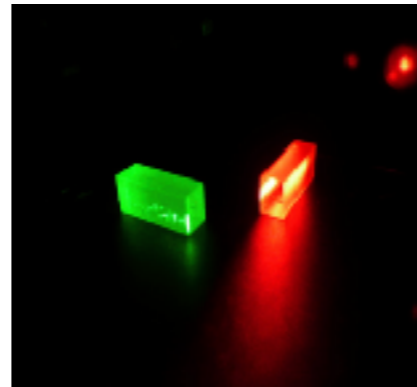
Distribution

Fiber optics & comp.



NL optics

Photonic crystals



DWDM
Demultiplexer

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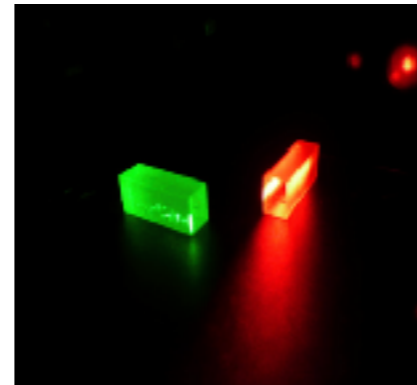
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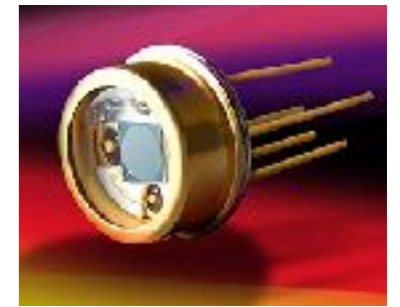
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Detection

Semiconductors



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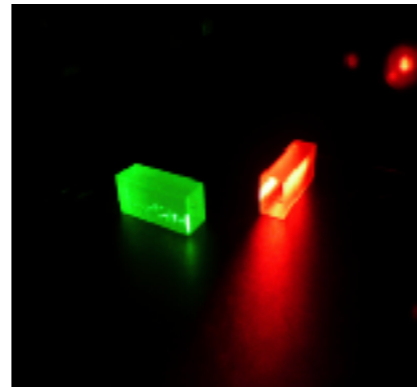
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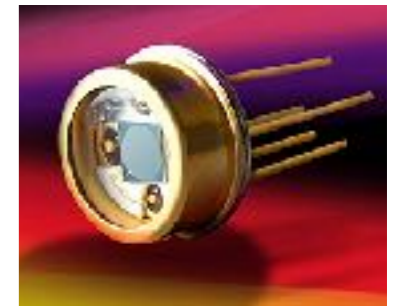
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Telecommunication networks



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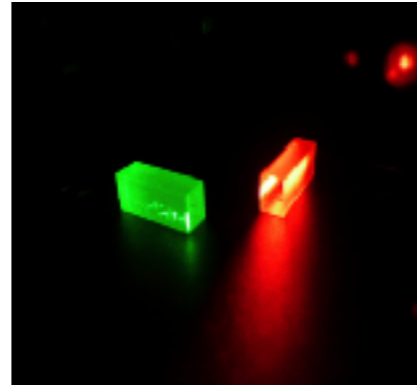
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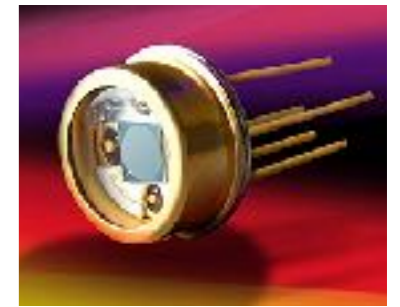
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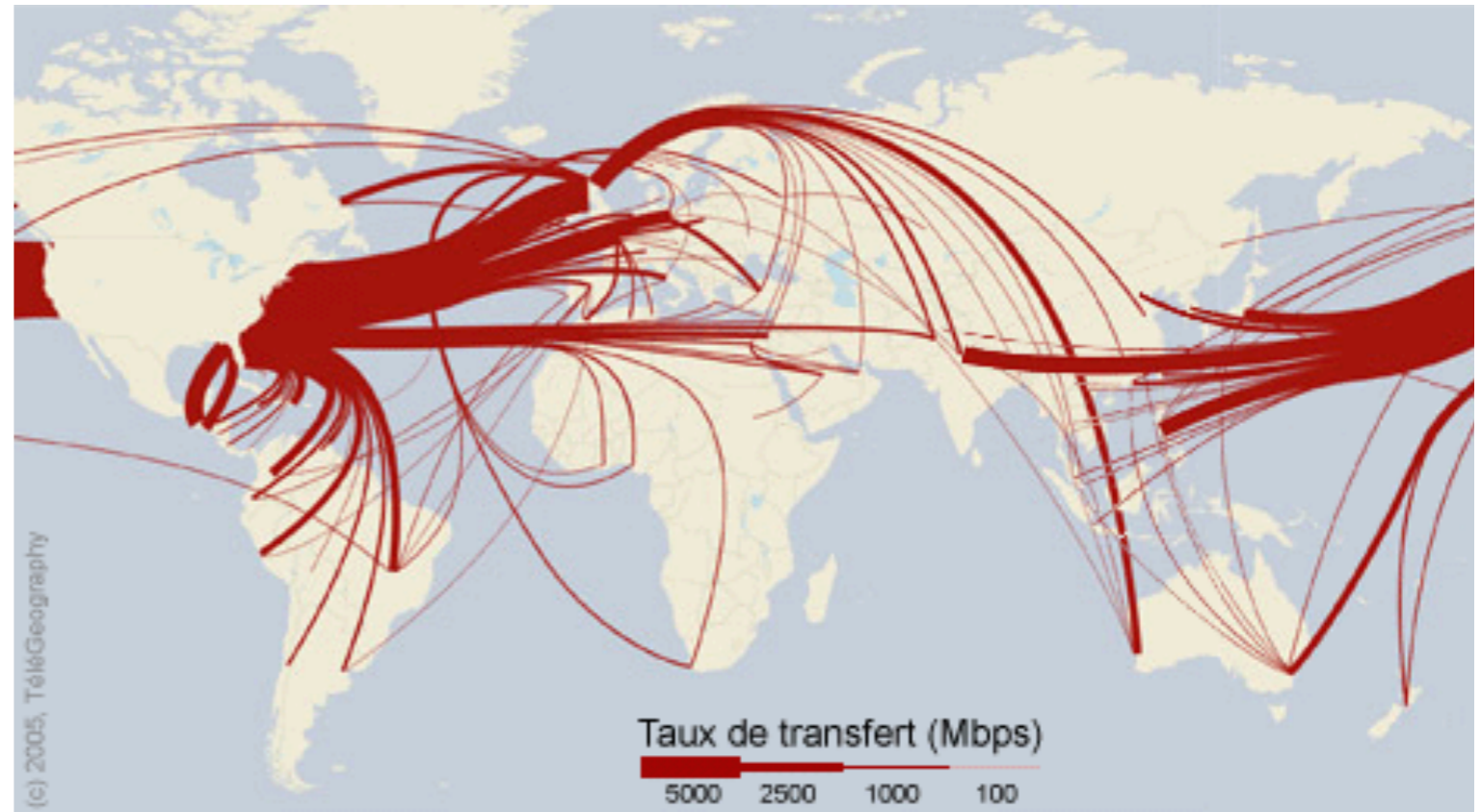
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Photonics is about light:

- Generation
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What about quantum photonics ?

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Quantum photonics is also about:

Generation, distribution, manipulation and detection of light.

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Q-communication



More secure

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Faster computation

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Q-metrology



Better accuracy

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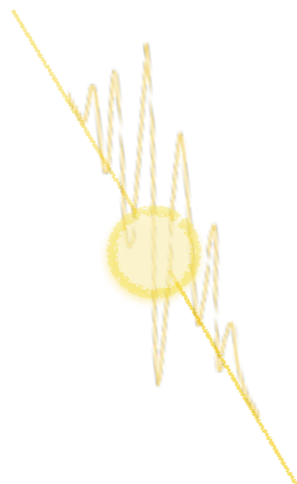
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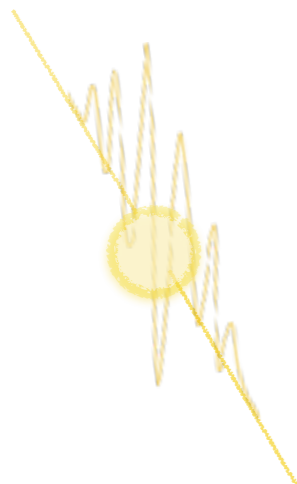
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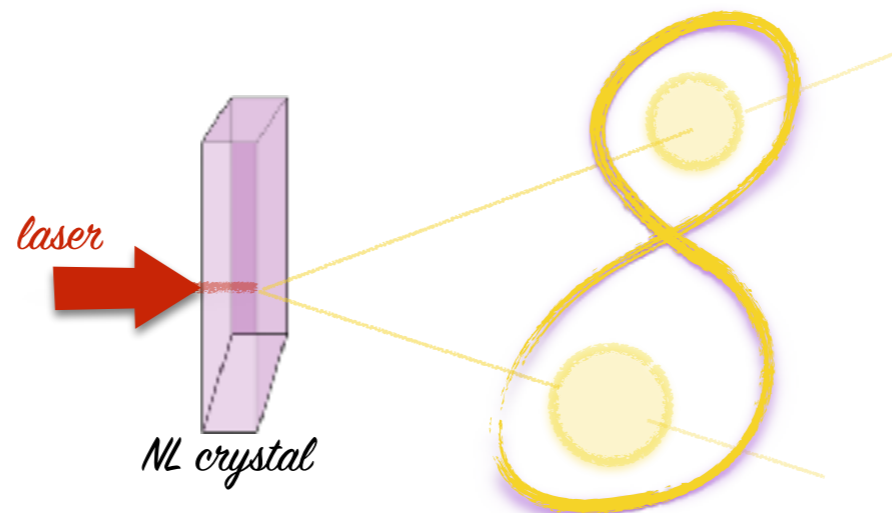


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Twin photons



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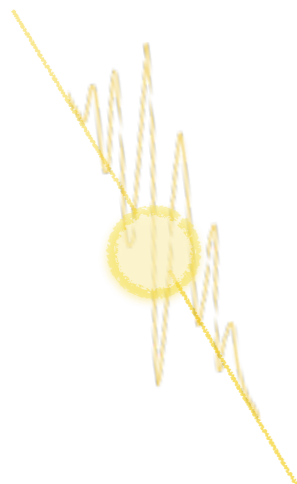
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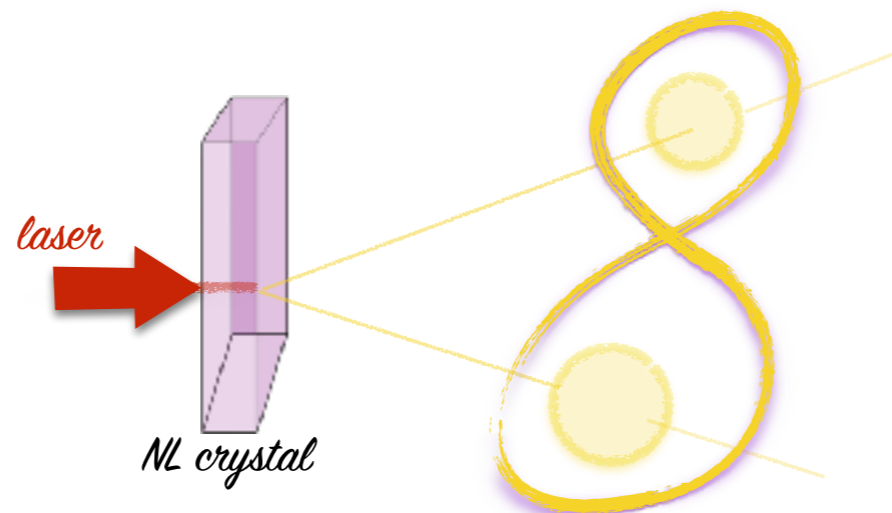


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and more ...

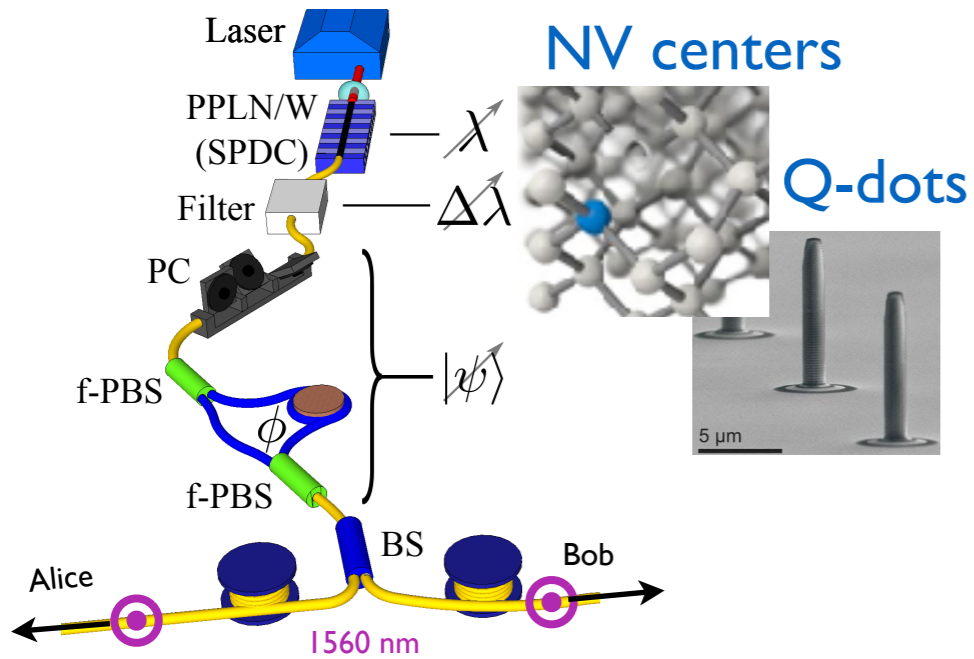
Entanglement & quantum applications

Entanglement

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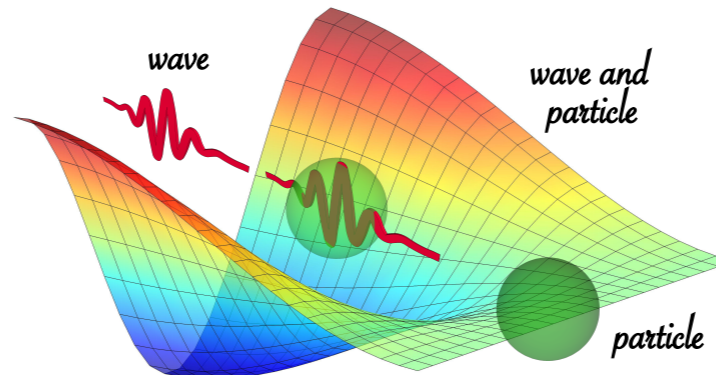
Terminal stations

Parametric sources



Fundamental studies

Non locality



Secured channels

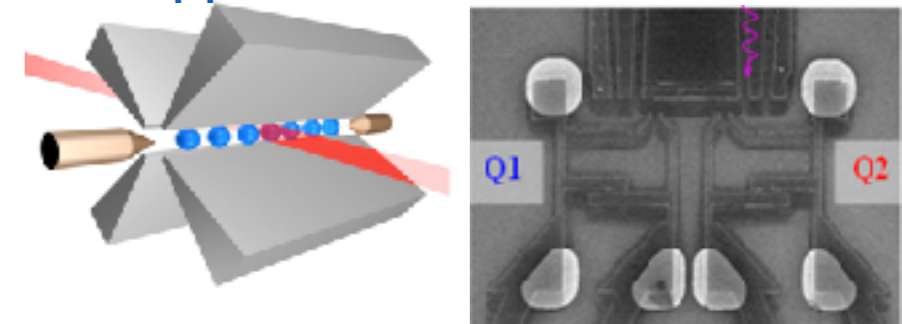
Q-key distribution



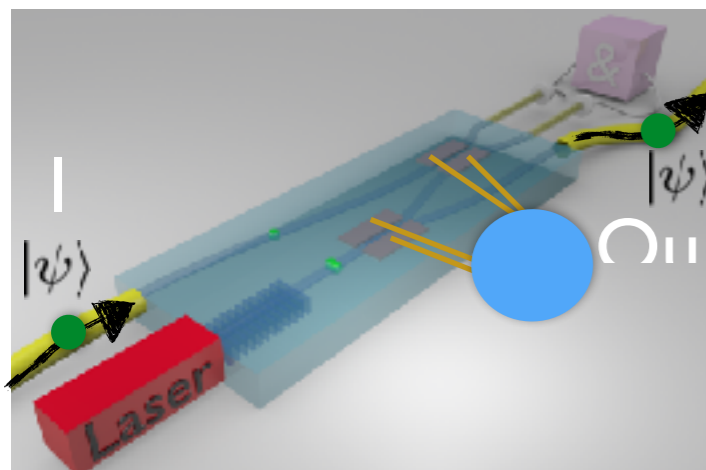
Entanglement

Processing

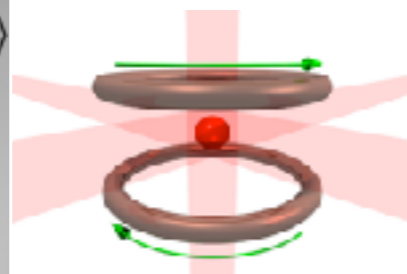
Trapped ions Supercond. circuits



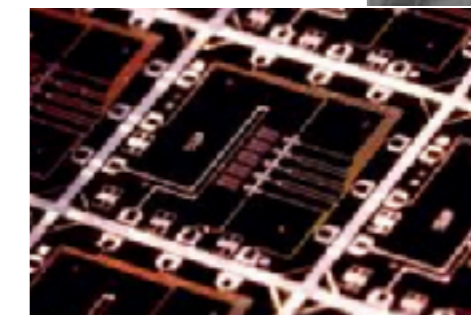
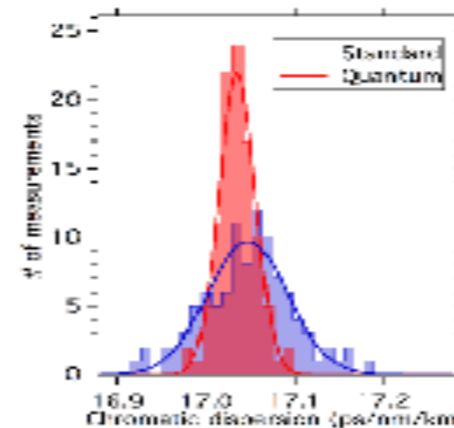
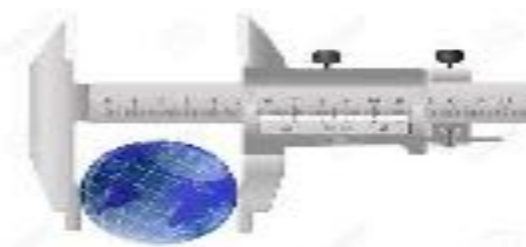
Relays, storage, & repeaters



Cold atoms

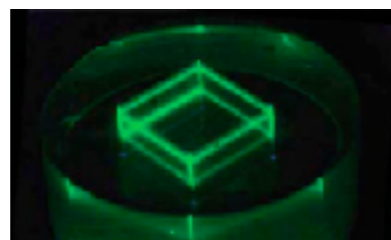


Q-Metrology



Spin qubits
Silicon

Rare-earth doped
crystals or fibers



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2. Fundamental tests on nonlocality

3. Generalised Bell inequality

4. Conclusion & outlook

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Entanglement in short

$$|\psi_{ab}\rangle = |\psi_a\rangle \otimes |\psi_b\rangle = \alpha_a \alpha_b |0_a\rangle |0_b\rangle + \alpha_a \beta_b |0_a\rangle |1_b\rangle + \beta_a \alpha_b |1_a\rangle |0_b\rangle + \beta_a \beta_b |1_a\rangle |1_b\rangle$$

2 qubits Bell state basis

$$\left\{ \begin{array}{l} |\Psi^+\rangle = \frac{1}{\sqrt{2}} (|01\rangle + |10\rangle) \\ |\Psi^-\rangle = \frac{1}{\sqrt{2}} (|01\rangle - |10\rangle) \\ |\Phi^+\rangle = \frac{1}{\sqrt{2}} (|00\rangle + |11\rangle) \\ |\Phi^-\rangle = \frac{1}{\sqrt{2}} (|00\rangle - |11\rangle). \end{array} \right.$$

Entanglement in short

- The tensor product of 2 qubits:

$$|\psi_{ab}\rangle = |\psi_a\rangle \otimes |\psi_b\rangle = \alpha_a \alpha_b |0_a\rangle |0_b\rangle + \alpha_a \beta_b |0_a\rangle |1_b\rangle + \beta_a \alpha_b |1_a\rangle |0_b\rangle + \beta_a \beta_b |1_a\rangle |1_b\rangle$$

- But there are 2 qubits states that cannot be written as such called entangled state:

$$\text{2 qubits Bell state basis} \left\{ \begin{array}{l} |\Psi^+\rangle = \frac{1}{\sqrt{2}} (|01\rangle + |10\rangle) \\ |\Psi^-\rangle = \frac{1}{\sqrt{2}} (|01\rangle - |10\rangle) \\ |\Phi^+\rangle = \frac{1}{\sqrt{2}} (|00\rangle + |11\rangle) \\ |\Phi^-\rangle = \frac{1}{\sqrt{2}} (|00\rangle - |11\rangle). \end{array} \right.$$

Bell violation with photon pair sources

VOLUME 49, NUMBER 2

PHYSICAL REVIEW LETTERS

12 JULY 1982

Experimental Realization of Einstein-Podolsky-Rosen-Bohm *Gedankenexperiment*: A New Violation of Bell's Inequalities

Alain Aspect, Philippe Grangier, and Gérard Roger

*Institut d'Optique Théorique et Appliquée, Laboratoire associé au Centre National de la Recherche Scientifique,
Université Paris-Sud, F-91406 Orsay, France*

(Received 30 December 1981)

The linear-polarization correlation of pairs of photons emitted in a radiative cascade of calcium has been measured. The new experimental scheme, using two-channel polarizers (i.e., optical analogs of Stern-Gerlach filters), is a straightforward transposition of Einstein-Podolsky-Rosen-Bohm *gedankenexperiment*. The present results, in excellent agreement with the quantum mechanical predictions, lead to the greatest violation of generalized Bell's inequalities ever achieved.

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First experimental realisation:

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Entanglement Photon Pair Sources (EPPS)

In a dielectric material, a pump field oscillating at a given angular frequency creates a deformation of the charge distribution in a crystal called polarisation density.

Polarization density:

$$\vec{P} = \epsilon_0 \chi_e \vec{E}.$$

Linear function of the electric field.

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But under strong pump fields and specific crystal symmetries:

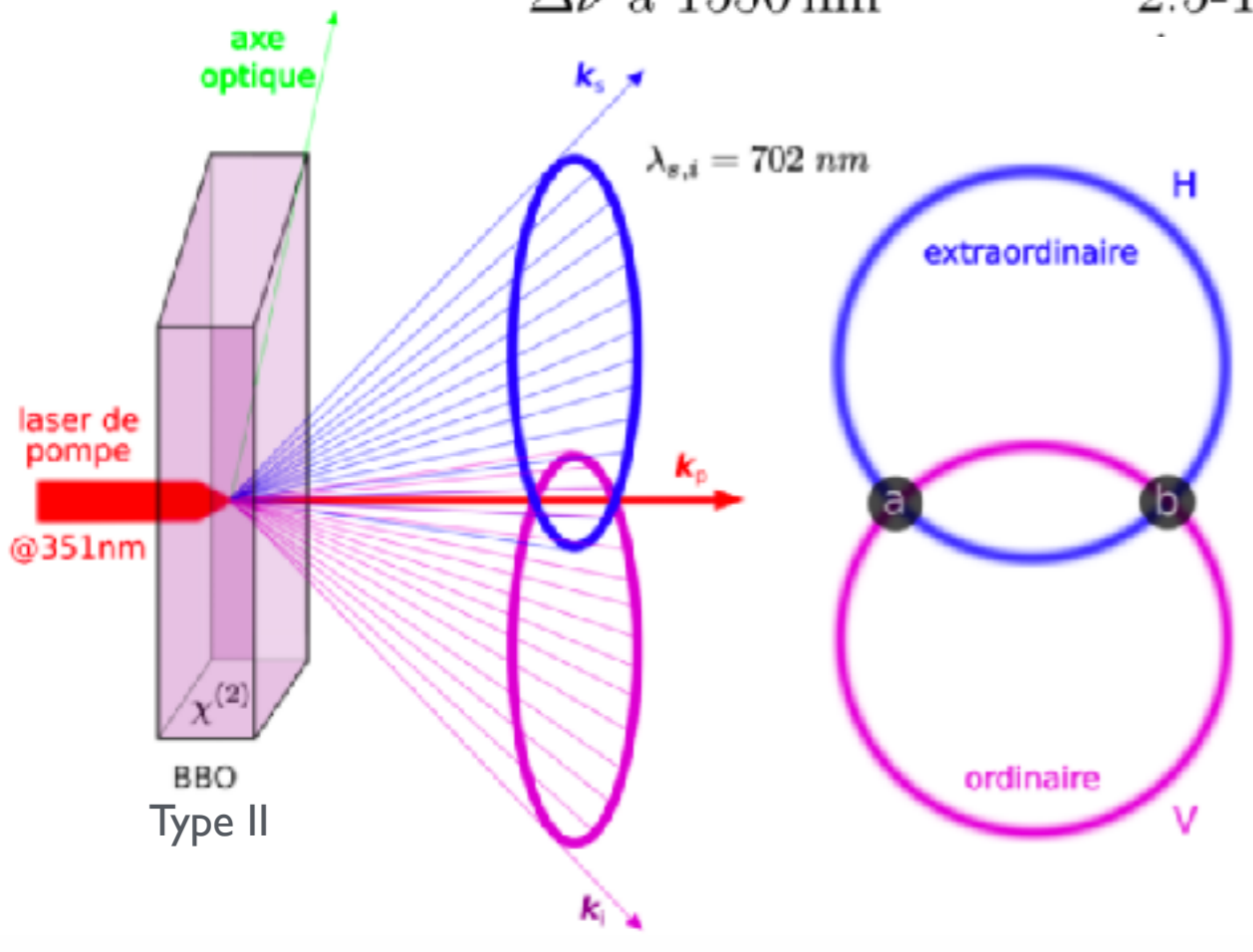
$$\vec{P}(\vec{E}) = \epsilon_0 \overleftrightarrow{\chi}^{(1)} \vec{E} + \epsilon_0 \overleftrightarrow{\chi}^{(2)} \vec{E} \vec{E} + \epsilon_0 \overleftrightarrow{\chi}^{(3)} \vec{E} \vec{E} \vec{E} + \dots = \epsilon_0 \overleftrightarrow{\chi}^{(1)} \vec{E} + \vec{P}^{NL}(\vec{E})$$

The polarisation density also depends on higher orders of \vec{E}

This non linear form allows the distribution to oscillate not only the frequency of the pump but also at linear combination of this frequency.

Entanglement Photon Pair Sources (EPPS)

Type d'interaction	type-0	type-I	type-II
Polarisation	$ V\rangle_p \rightarrow V\rangle_s V\rangle_i$	$ V\rangle_p \rightarrow H\rangle_s H\rangle_i$	$ H\rangle_p \rightarrow H\rangle_s V\rangle_i$
$\chi^{(2)}$ coeff. du LiNbO3	$d_{33} \approx 30 \text{ pm/V}$	$d_{31} \approx -5 \text{ pm/V}$	$d_{24} \approx 10 \text{ pm/V}$
η_{SPDC} dans	$10^{-6} - 10^{-5}$	10^{-7}	10^{-9}
$\Delta\lambda$ à 1550 nm	20-100 nm	20-100 nm	0.8-3 nm
$\Delta\nu$ à 1550 nm	2.5-12.5 THz	2.5-12.5 THz	0.1-0.375 THz

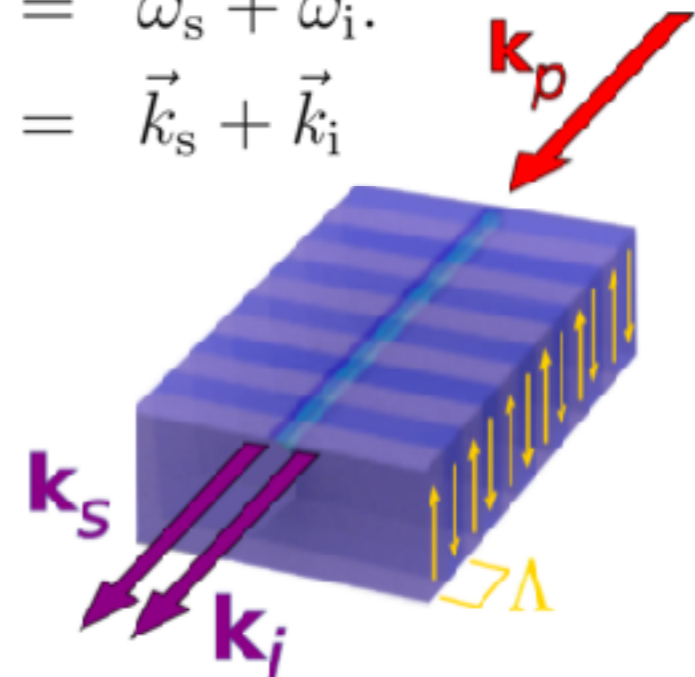


Kwiat et al., (1995)

Conservation laws:

$$\omega_p = \omega_s + \omega_i.$$

$$\vec{k}_p = \vec{k}_s + \vec{k}_i$$



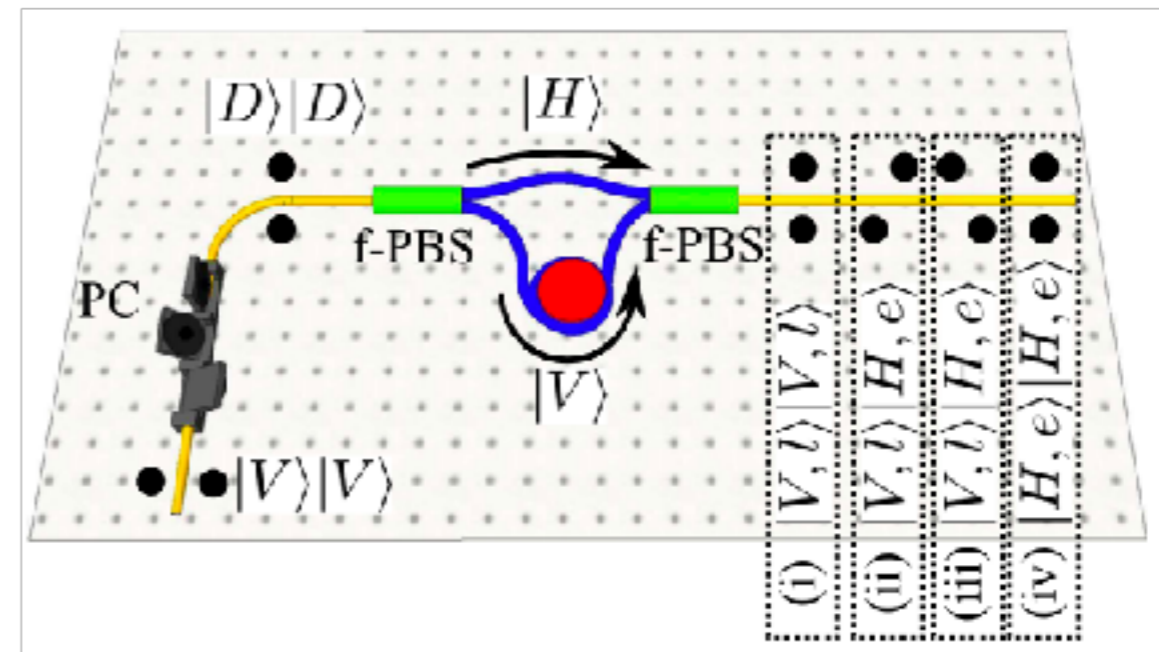
$$\vec{k}_p = \vec{k}_s + \vec{k}_i + \frac{2\pi}{\Lambda} \cdot \vec{u}.$$

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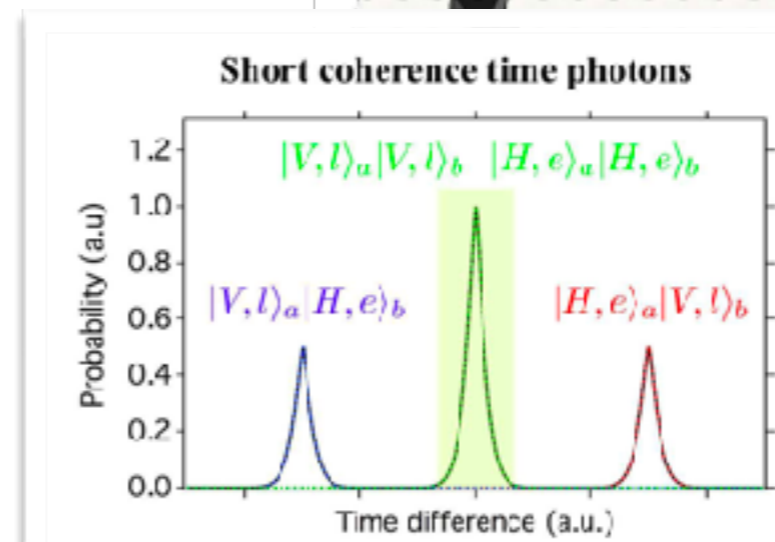
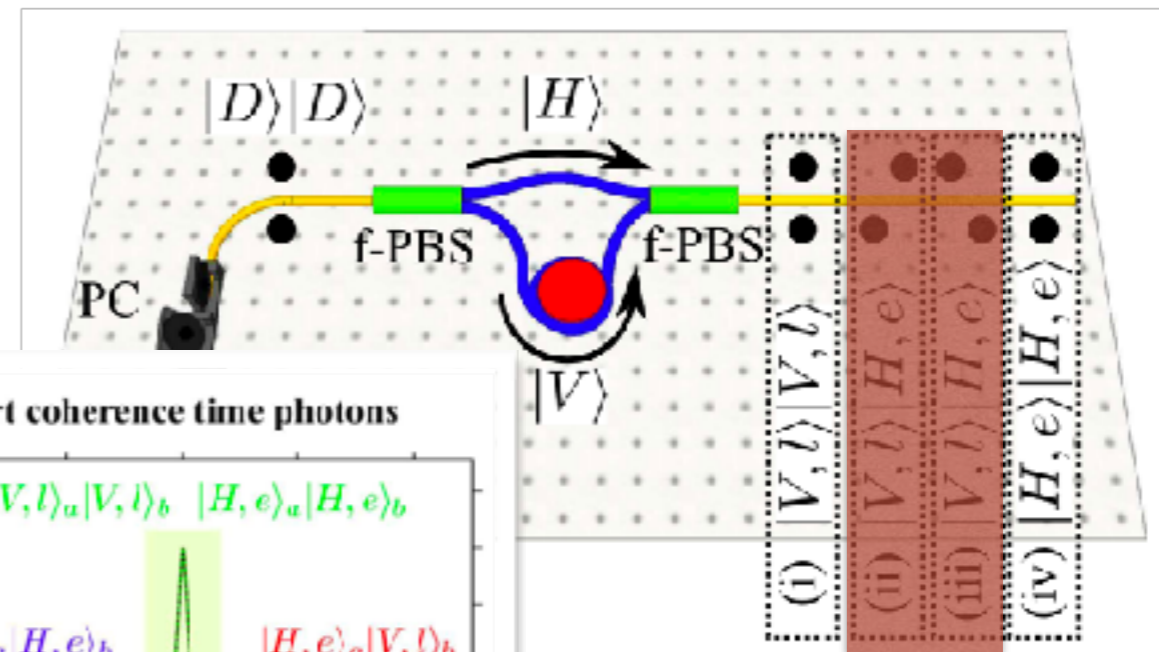
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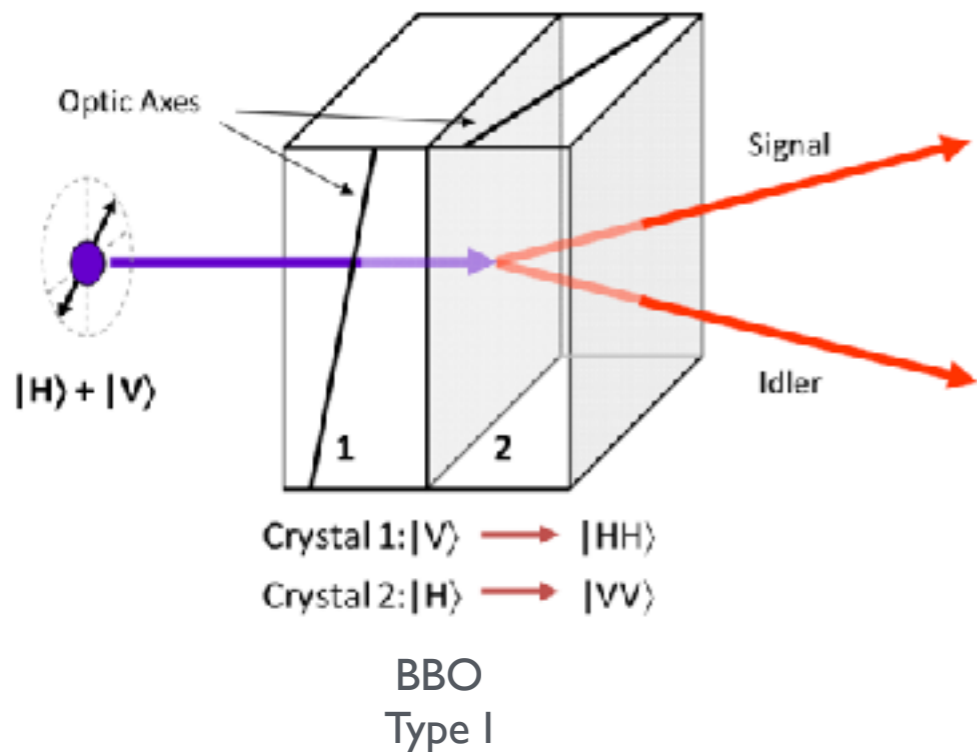
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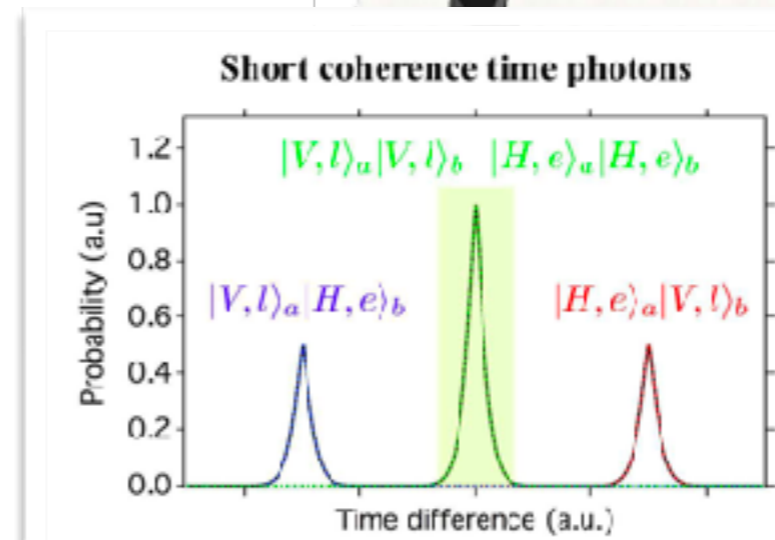
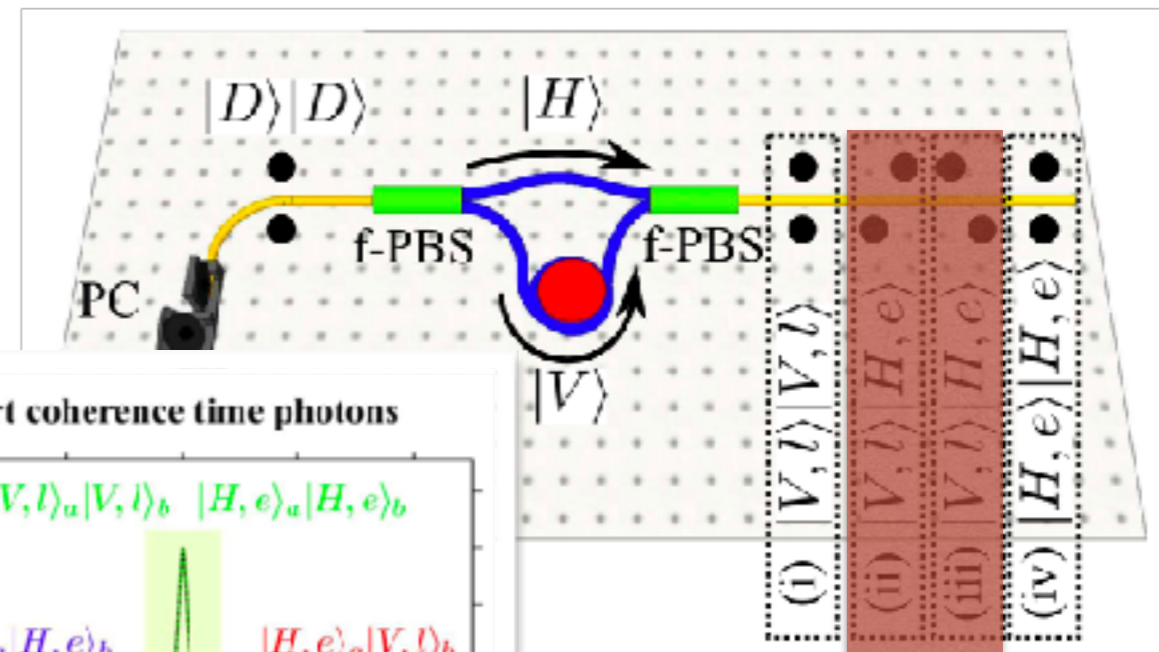


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$$|\Psi\rangle = \frac{|HH\rangle + e^{i\phi}|VV\rangle}{\sqrt{2}}$$



OUTLINE

1. Introduction

2. Fundamental tests on nonlocality

3. Generalised Bell inequality

4. Conclusion & outlook

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- Correlations nature & origine
- Bell non-locality
- Eberhardt inequality

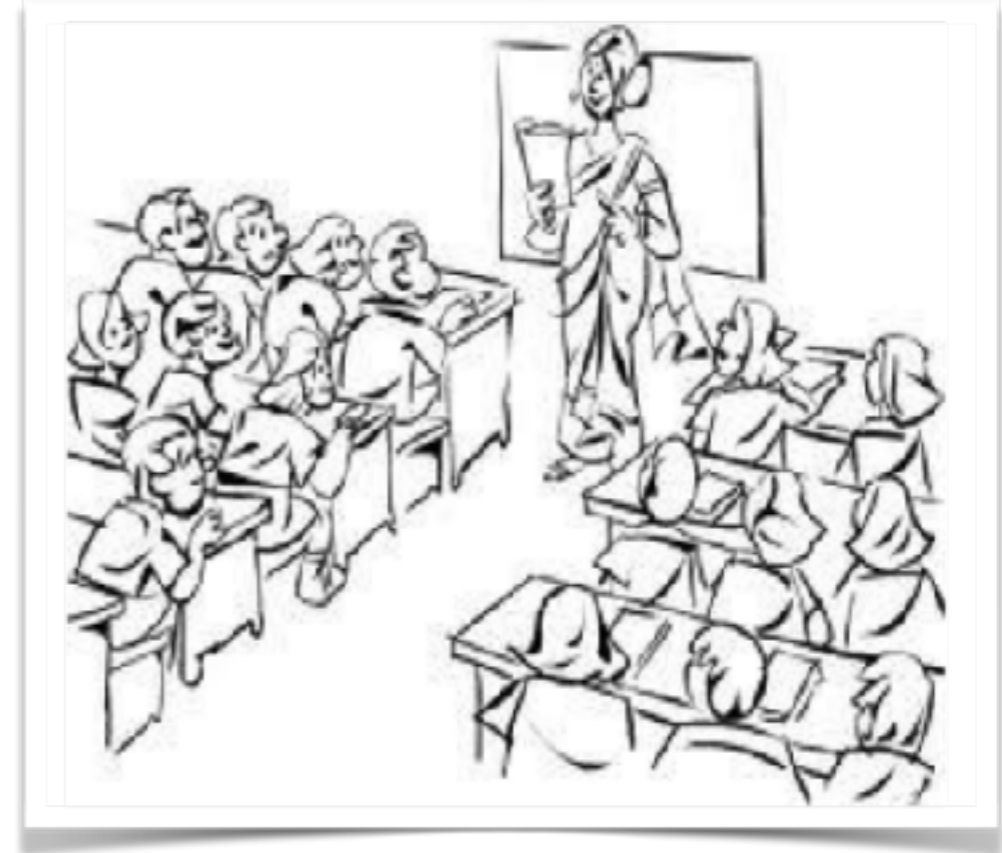
3. Q

4. Conclusion & outlook

Correlations in the classical world



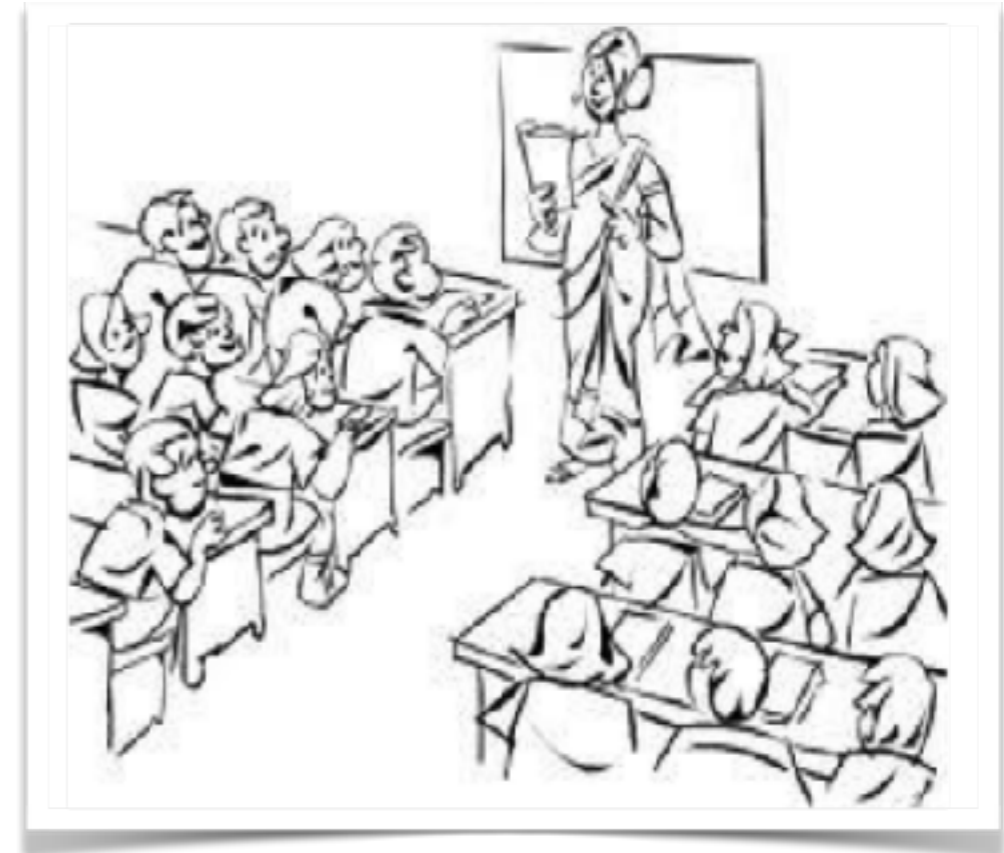
systems:
professor
+
student



Correlations in the classical world



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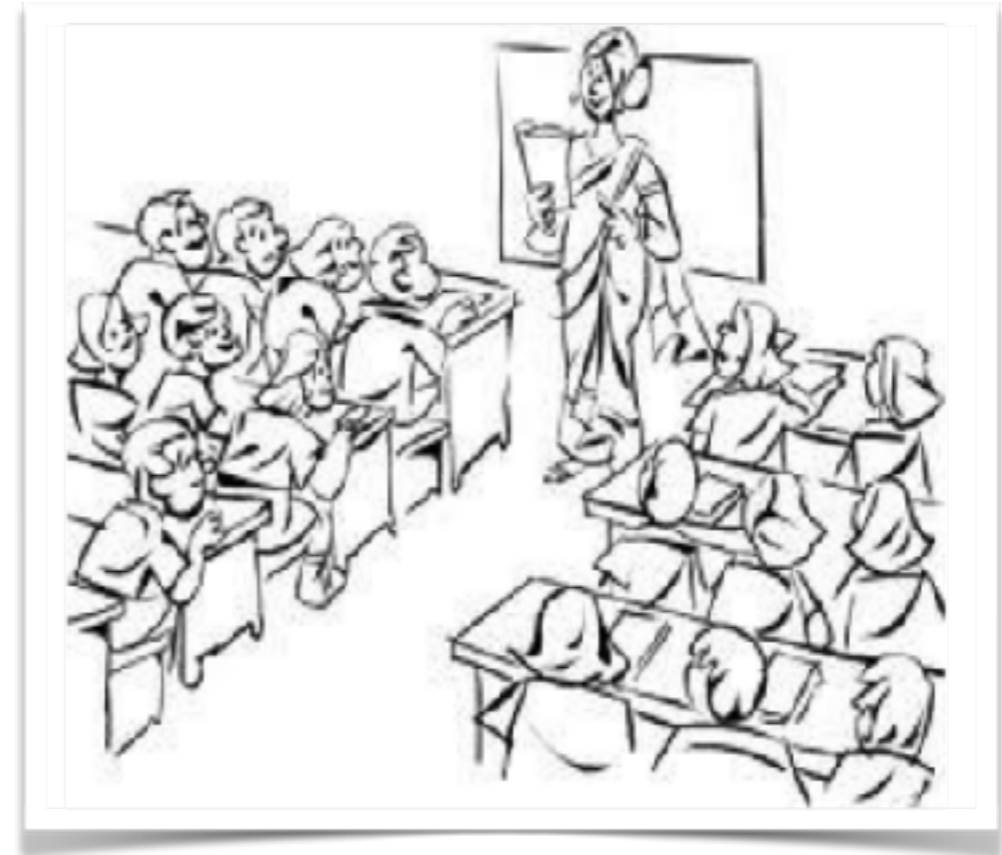


Two
possible origins
for classical
correlations

Correlations in the classical world



systems:
professor
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student



Through a **signal** between two subsystems:

- The professor yell and the students calm down.

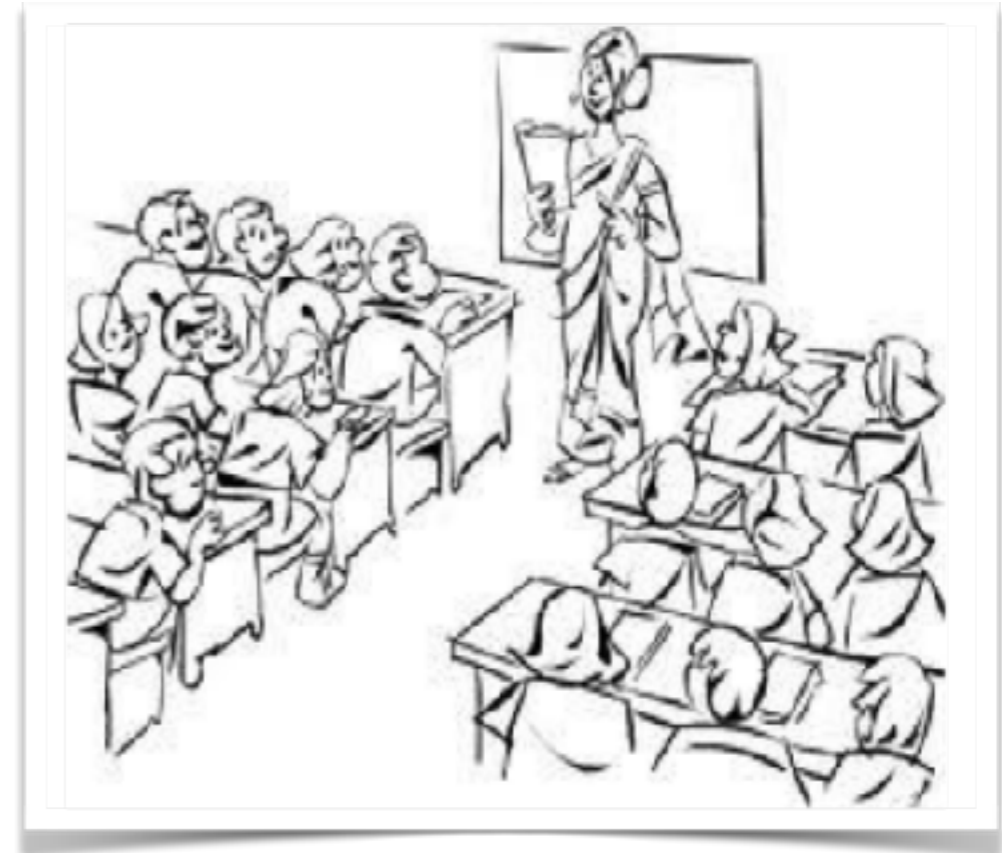


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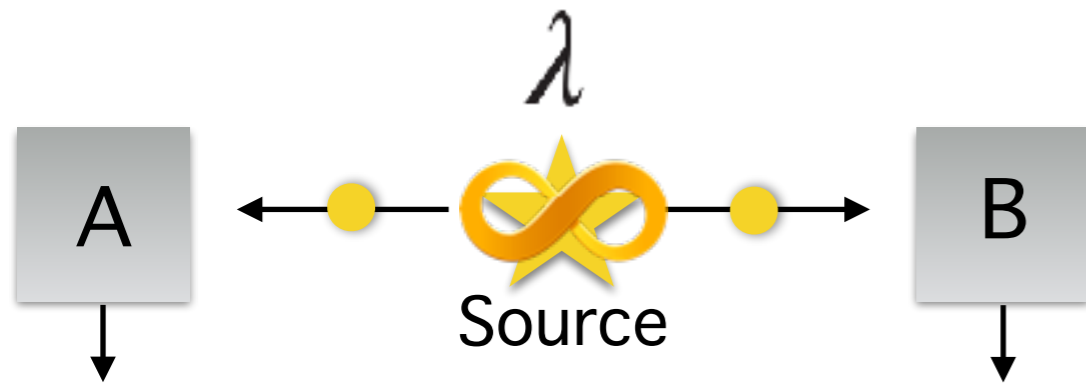


Two possible origins for classical correlations

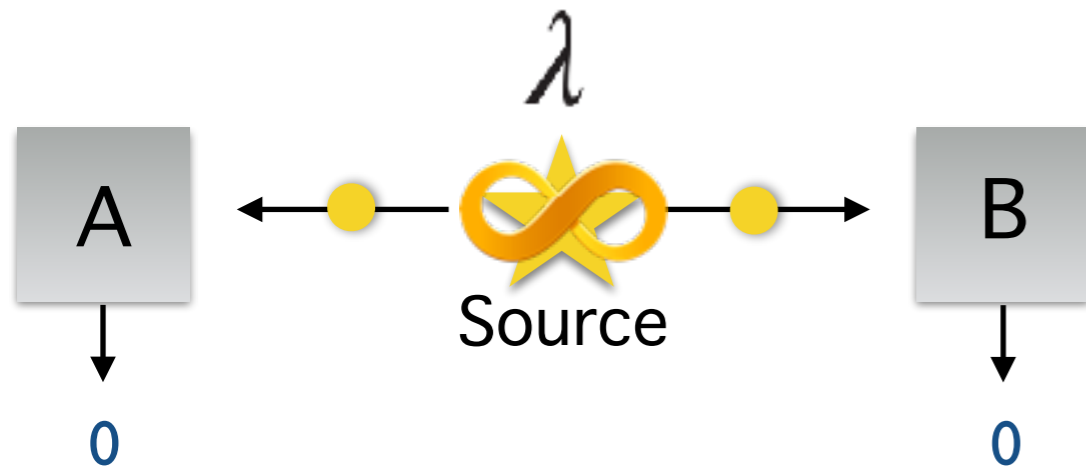
With a **pre-established strategy**:

- The professor establish rules at the beginning of the year.
 - # No chit-chat.
 - # No changing seat.
 - # Listening quietly.

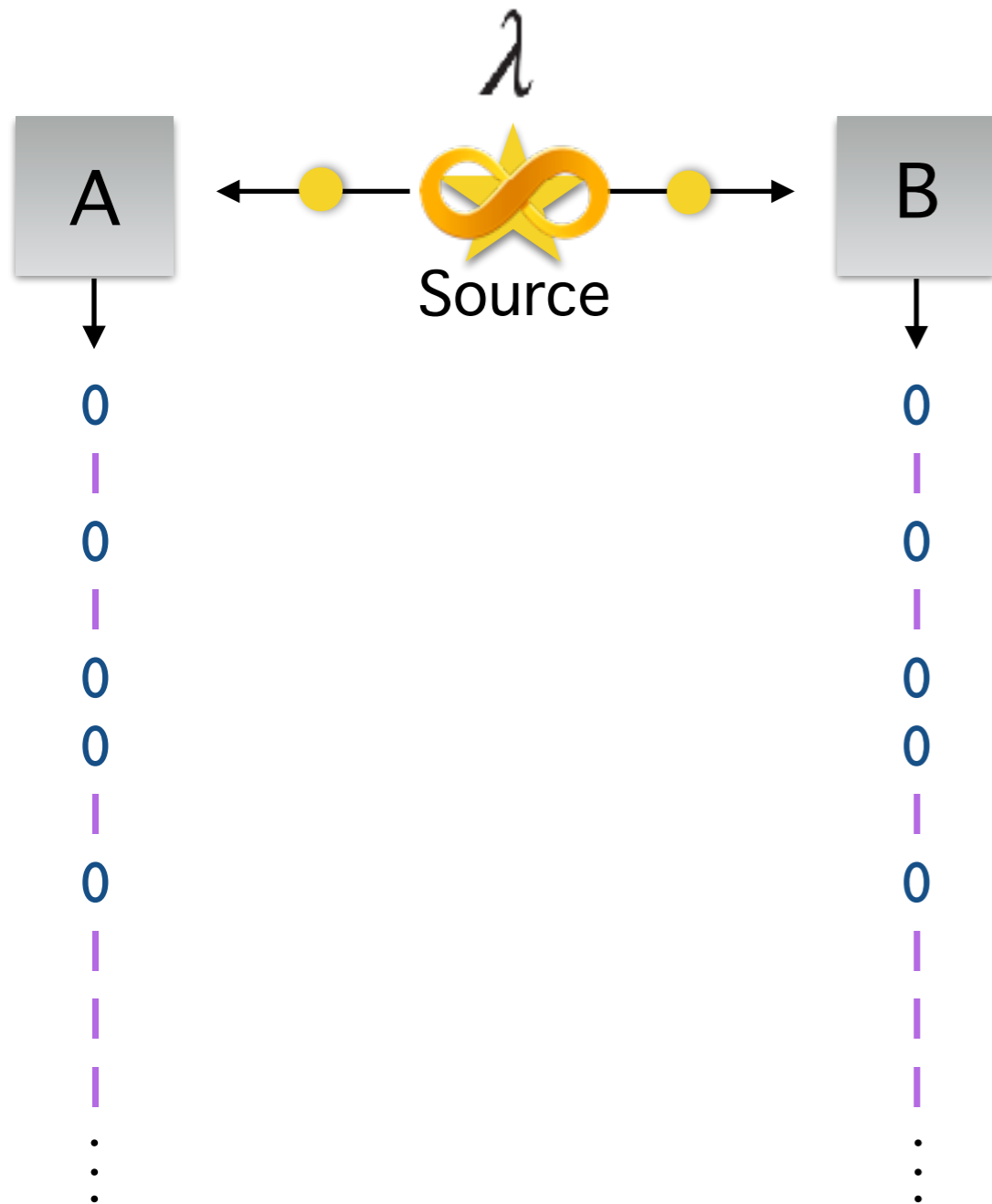
Quantum correlations



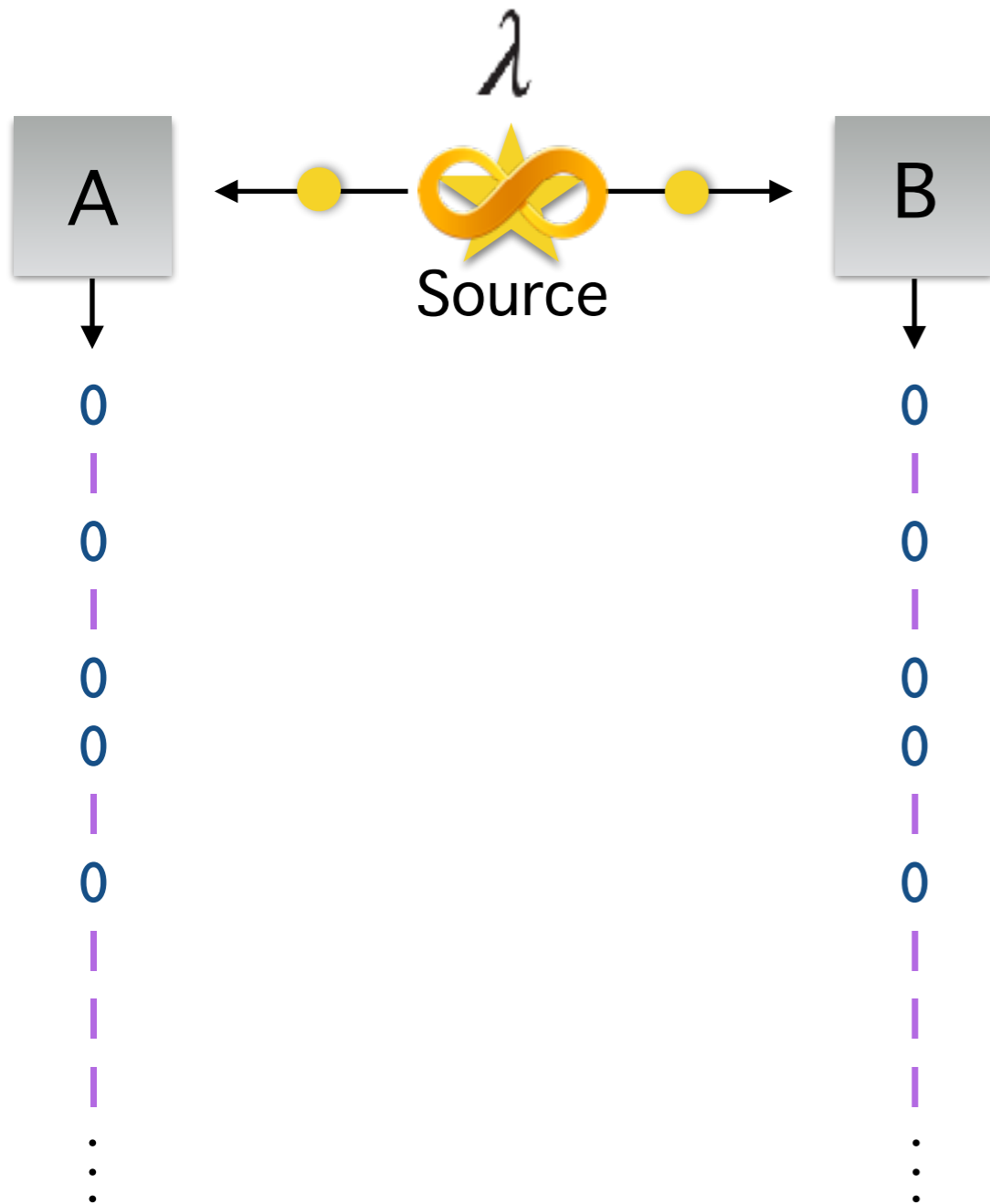
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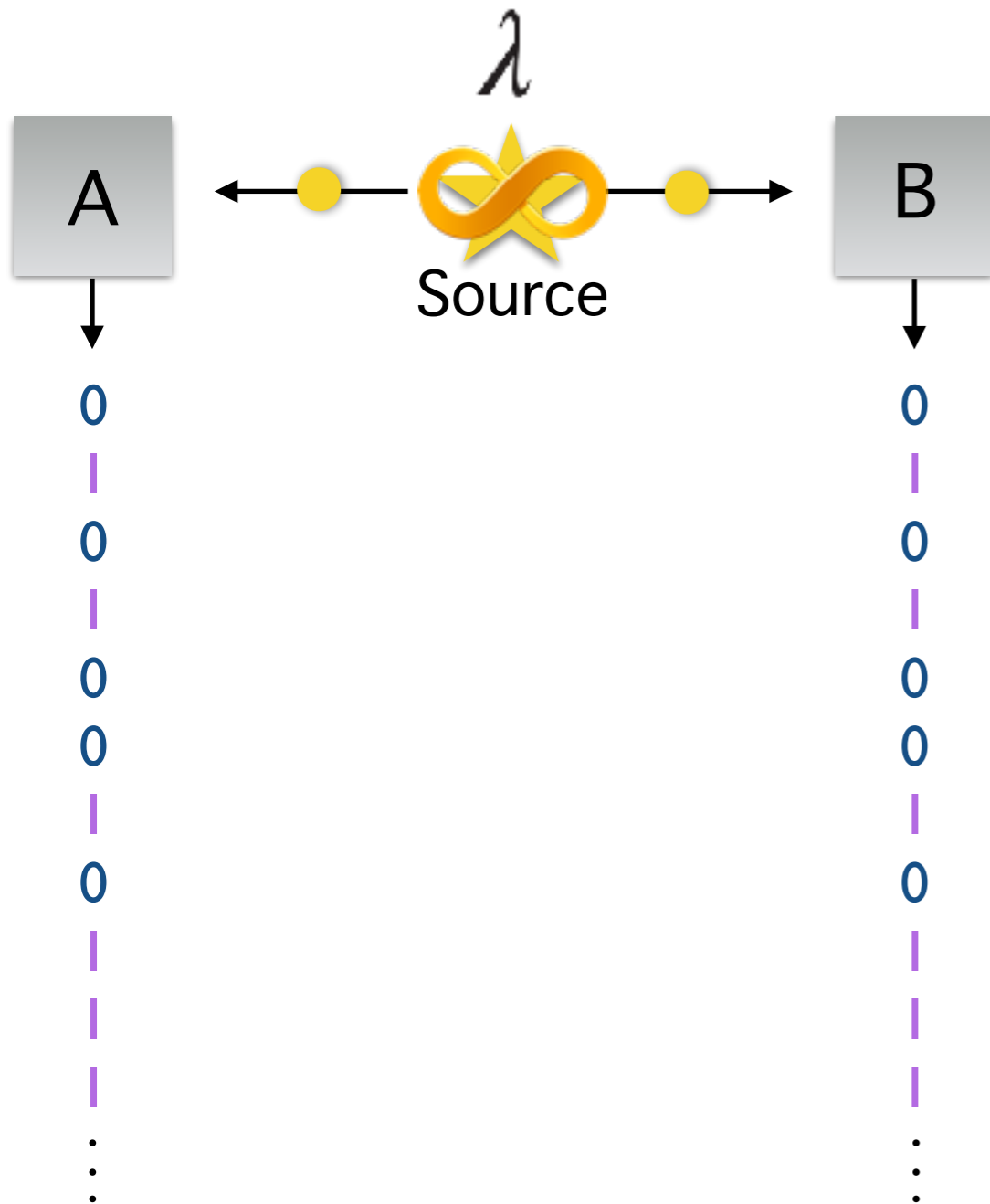
Quantum correlations



A and B gets the same answer,
Whatever the question!
(exclude strategy)
Whatever the distance!
(exclude signaling)

« *Spooky* » action
at a distance

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Strong Q-correlations
provided by entangled states

Entanglement & original Bell test (1964)



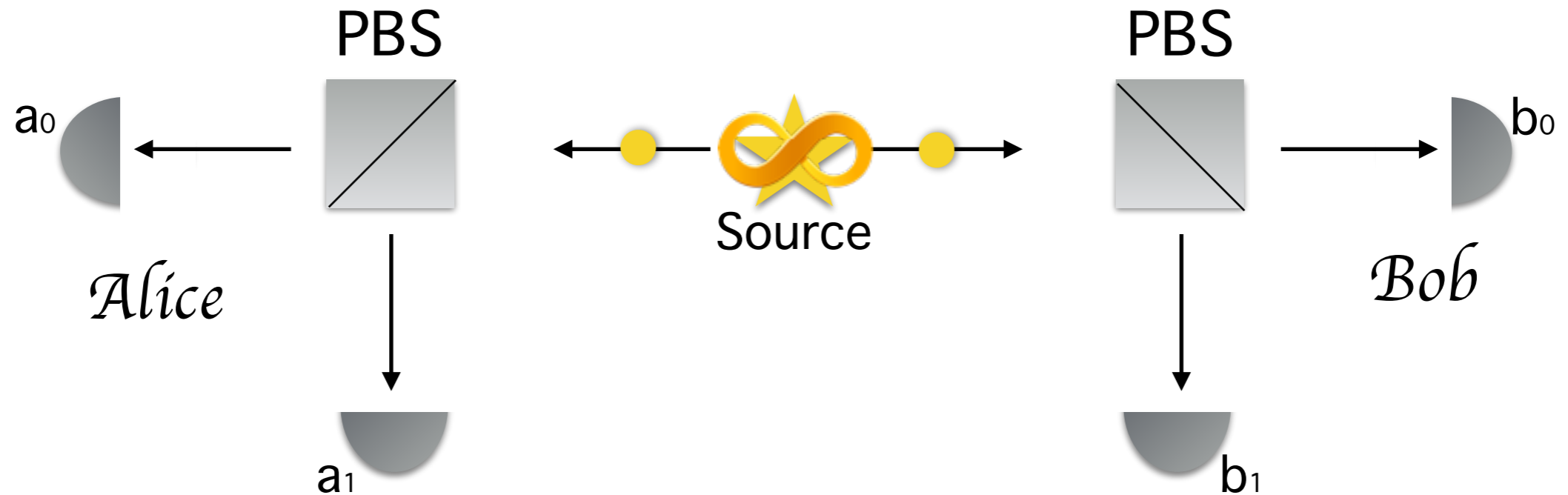
Entanglement & original Bell test (1964)

$$|\Psi\rangle = 1/\sqrt{2} (|H_a H_b\rangle + |V_a V_b\rangle)$$



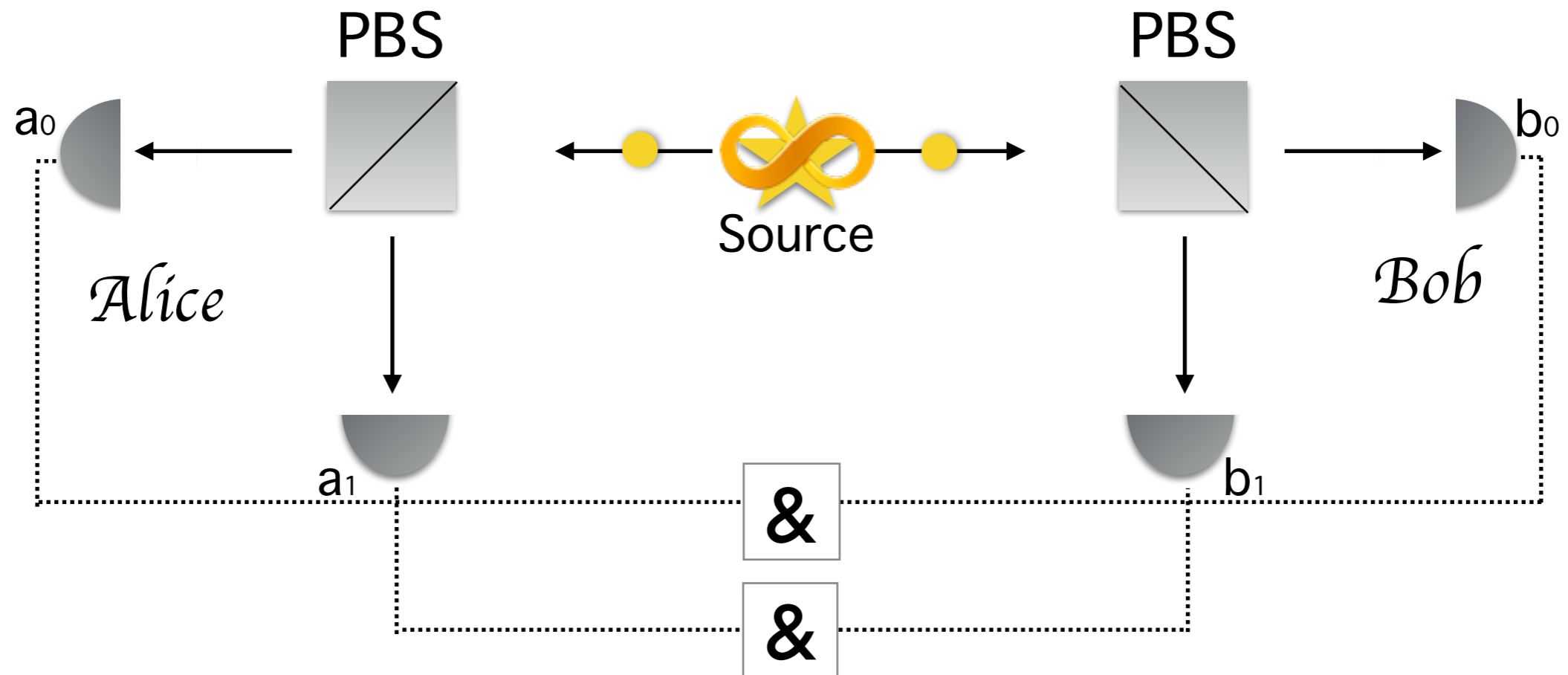
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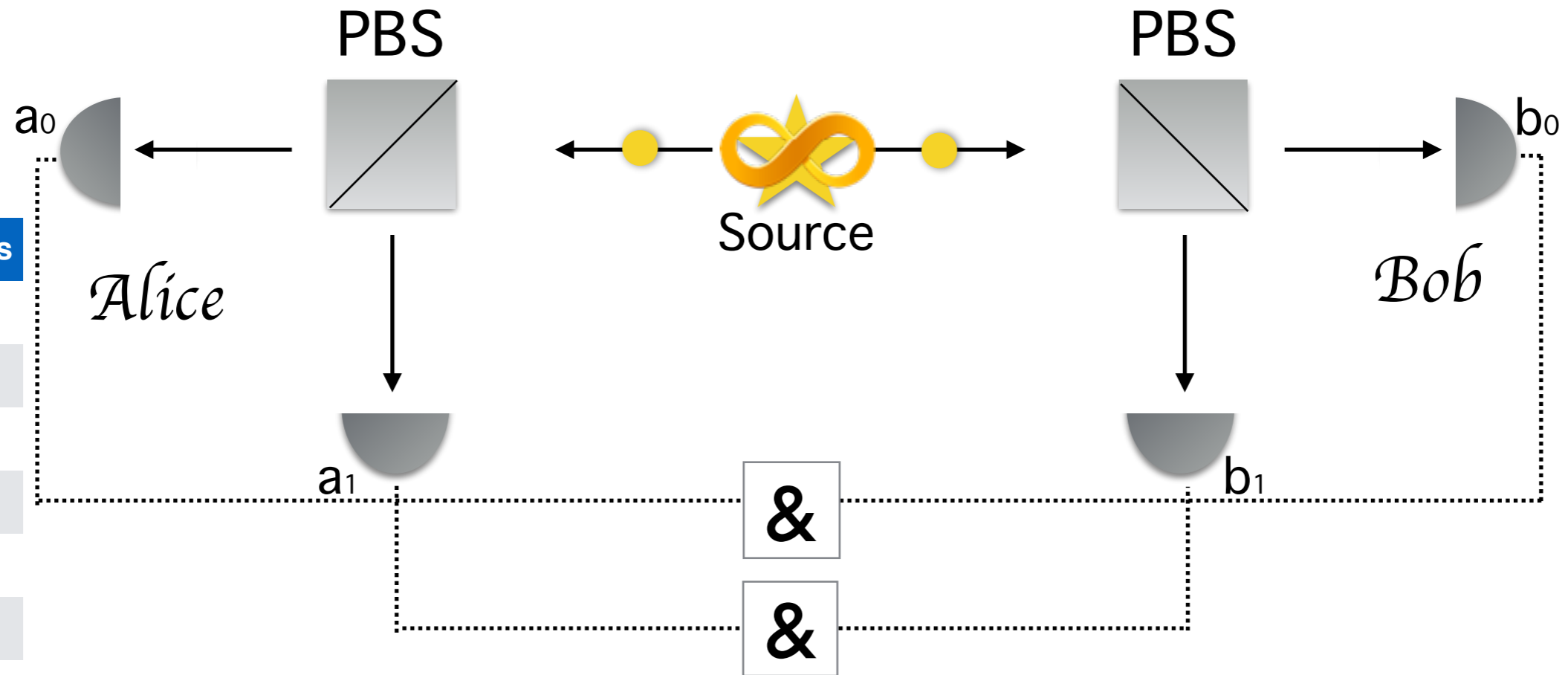
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Entanglement & original Bell test (1964)

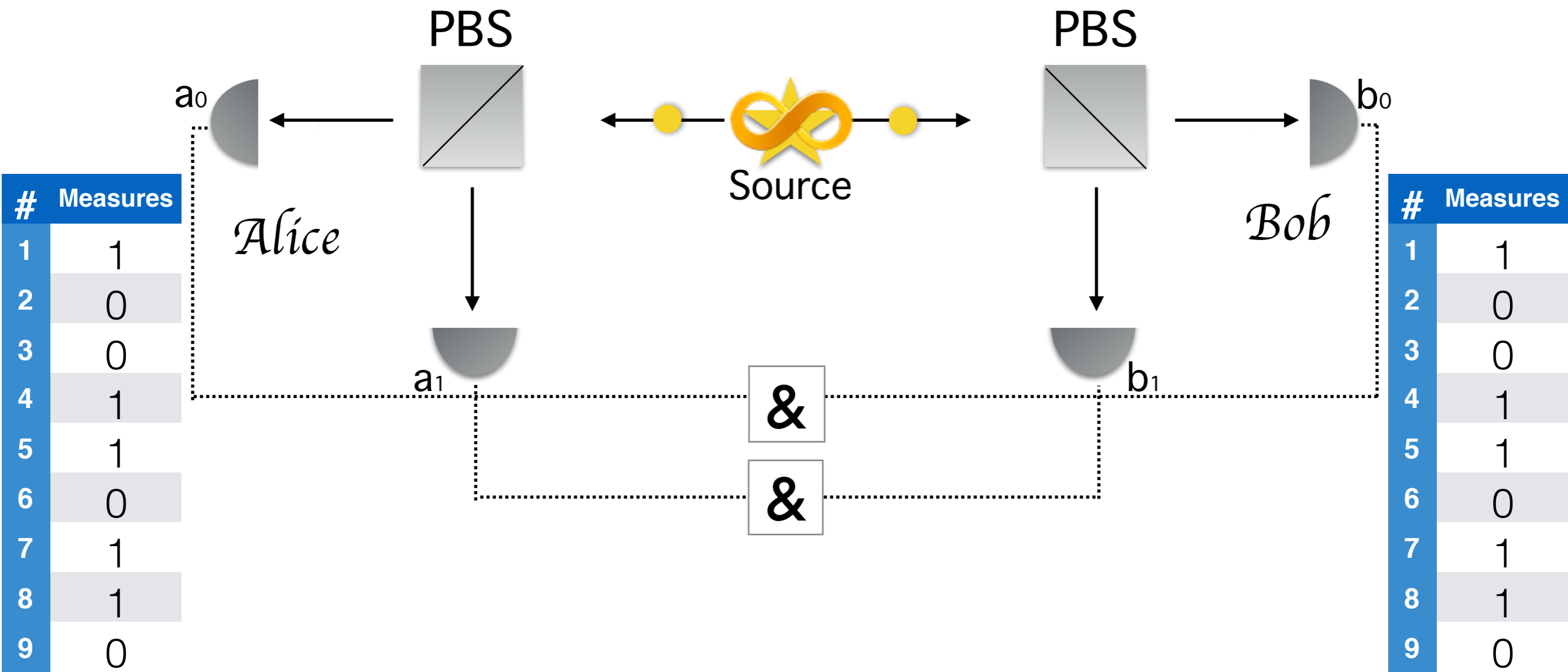
$$|\Psi\rangle = 1/\sqrt{2} (|H_a H_b\rangle + |V_a V_b\rangle)$$



#	Measures
1	1
2	0
3	0
4	1
5	1
6	0
7	1
8	1
9	0

Entanglement & original Bell test (1964)

$$|\Psi\rangle = 1/\sqrt{2} (|H_a H_b\rangle + |V_a V_b\rangle)$$

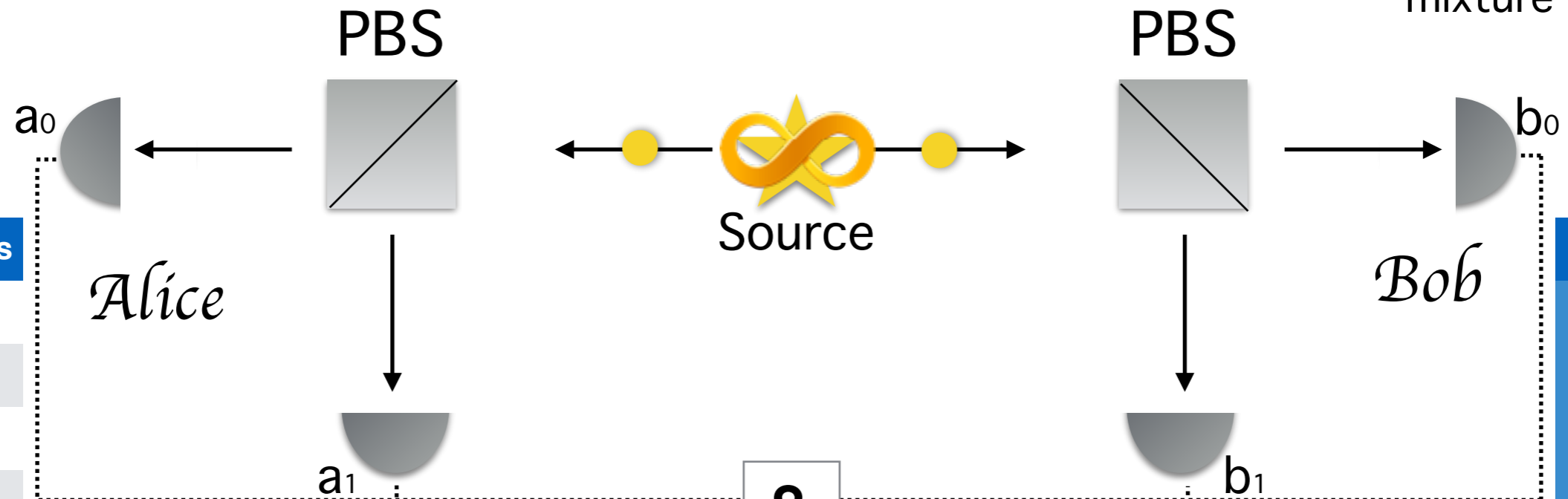


Entanglement & original Bell test (1964)

$$|\Psi\rangle = 1/\sqrt{2} (|H_a H_b\rangle + |V_a V_b\rangle)$$

$$\Psi = \begin{cases} P = \frac{1}{2} \rightarrow |HH\rangle \\ P = \frac{1}{2} \rightarrow |VV\rangle \end{cases}$$

mixture



#	Measures
1	1
2	0
3	0
4	1
5	1
6	0
7	1
8	1
9	0

#	Measures
1	1
2	0
3	0
4	1
5	1
6	0
7	1
8	1
9	0

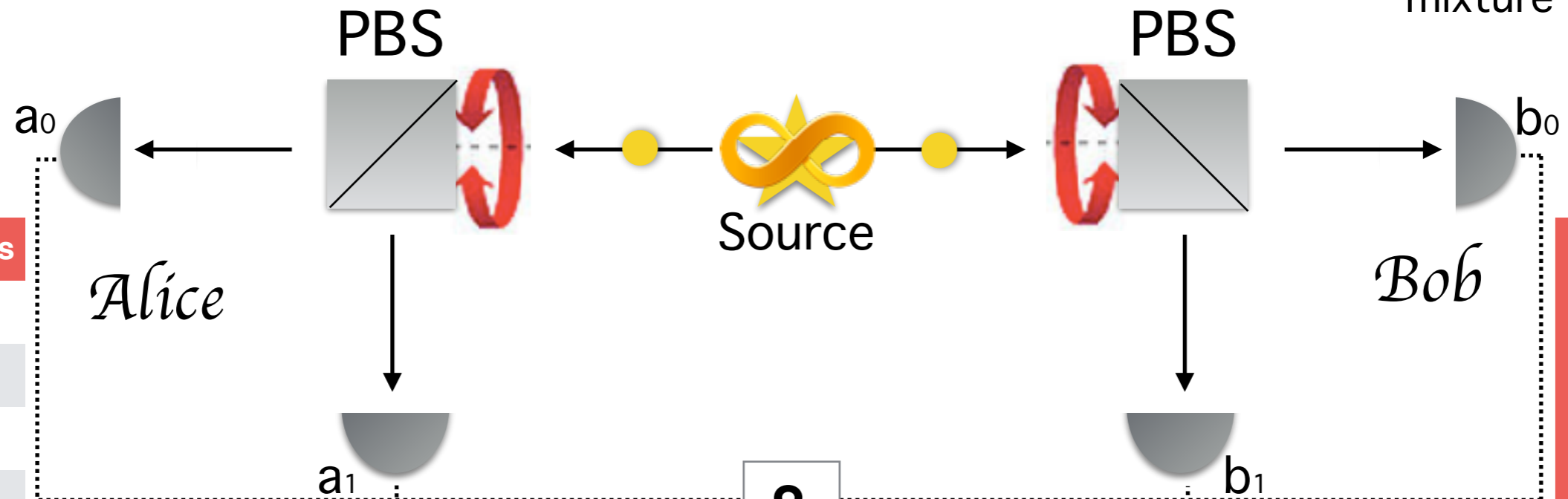


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#	Measures
1	0
2	0
3	1
4	1
5	1
6	0
7	0
8	1
9	1



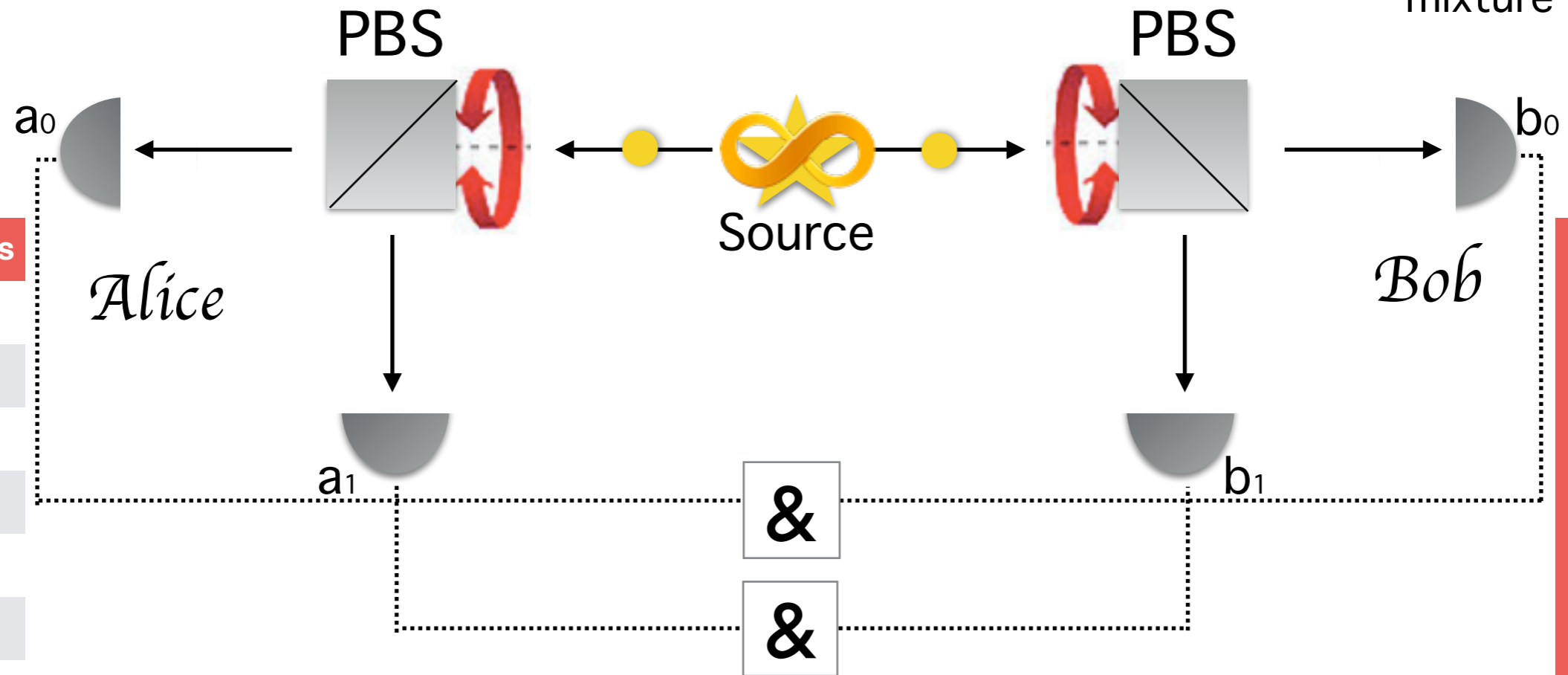
#	Measures
1	0
2	0
3	1
4	1
5	1
6	0
7	0
8	1
9	1

Entanglement & original Bell test (1964)

$$|\Psi\rangle = 1/\sqrt{2} (|H_a H_b\rangle + |V_a V_b\rangle) \neq$$

$$\Psi = \begin{cases} P = \frac{1}{2} \rightarrow |HH\rangle \\ P = \frac{1}{2} \rightarrow |VV\rangle \end{cases}$$

mixture



#	Measures
1	0
2	0
3	1
4	1
5	1
6	0
7	0
8	1
9	1

#	Measures
1	0
2	0
3	1
4	1
5	1
6	0
7	0
8	1
9	1

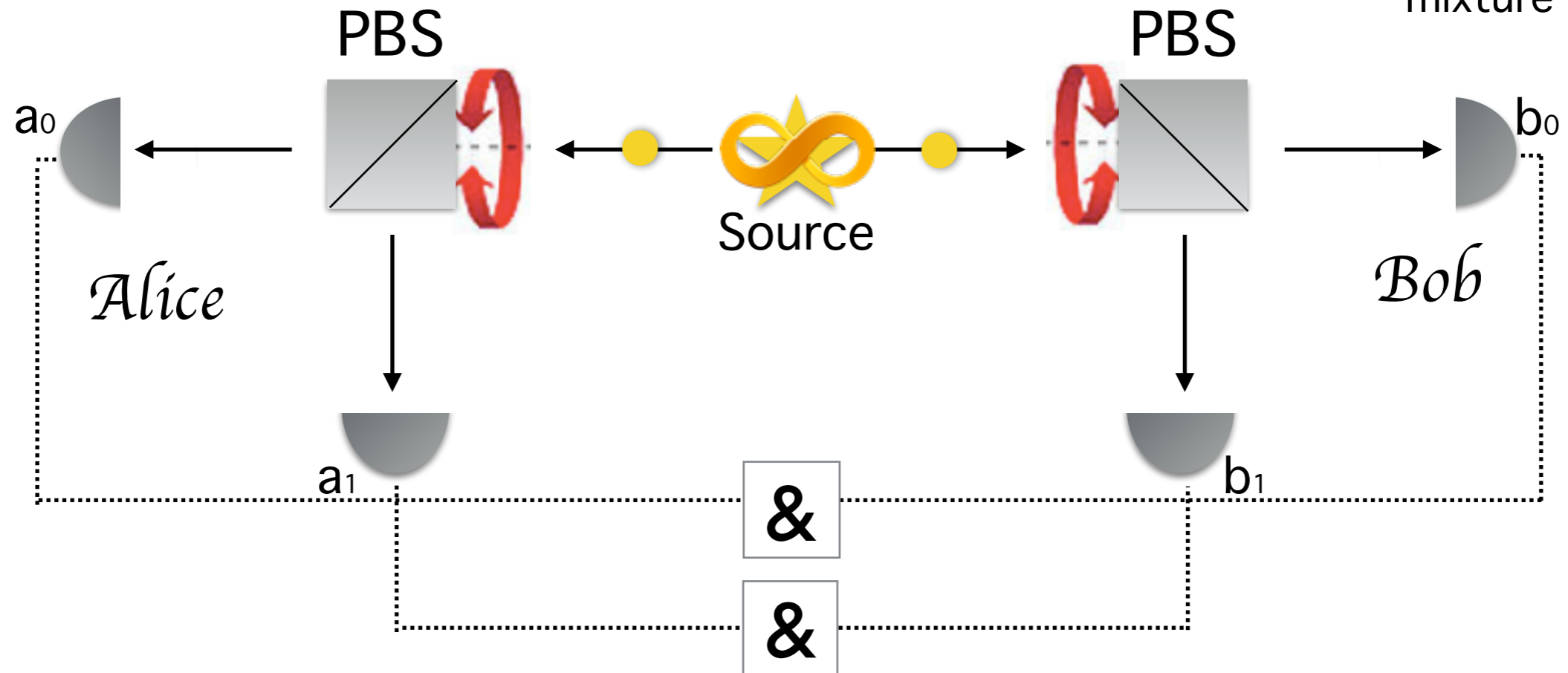
correlations
invariant through rotation

Entanglement & original Bell test (1964)

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mixture



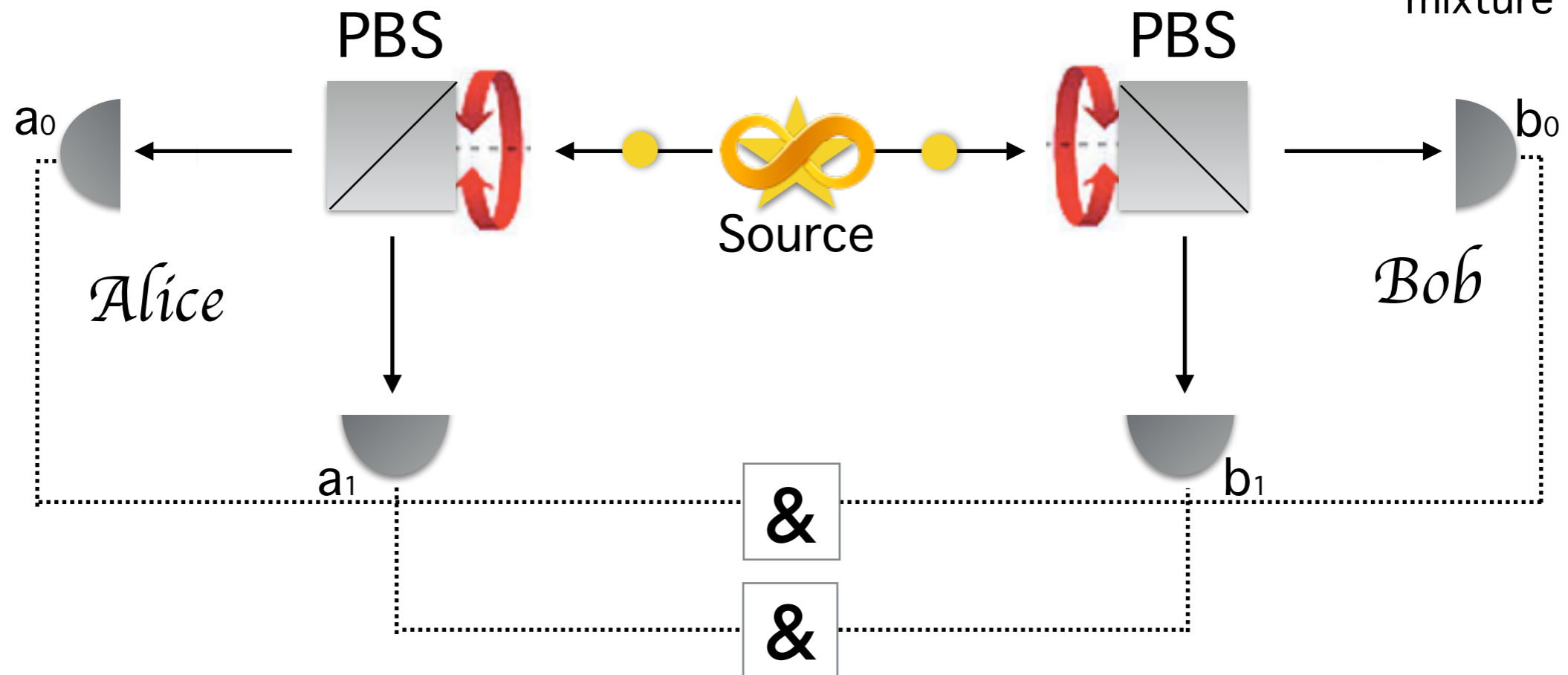
correlations
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Entanglement & original Bell test (1964)

$$|\Psi\rangle = 1/\sqrt{2} (|H_a H_b\rangle + |V_a V_b\rangle) \neq$$

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mixture

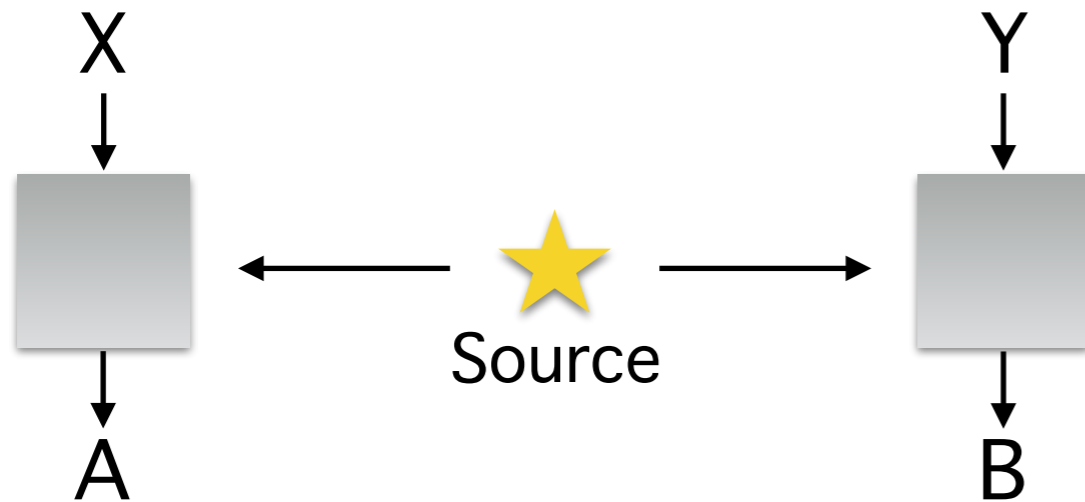


correlations
invariant through rotation



most
unintuitive

Bell *(non)locality* (1964)

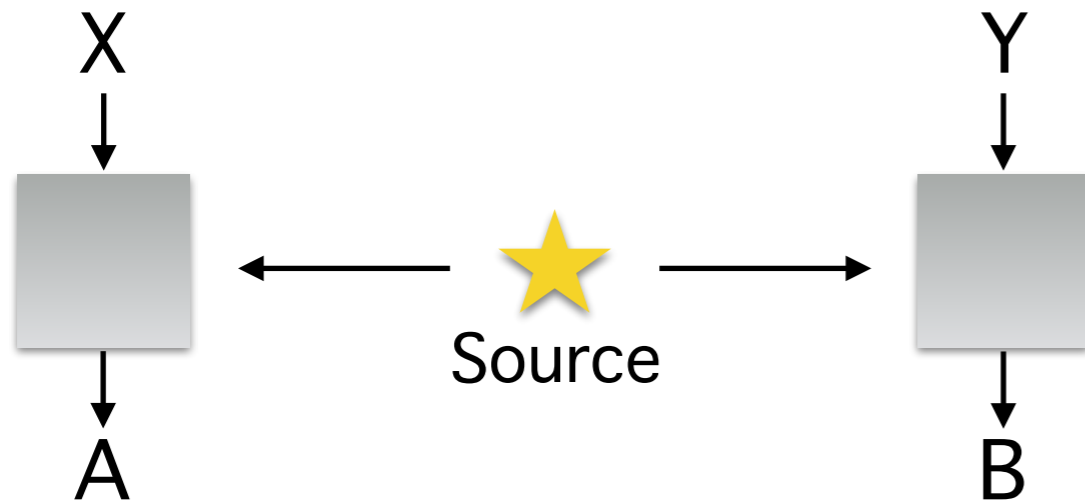


(X, Y) : settings

(A, B) : outcomes

$$p(ab|xy) \neq p(a|x)p(b|y)$$

Bell *(non)locality* (1964)

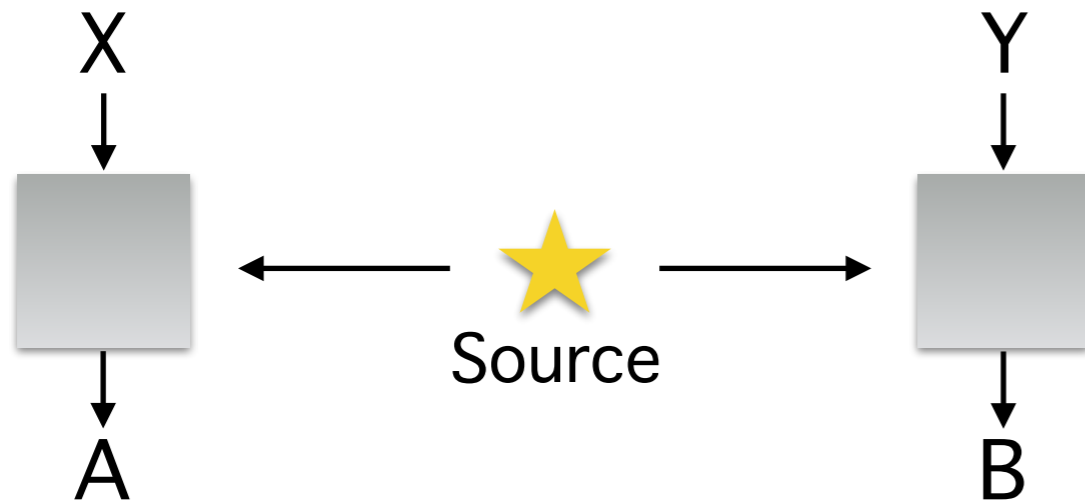


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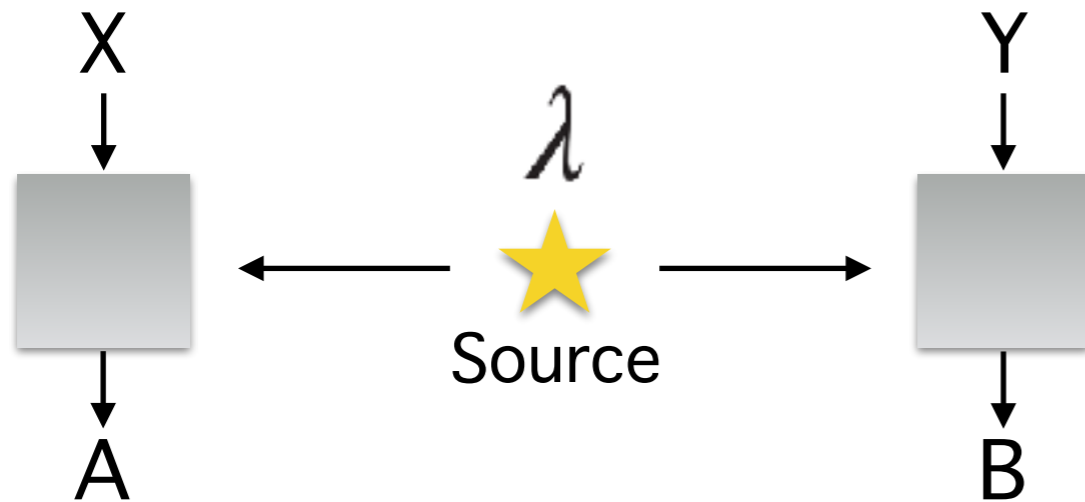
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$$p(ab|xy) \neq p(a|x)p(b|y)$$

Locality formalized:

Bell (*non*)locality (1964)



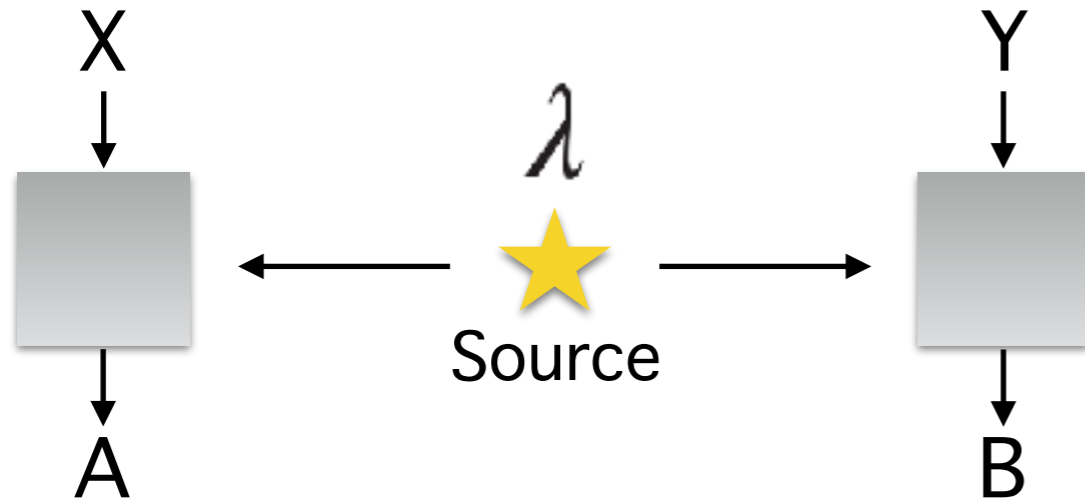
(X, Y) : settings
 (A, B) : outcomes

$$p(ab|xy) \neq p(a|x)p(b|y)$$

Locality formalized:

$$p(ab|xy, \lambda) = p(a|x, \lambda)p(b|y, \lambda)$$

Bell (*non*)locality (1964)



(X,Y) : settings

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$$p(ab|xy) \neq p(a|x)p(b|y)$$

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Outcome independence:

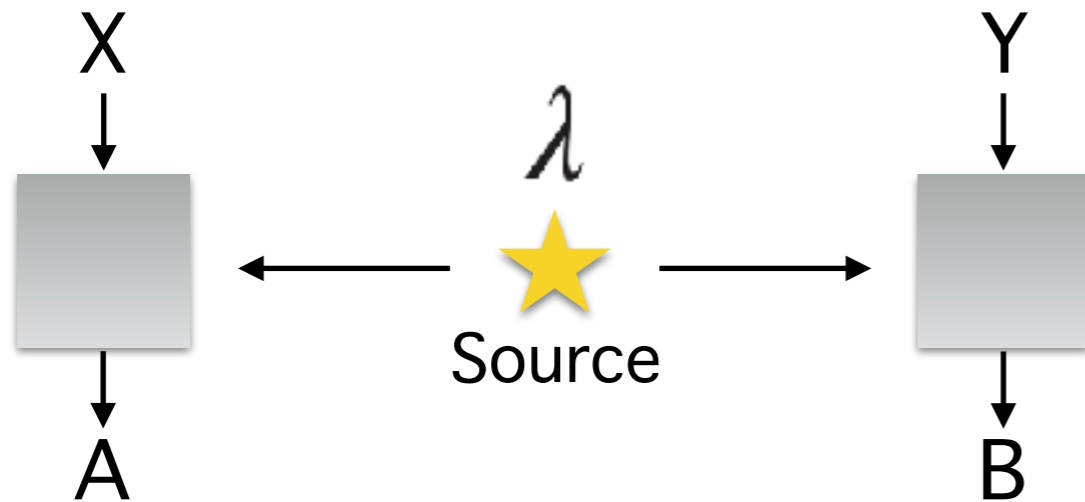
$$p(ab|xy\lambda) = p(a|xy\lambda)p(b|xy\lambda)$$

parameter independence:

$$p(a|xy\lambda) = p(a|x\lambda)$$

$$p(b|xy\lambda) = p(b|y\lambda)$$

Bell (*non*)locality (1964)



(X,Y) : settings

(A,B) : outcomes

$$p(ab|xy) \neq p(a|x)p(b|y)$$

Locality formalized:

$$p(ab|xy, \lambda) = p(a|x, \lambda)p(b|y, \lambda)$$

$$p(ab|xy) = \int_{\Lambda} d\lambda q(\lambda) p(a|x, \lambda) p(b|y, \lambda)$$

Outcome independence:

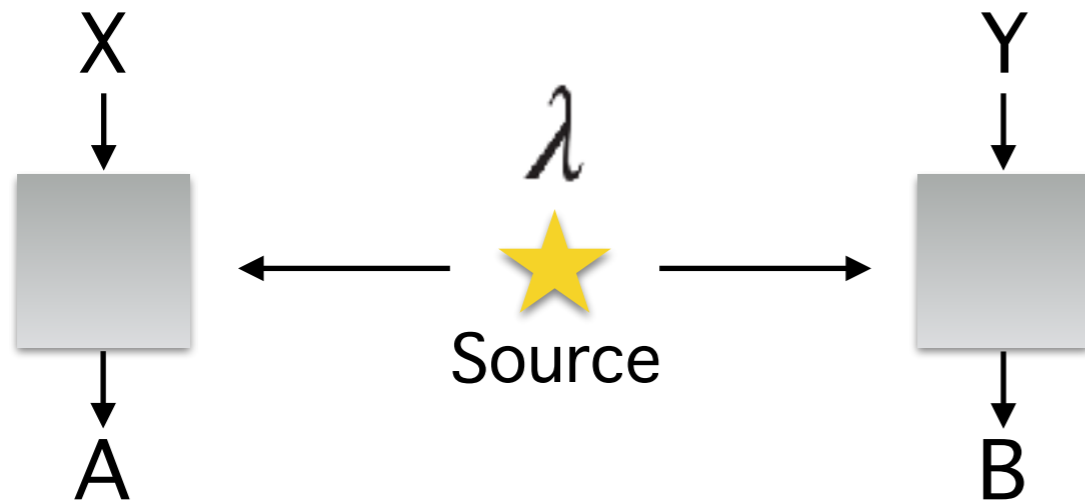
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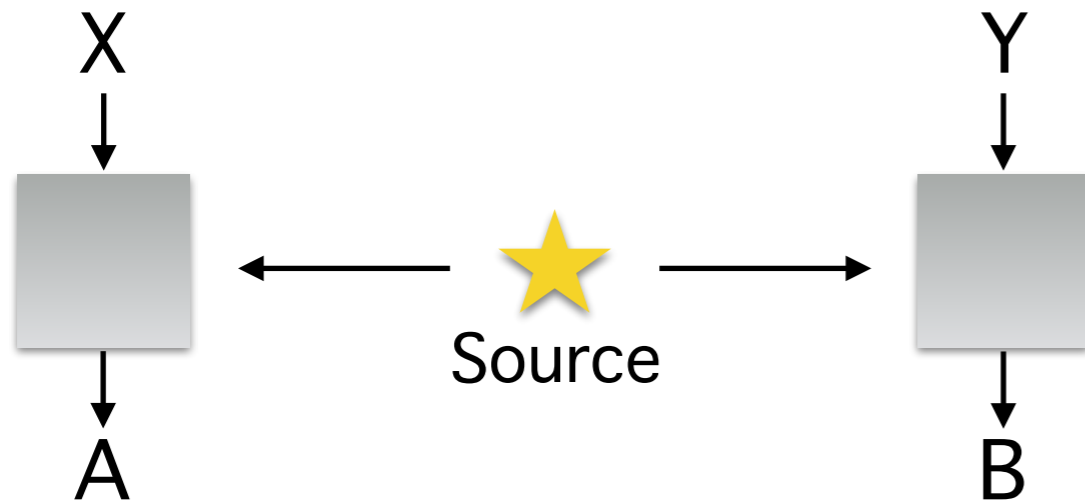
$$p(a|xy\lambda) = p(a|x\lambda)$$

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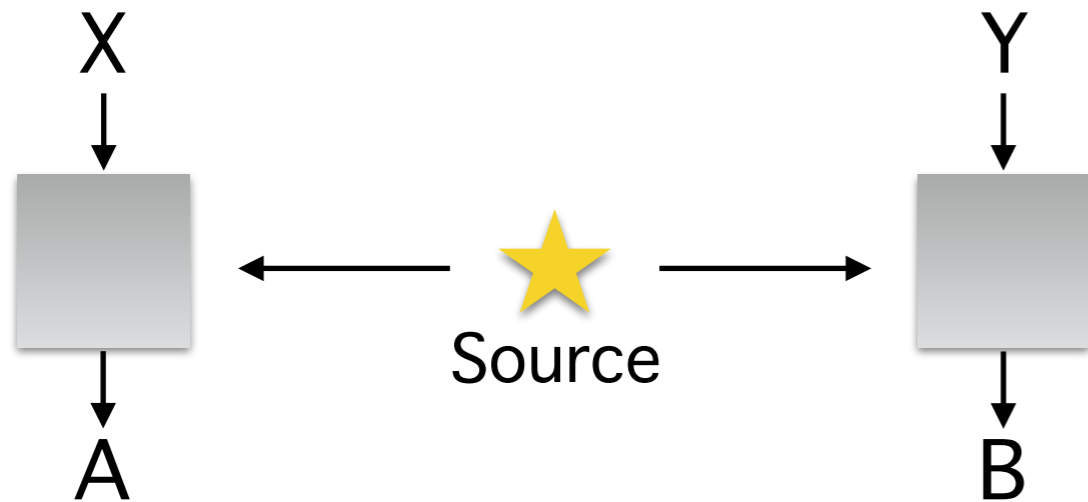
measurement independence:

$$p(xy|\lambda) = p(xy)$$

Bell-CHSH inequality (1969)



Bell-CHSH inequality (1969)

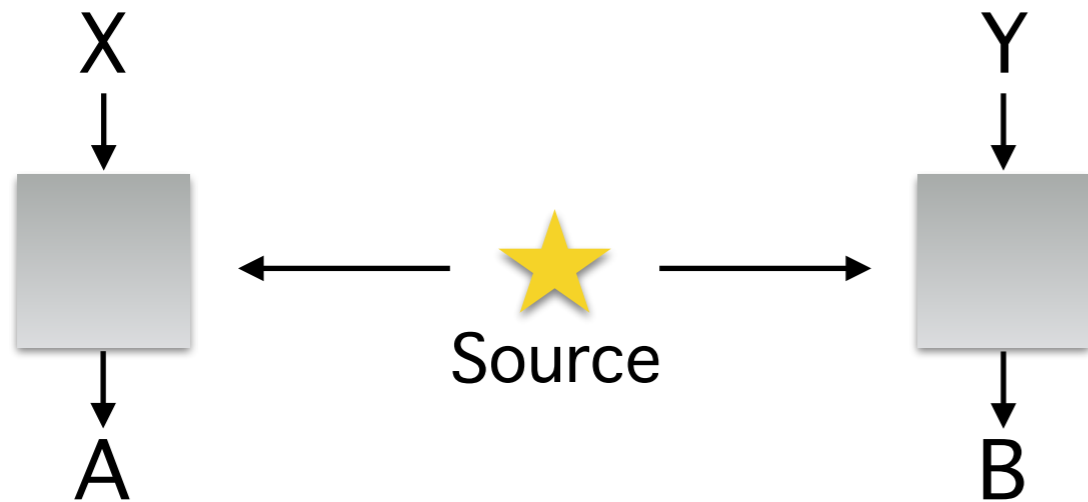


Simple case (binary):

$$x, y \in \{0, 1\}$$

$$a, b \in \{-1, +1\}$$

Bell-CHSH inequality (1969)



Simple case (binary):

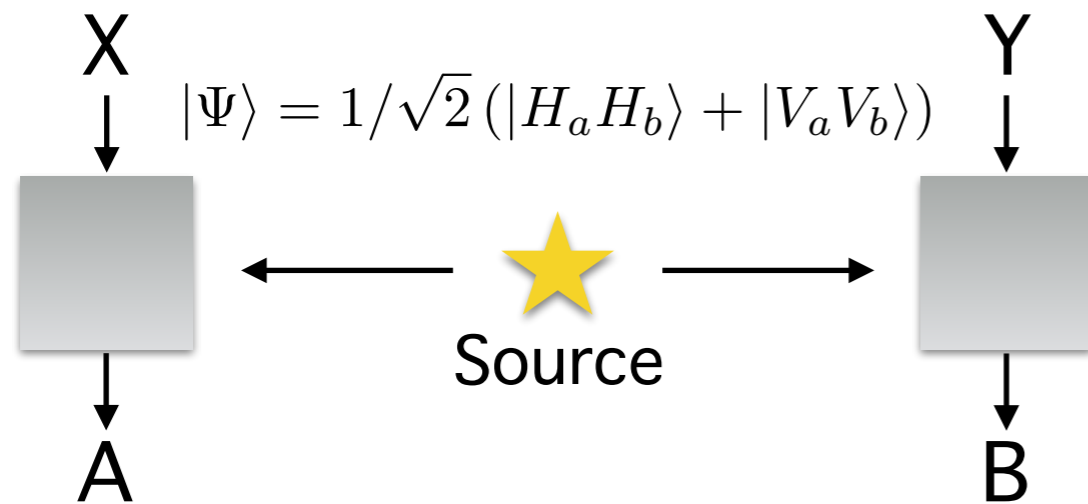
$$x, y \in \{0, 1\}$$

$$a, b \in \{-1, +1\}$$

$$S = |\langle a_0 b_0 \rangle + \langle a_0 b_1 \rangle + \langle a_1 b_0 \rangle - \langle a_1 b_1 \rangle| \leq 2$$

**Bell-CHSH
inequality**

Bell-CHSH inequality (1969)



Simple case (binary):

$$x, y \in \{0, 1\}$$

$$a, b \in \{-1, +1\}$$

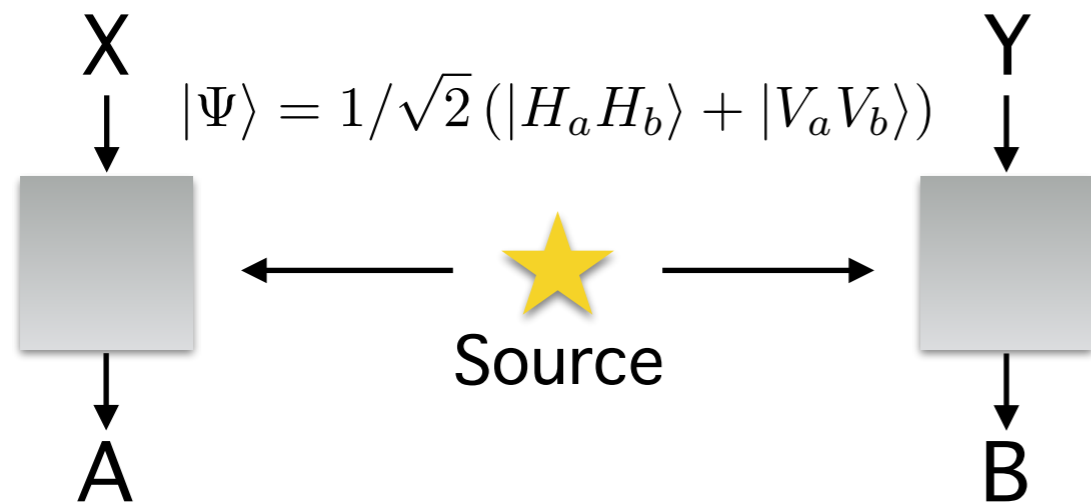
$$S = \left| \underbrace{\langle a_0 b_0 \rangle}_{1/\sqrt{2}} + \underbrace{\langle a_0 b_1 \rangle}_{1/\sqrt{2}} + \underbrace{\langle a_1 b_0 \rangle}_{1/\sqrt{2}} - \underbrace{\langle a_1 b_1 \rangle}_{-1/\sqrt{2}} \right| \leq 2$$

**Bell-CHSH
inequality**

quantum correlations with
proper settings:

$$S = 2\sqrt{2} > 2$$

Bell-CHSH inequality (1969)



Simple case (binary):

$$x, y \in \{0, 1\}$$

$$a, b \in \{-1, +1\}$$

$$S = \left| \frac{1}{\sqrt{2}} \langle a_0 b_0 \rangle + \frac{1}{\sqrt{2}} \langle a_0 b_1 \rangle + \frac{1}{\sqrt{2}} \langle a_1 b_0 \rangle - \frac{1}{\sqrt{2}} \langle a_1 b_1 \rangle \right| \leq 2$$

Bell-CHSH inequality

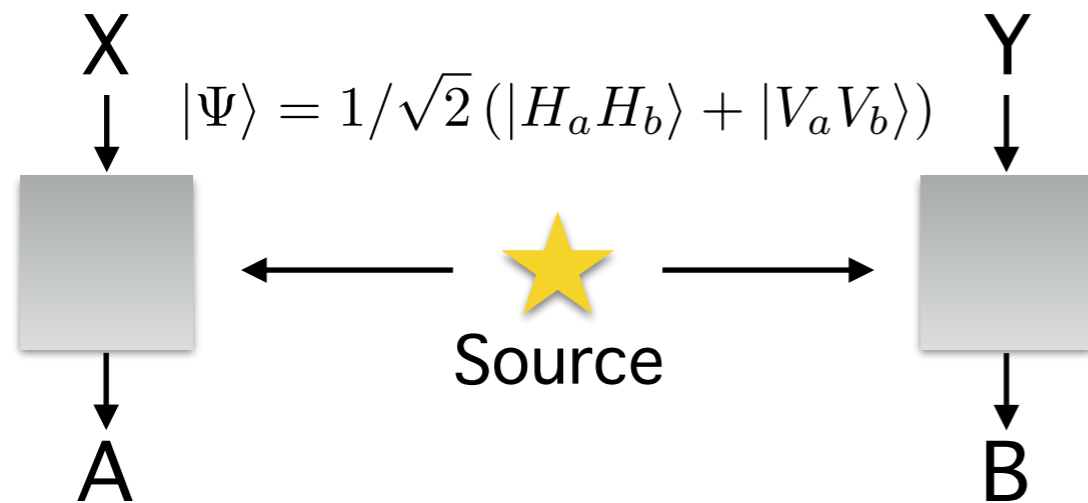
quantum correlations with proper settings:

$$S = 2\sqrt{2} > 2$$

Detection Loophole !

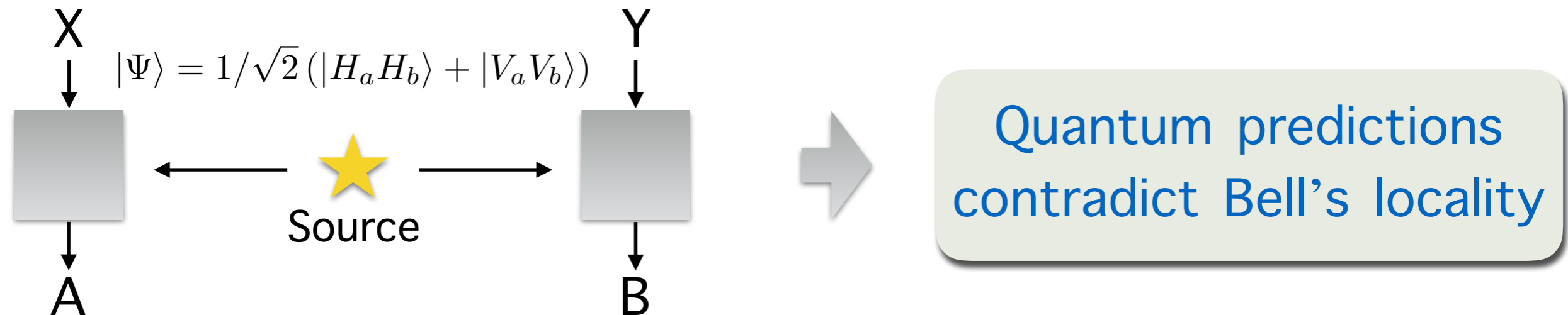
$$\eta_{\text{thr}} = 2(\sqrt{2}-1) \approx 0.83$$

Bell-CHSH inequality (1969)



Quantum predictions
contradict Bell's locality

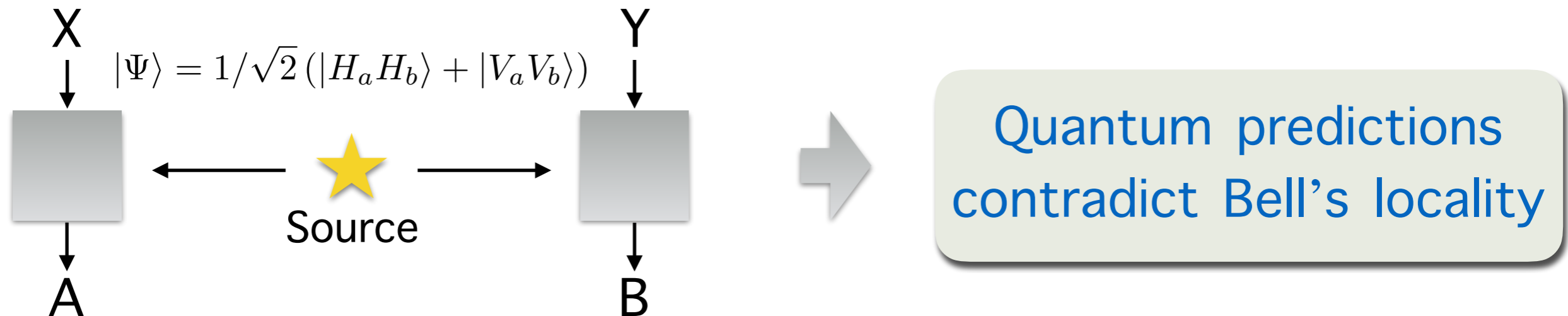
Bell-CHSH inequality (1969)



Power of entanglement

- Pair of particles \longrightarrow well defined properties
- Individual particles \longrightarrow NO well defined properties

Bell-CHSH inequality (1969)



Power of entanglement

- Pair of particles \longrightarrow well defined properties
- Individual particles \longrightarrow NO well defined properties

Device Independent Applications
(Device Independent Quantum Information Processing)

Eberhardt inequality & Fair sampling

$$N^{++}(a,b) - N^{+0}(a,b') - N^{0+}(a',b) - N^{++}(a',b') \leq 0$$

Eberhardt inequality & Fair sampling

Not coincidences clics

$$\boxed{N^{++}(a,b)} - \boxed{N^{+0}(a,b') - N^{0+}(a',b)} - \boxed{N^{++}(a',b')} \leq 0$$

Coincidences clics

Coincidences clics

Eberhardt inequality & Fair sampling

Not coincidences clics

$$\boxed{N^{++}(a,b)} - \boxed{N^{+0}(a,b') - N^{0+}(a',b)} - \boxed{N^{++}(a',b')} \leq 0$$

Coincidences clics

Coincidences clics

Which is actually the CH-inequality:

$$N^{++}(a,b) + N^{++}(a,b') + N^{++}(a',b) - N^{++}(a',b') \\ - S(a) - S(b) \leq 0$$

With:

$$S(a,j) = N^{+0}(a,j) + N^{++}(a,j)$$

Eberhardt inequality & Fair sampling

Not coincidences clics

$$\boxed{N^{++}(a,b)} - \boxed{N^{+0}(a,b') - N^{0+}(a',b)} - \boxed{N^{++}(a',b')} \leq 0$$

Coincidences clics

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Which is actually the CH-inequality:

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With:

$$S(a,j) = N^{+0}(a,j) + N^{++}(a,j)$$



$$\eta^2 \times [P^{++}(a,b) + P^{++}(a,b') + P^{++}(a',b) - P^{++}(a',b')] - \eta \times [P^+(a) + P^+(b)] \equiv J \leq 0$$

Eberhardt inequality & Fair sampling

Not coincidences clics

$$\boxed{N^{++}(a,b)} - \boxed{N^{+0}(a,b') - N^{0+}(a',b)} - \boxed{N^{++}(a',b')} \leq 0$$

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Single >> Coincidences

Eberhardt inequality & Fair sampling

$$|\Psi_E\rangle = (1+r^2)^{-1/2} \{ |x_A, y_B\rangle + r |y_A, x_B\rangle \}$$

Coincidence: $P^{++}(a,b) = (1+r^2)^{-1} [\cos(a)\sin(b) + r \sin(a)\cos(b)]^2$

Singles: $P^+(a) = (1+r^2)^{-1} [\cos^2(a) + r^2 \sin^2(a)]$

$$P^+(b) = (1+r^2)^{-1} [r^2 \cos^2(b) + \sin^2(b)]$$

Eberhardt inequality & Fair sampling

- Use non maximally entangled state:

$$|\Psi_E\rangle = (1+r^2)^{-1/2} \{ |x_A, y_B\rangle + r |y_A, x_B\rangle \}$$

Coincidence: $P^{++}(a,b) = (1+r^2)^{-1} [\cos(a)\sin(b) + r \sin(a)\cos(b)]^2$

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With:
 $\cos(a) \approx 0$
 $\sin(b) \approx 0,$

Eberhardt inequality & Fair sampling

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- Use non maximally entangled state:

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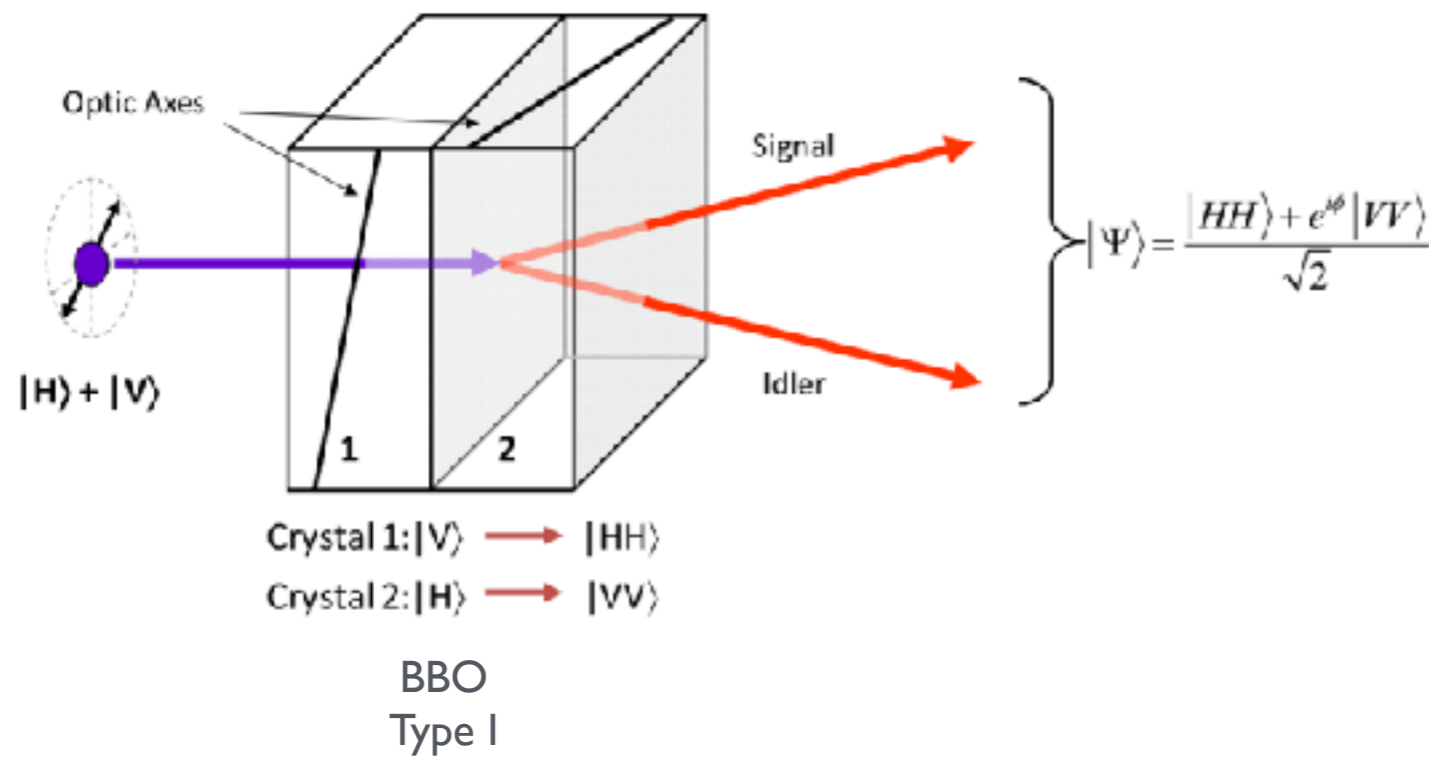


$$J \approx 3\eta r^2 - 2r^2 \leq 0$$

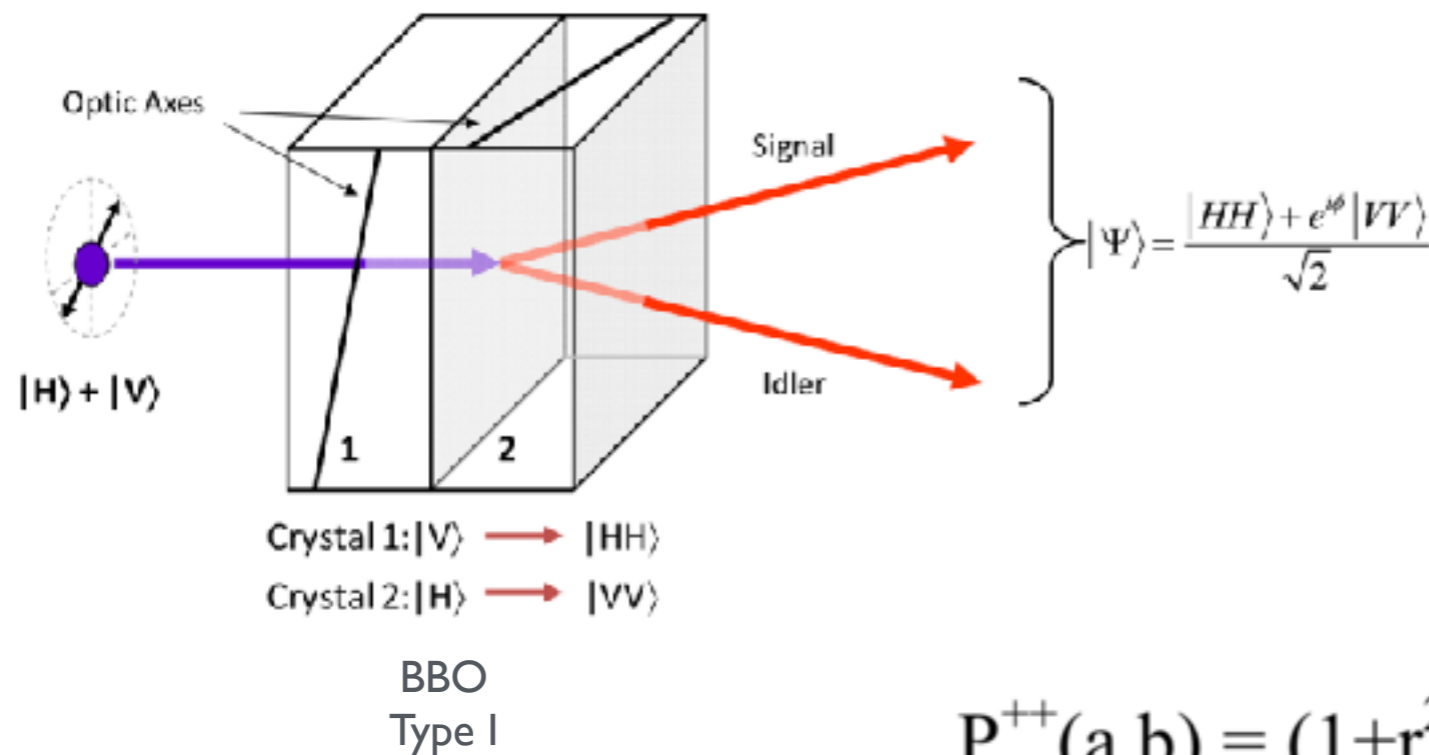
Detection Loophole !

$$\eta_{\text{thr}} = 2/3$$

Non-maximally entangled states



Non-maximally entangled states



$$P^{++}(a,b) = (1+r^2)^{-1} [\cos(a)\cos(b) + r \sin(a)\sin(b)]^2$$

$$P^+(a) = (1+r^2)^{-1} [\cos^2(a) + r^2 \sin^2(a)]$$

OUTLINE

1. Introduction

2. Fundamental tests on nonlocality

3. Generalised Bell inequality

4. Conclusion & outlook

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- **Measurement Dependent Locality (MDL)**
 - Theoretical framework
 - Experimental demonstration
- **Limited Detection Locality (LDL)**
 - Theoretical framework
 - Experimental demonstration

4. C

MDL inequality

Arbitrarily small amount of measurement independence is sufficient to manifest quantum nonlocality

Gilles Pütz,^{1,*} Denis Rosset,¹ Tomer Jack Barnea,¹ Yeong-Cherng Liang,² and Nicolas Gisin¹

¹*Group of Applied Physics, University of Geneva, CH-1211 Geneva 4, Switzerland.*

²*Institute for Theoretical Physics, ETH Zurich, 8093 Zurich, Switzerland.*

(Dated: October 6, 2018)

The use of Bell's theorem in any application or experiment relies on the assumption of free choice or, more precisely, measurement independence, meaning that the measurements can be chosen freely. Here, we prove that even in the simplest Bell test — one involving 2 parties each performing 2 binary-outcome measurements — an *arbitrarily small amount* of measurement independence is *sufficient* to manifest quantum nonlocality. To this end, we introduce the notion of measurement dependent locality and show that the corresponding correlations form a convex polytope. These correlations can thus be characterized efficiently, e.g., using a finite set of Bell-like inequalities — an observation that enables the systematic study of quantum nonlocality and related applications under limited measurement independence.

$$\ell P(0000) - h(P(0101) + P(1010) + P(0011)) \stackrel{MDL}{\leq} 0.$$

MDL inequality

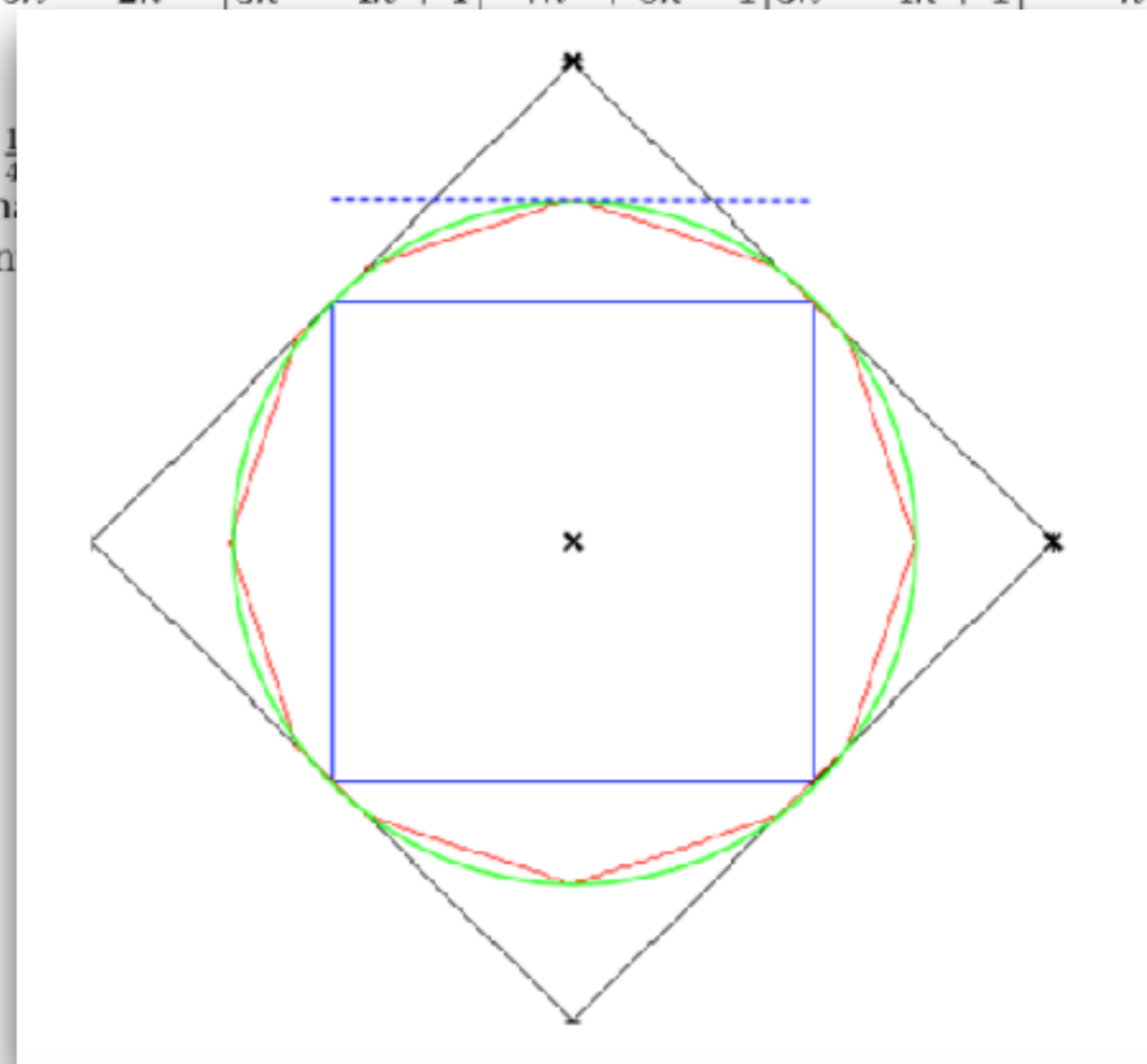
1	$P_{A X}(0 0)$	$P_{A X}(0 1)$	$P_{B Y}(0 0)$	$P(00 00)$	$P(00 10)$	$P_{B Y}(0 1)$	$P(00 01)$	$P(00 11)$
$12h^2 - 11h + 2$	$2h - 1$	$4h - 1$	$2h - 1$	$2h$	$2 - 6h$	$4h - 1$	$2 - 6h$	$-2h$
$12h^2 - 11h + 2$	$4h - 1$	$3h - 1$	$4h - 1$	$-h$	$1 - 3h$	$3h - 1$	$1 - 3h$	$1 - 3h$
$11h^2 - 8h + 1$	$-4h^2 + 5h - 1$	$5h^2 - 4h + 1$	$-4h^2 + 5h - 1$	$-3h^2 - 2h + 1$	$3h^2 - 2h$	$5h^2 - 4h + 1$	$3h^2 - 2h$	$-9h^2 + 9h - 2$
$8h^2 - 7h + 1$	$4h^2$	0	$-4h^2 + 5h - 1$	$-h$	$1 - 3h$	$-4h^2 + 2h$	$-h$	$3h - 1$
$13h^2 - 8h + 1$	$-8h^2 + 6h - 1$	$-5h^2 + 2h$	$-h^2 + h$	$5h^2 - 2h$	$h^2 - h$	0	$3h^2 - 4h + 1$	$-3h^2 + 4h - 1$
$20h^2 - 13h + 2$	$-8h^2 + 6h - 1$	$-7h^2 + 5h - 1$	$-8h^2 + 6h - 1$	$5h^2 - 2h$	$3h^2 - 4h + 1$	$-7h^2 + 5h - 1$	$3h^2 - 4h + 1$	$-h^2 + h$
$1 - 4h$	$3h - 1$	0	$3h - 1$	$1 - 3h$	h	0	h	$-h$

TABLE I. Conjectured families of MDL inequalities for $h \in]\frac{1}{4}, \frac{1}{3}[$. The Table contains the coefficients belonging to each term (given in the first row). We denote by $P_{A|X}(a|x)$ the marginal distribution over Alice's output A conditioned on her input X and similarly for Bob. The expression being ≤ 0 is a representative MDL inequality from each family.

MDL inequality

1	$P_{A X}(0 0)$	$P_{A X}(0 1)$	$P_{B Y}(0 0)$	$P(00 00)$	$P(00 10)$	$P_{B Y}(0 1)$	$P(00 01)$	$P(00 11)$
$12h^2 - 11h + 2$	$2h - 1$	$4h - 1$	$2h - 1$	$2h$	$2 - 6h$	$4h - 1$	$2 - 6h$	$-2h$
$12h^2 - 11h + 2$	$4h - 1$	$3h - 1$	$4h - 1$	$-h$	$1 - 3h$	$3h - 1$	$1 - 3h$	$1 - 3h$
$11h^2 - 8h + 1$	$-4h^2 + 5h - 1$	$5h^2 - 4h + 1$	$-4h^2 + 5h - 1$	$-3h^2 - 2h + 1$	$3h^2 - 2h$	$5h^2 - 4h + 1$	$3h^2 - 2h$	$-9h^2 + 9h - 2$
$8h^2 - 7h + 1$	$4h^2$	0	$-4h^2 + 5h - 1$	$-h$	$1 - 3h$	$-4h^2 + 2h$	$-h$	$3h - 1$
$13h^2 - 8h + 1$	$-8h^2 + 6h - 1$	$-5h^2 + 2h$	$-h^2 + h$	$5h^2 - 2h$	$h^2 - h$	0	$3h^2 - 4h + 1$	$-3h^2 + 4h - 1$
$20h^2 - 13h + 2$	$-8h^2 + 6h - 1$	$-7h^2 + 5h - 1$	$-8h^2 + 6h - 1$	$5h^2 - 2h$	$3h^2 - 4h + 1$	$-7h^2 + 5h - 1$	$3h^2 - 4h + 1$	$-h^2 + h$
$1 - 4h$	$3h - 1$	0	$3h - 1$					

TABLE I. Conjectured families of MDL inequalities for $h \in]\frac{1}{3}, \frac{1}{2}[$ (given in the first row). We denote by $P_{A|X}(a|x)$ the marginal and similarly for Bob. The expression being ≤ 0 is a representation



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Any pure non maximally entangled state* & every $\ell > 0$

Experimental realization

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$$\ell P(0000) - (1 - 3\ell)(P(0101) + P(1010) + P(0011)) \stackrel{\text{MDL}}{\leq} 0$$

$$|\Psi\rangle = \frac{1}{\sqrt{3}}(|01\rangle + |10\rangle - |11\rangle)$$

input 0: $\left\{ \frac{|0\rangle + |1\rangle}{\sqrt{2}}, \frac{|0\rangle - |1\rangle}{\sqrt{2}} \right\}$ & input 1: $\{|0\rangle, |1\rangle\}$

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Alice
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$\approx 0,083$

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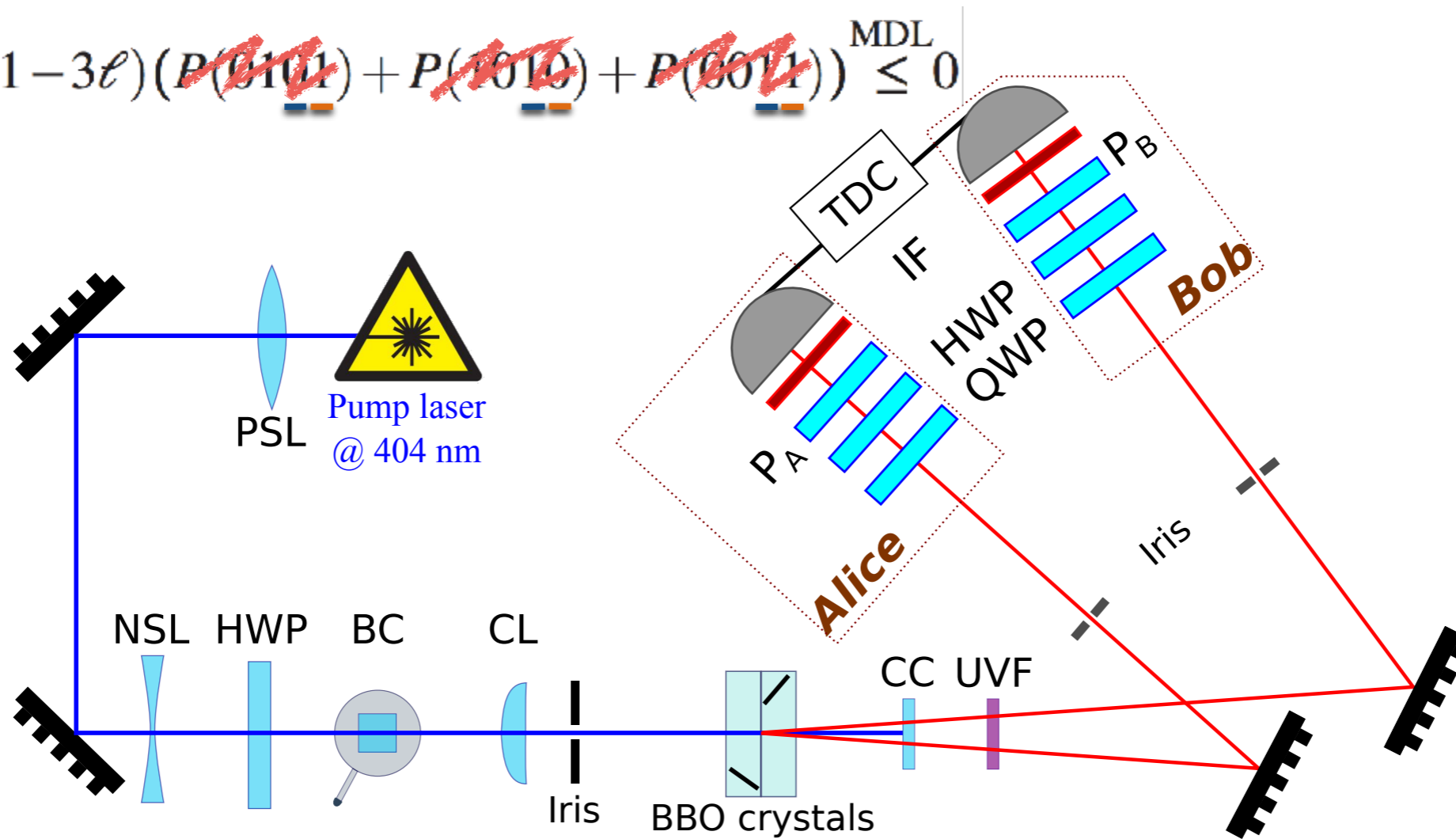
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$$|\Psi\rangle = \frac{1}{\sqrt{3}} (|01\rangle + |10\rangle - |11\rangle) \stackrel{\text{golden state}}{=} \frac{1}{\sqrt{3}} \left(\frac{\sqrt{5}-1}{2} |H_A, H_B\rangle + \frac{\sqrt{5}+1}{2} |V_A, V_B\rangle \right)$$

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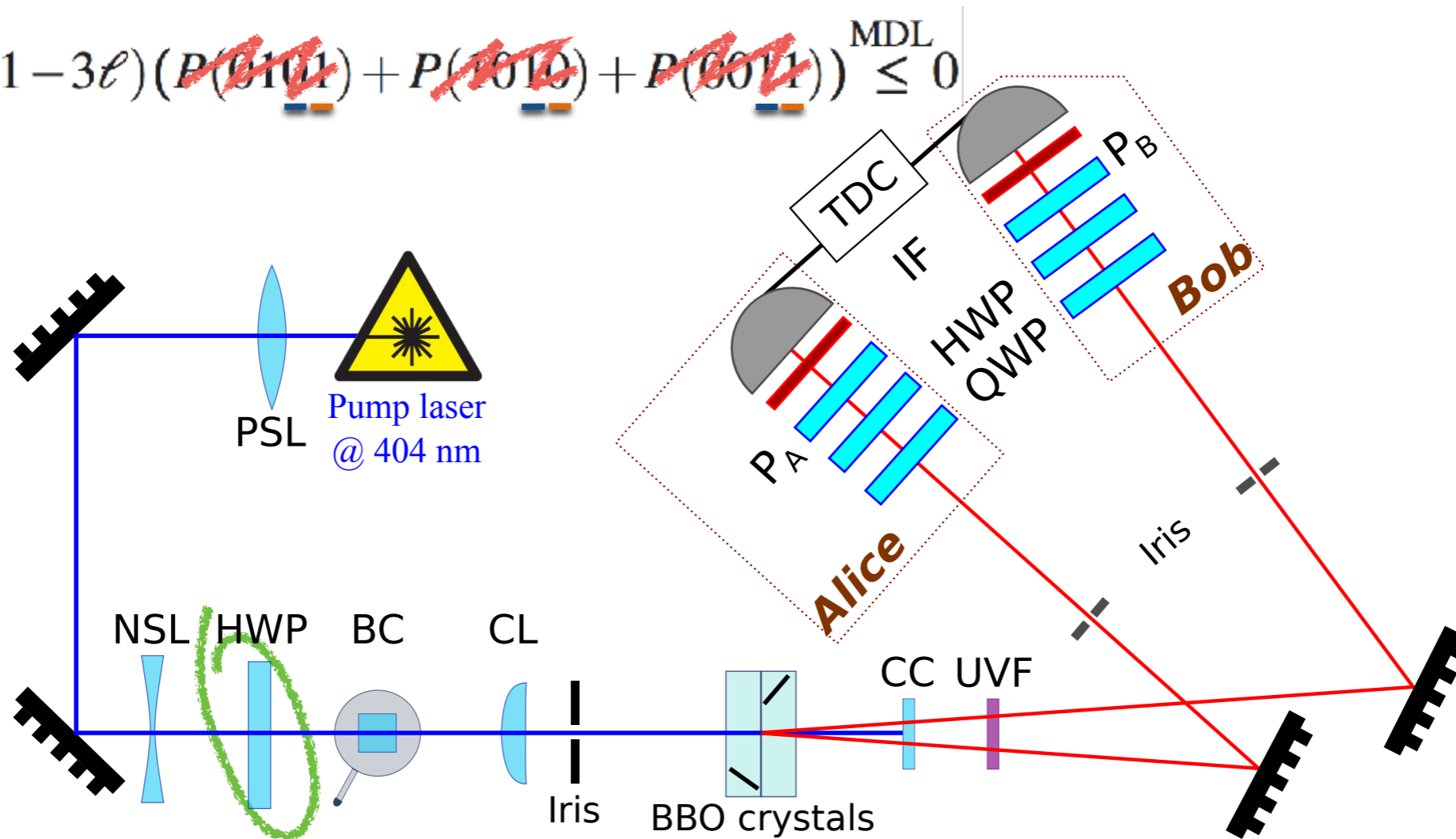


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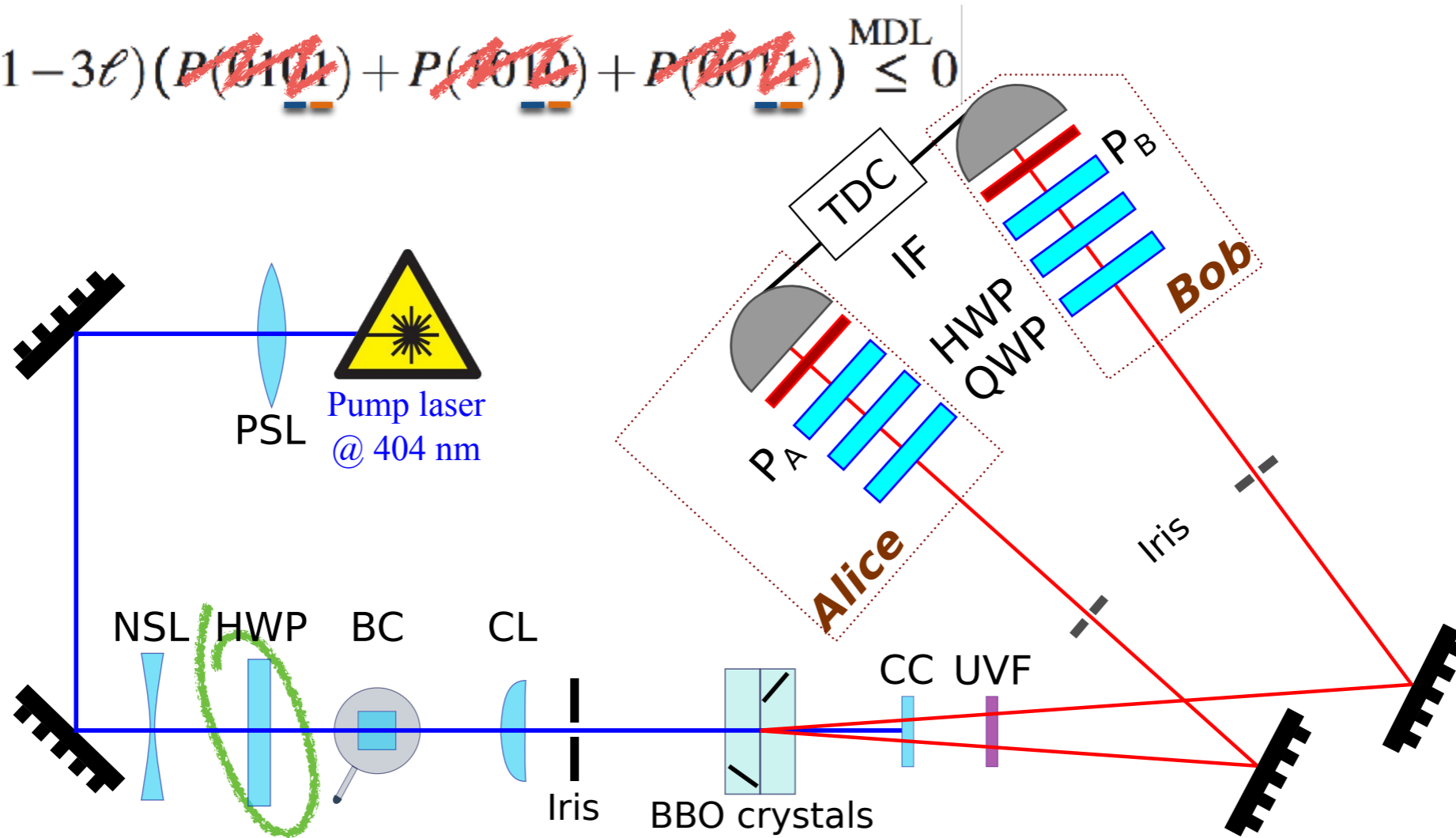


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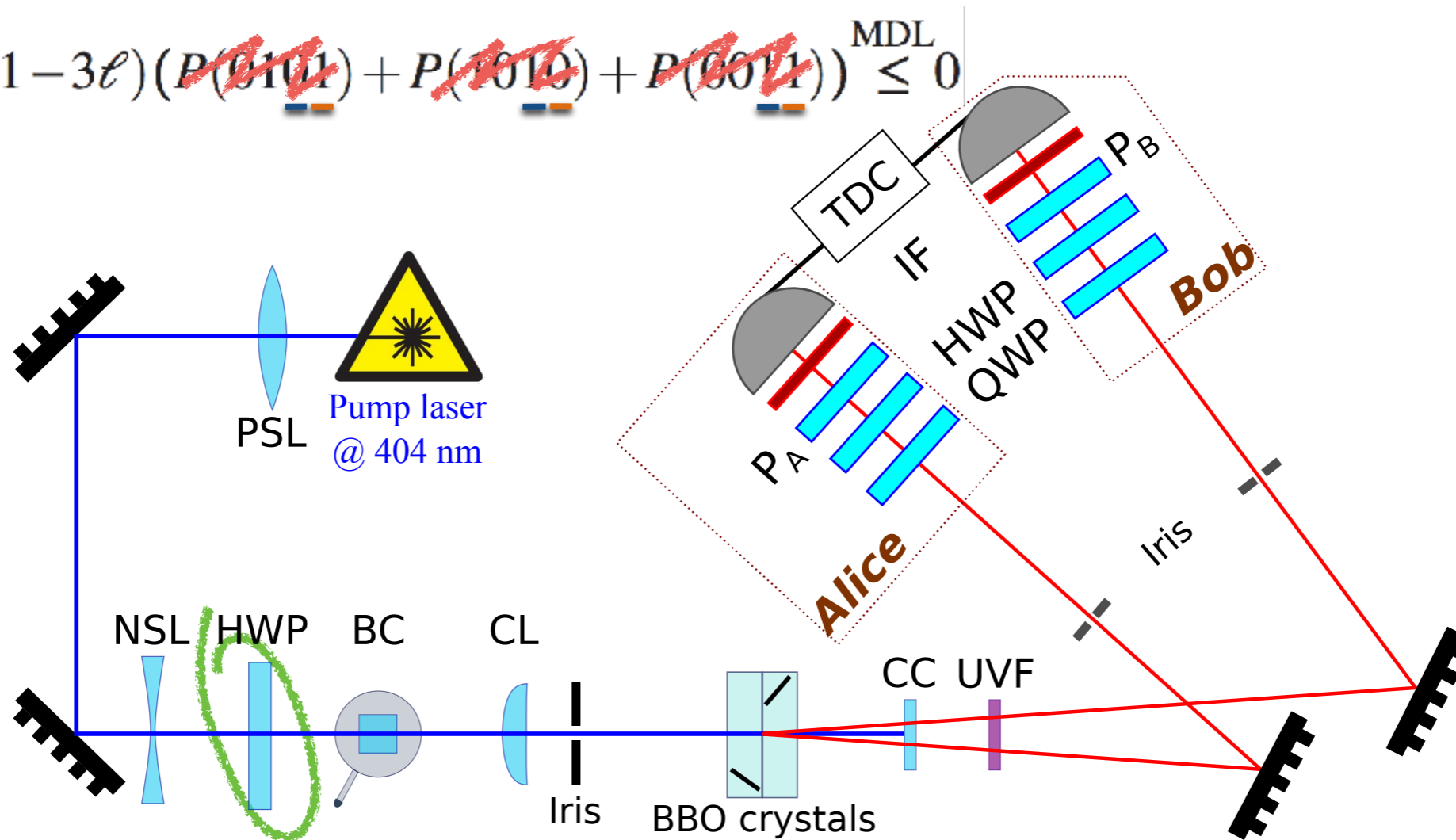
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Projectors:

$$\theta = \arccos \sqrt{1/2 + 1/\sqrt{5}}$$

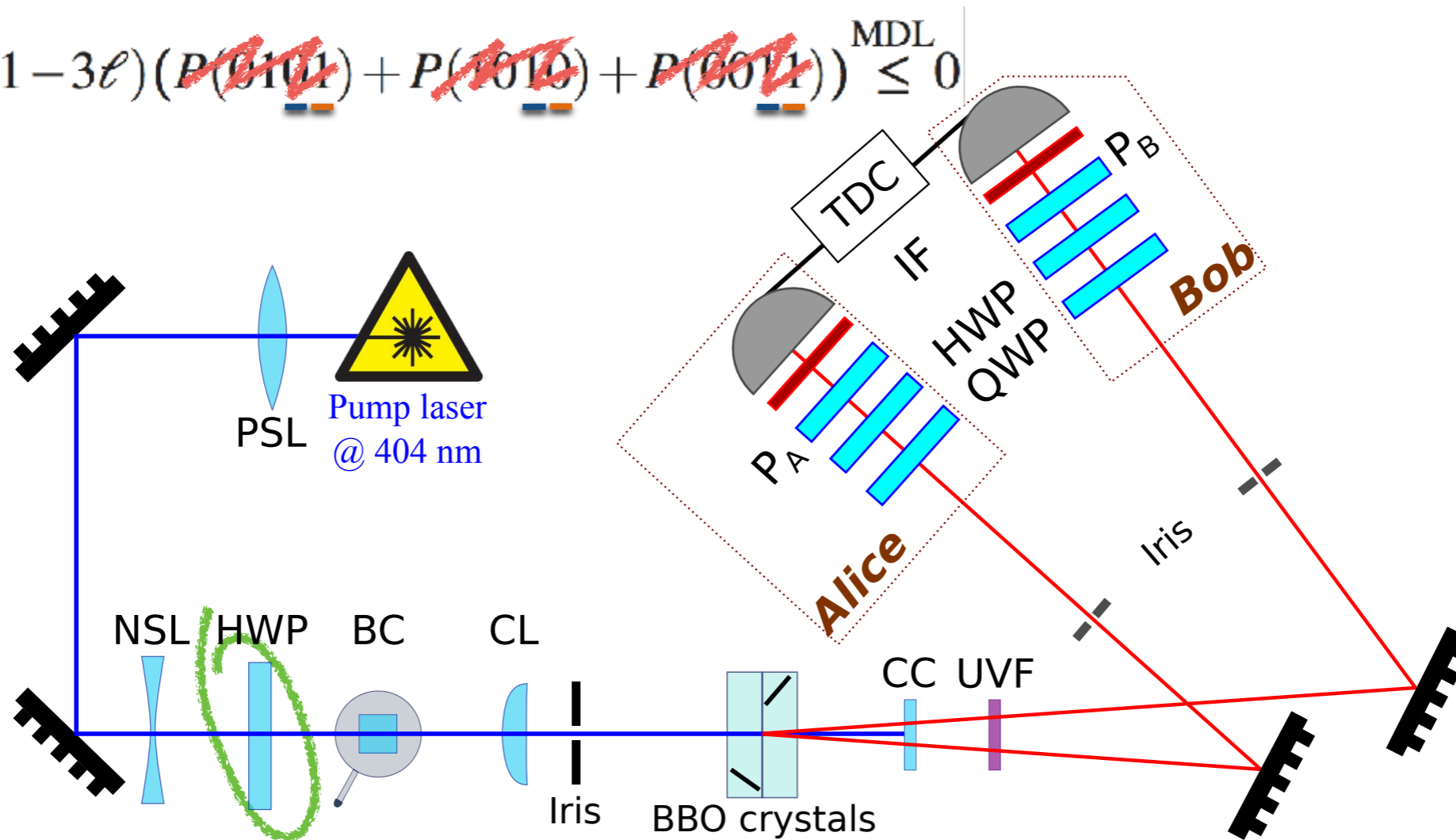
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Results

Analyseurs dans la base $\{A_0, B_0\}$

Alice α (°)	Bob β (°)	Coïncidences (/30 s)	Bruit (/30 s)
13,3	-13,3	2939	14
13,3	76,7	2926	27
103,3	-13,3	3040	48
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Analyseurs dans la base $\{A_0, B_1\}$

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13,3	301,7	129	26
13,3	31,7	5394	9
103,3	301,7	25240	163
103,3	31,7	5895	72

Analyseurs dans la base $\{A_1, B_0\}$

Alice α (°)	Bob β (°)	Coïncidences (/30 s)	Bruit (/30 s)
58,3	-13,3	114	32
58,3	76,7	21780	155
328,3	-13,3	6247	15
328,3	76,7	6552	78

Analyseurs dans la base $\{A_1, B_1\}$

Alice α (°)	Bob β (°)	Coïncidences (/30 s)	Bruit (/30 s)
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328,3	301,7	13405	72
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58,3	301,7	12898	112

Theory: Puetz *et al*, PRL, **113** 190402 (2014)

Experiment: Aktas *et al*, PRL, **114** 220404 (2015)

Results

Results

Alice (x)	Bob (y)	Coïncidences	Bruit	$P^{\text{raw}}(ab xy)$	$P^{\text{net}}(ab xy)$
0	0	2939 / 35183	14 / 269	$P(00 00) = 0,0835(10)$	$P(00 00) = 0,0838(15)$
0	1	129 / 36658	26 / 270	$P(01 01) = 0,0035(3)$	$P(01 01) = 0,0028(3)$
1	0	114 / 34693	32 / 280	$P(10 10) = 0,0033(3)$	$P(10 10) = 0,0024(3)$
1	1	130 / 36962	23 / 276	$P(00 11) = 0,0035(3)$	$P(00 11) = 0,0027(3)$

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- 1) Standard Bell test cannot do better than $\ell > 0.149$
- 2) Detector's noise is not the main limitation.

Theory: Puetz *et al*, PRL, **113** 190402 (2014)

Experiment: Aktas *et al*, PRL, **114** 220404 (2015)

Limited Detection Locality

Detection loophole:

Granting **nonlocality**
with **Bell's Inequality**



overall detection
efficiency $\geq 2/3$



- Hensen *et al*, Nature **526**, 682 (2015)
- Shalm *et al*, Phys. Rev. Lett. **115**, 250402 (2015)
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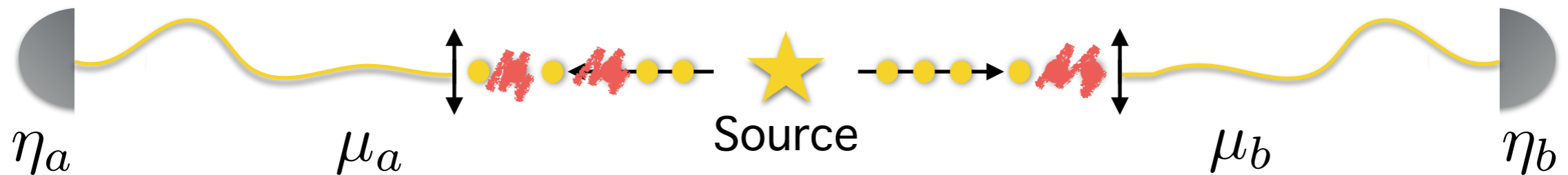
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Suppose that there exist fixed η_{min} and η_{max} with

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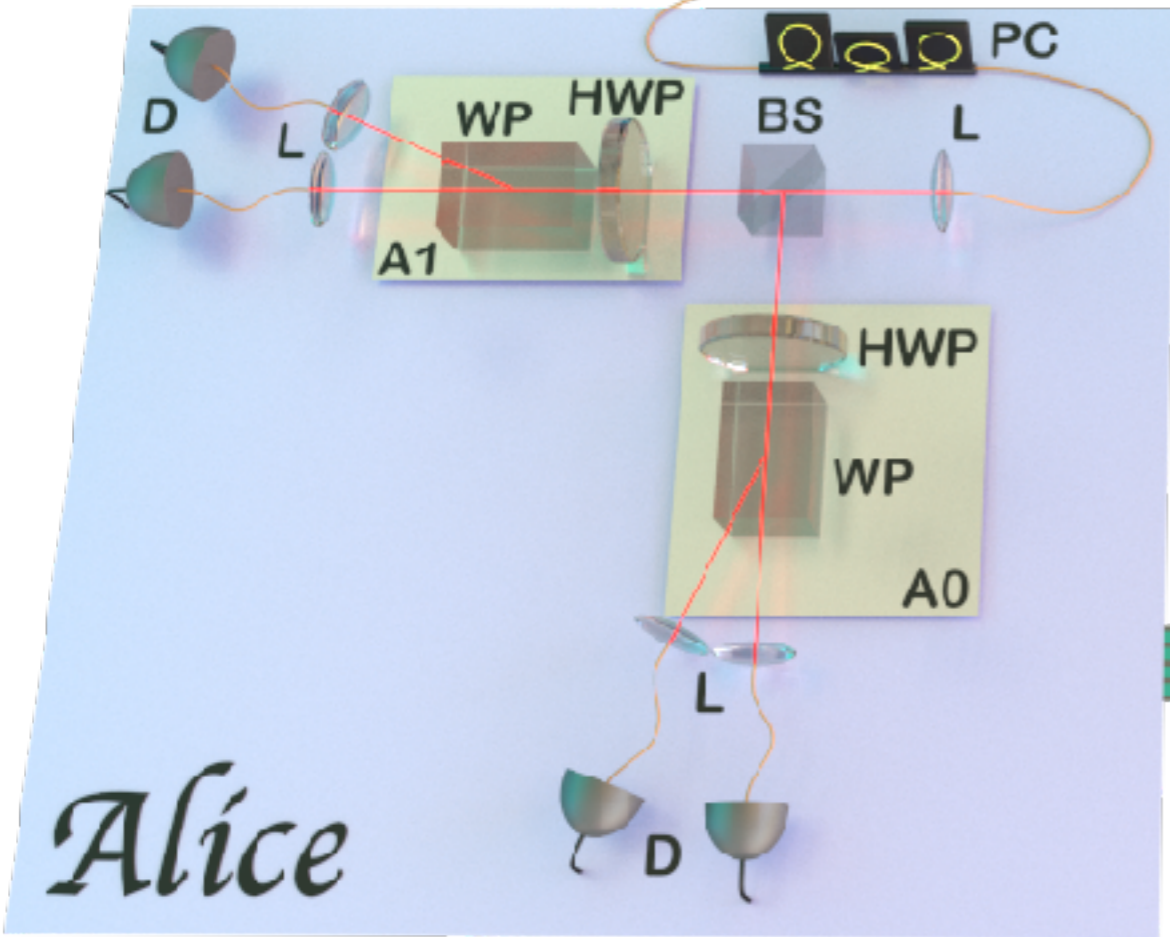
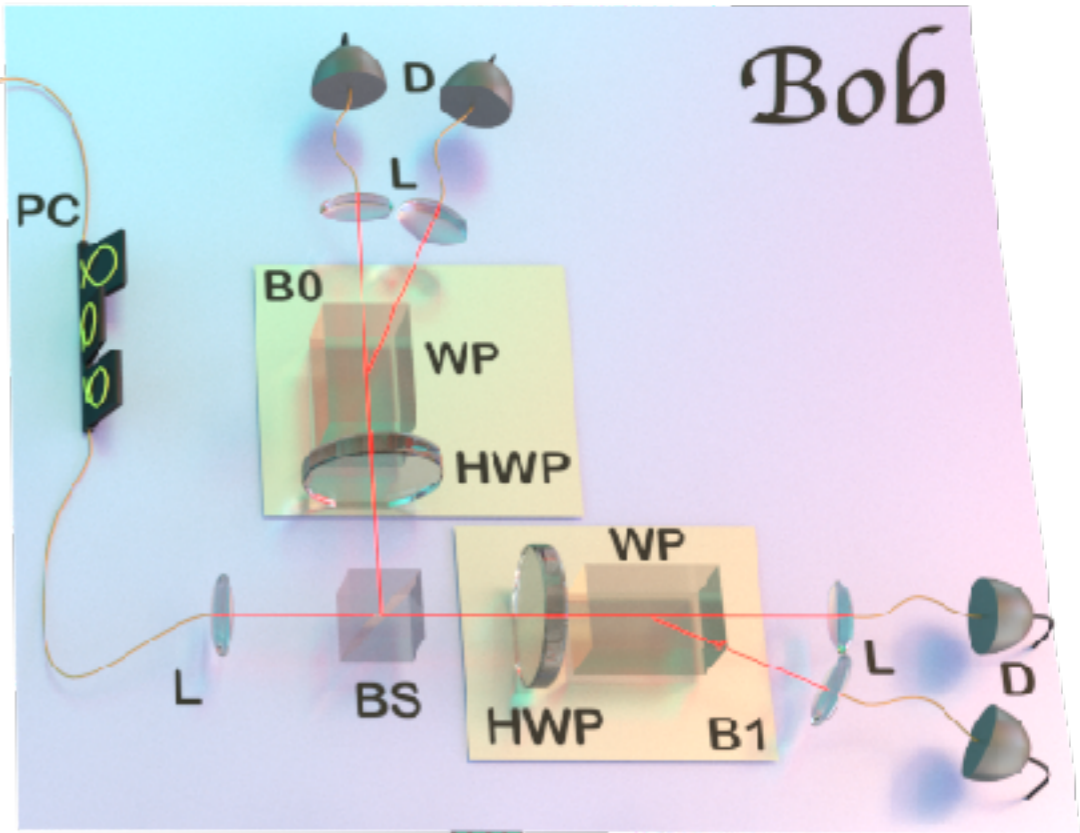
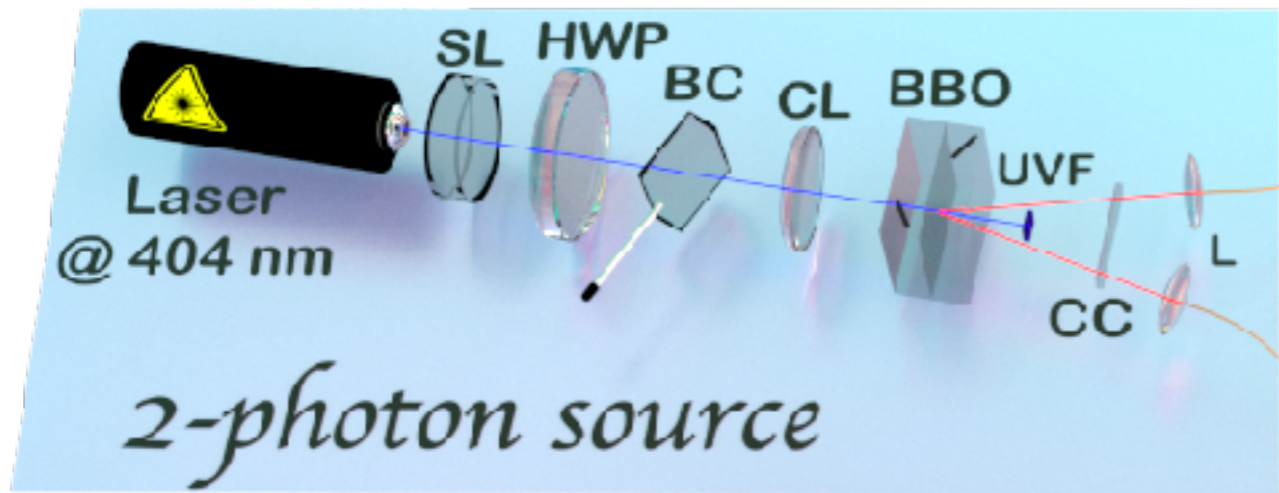


Bell-like inequality:

$$\eta_{min}^2 P(00|00) - \eta_{min}\eta_{max} P(01|01) - \eta_{min}\eta_{max} P(10|10) - \eta_{max}^2 P(00|11) \stackrel{LDL}{\leq} 0$$

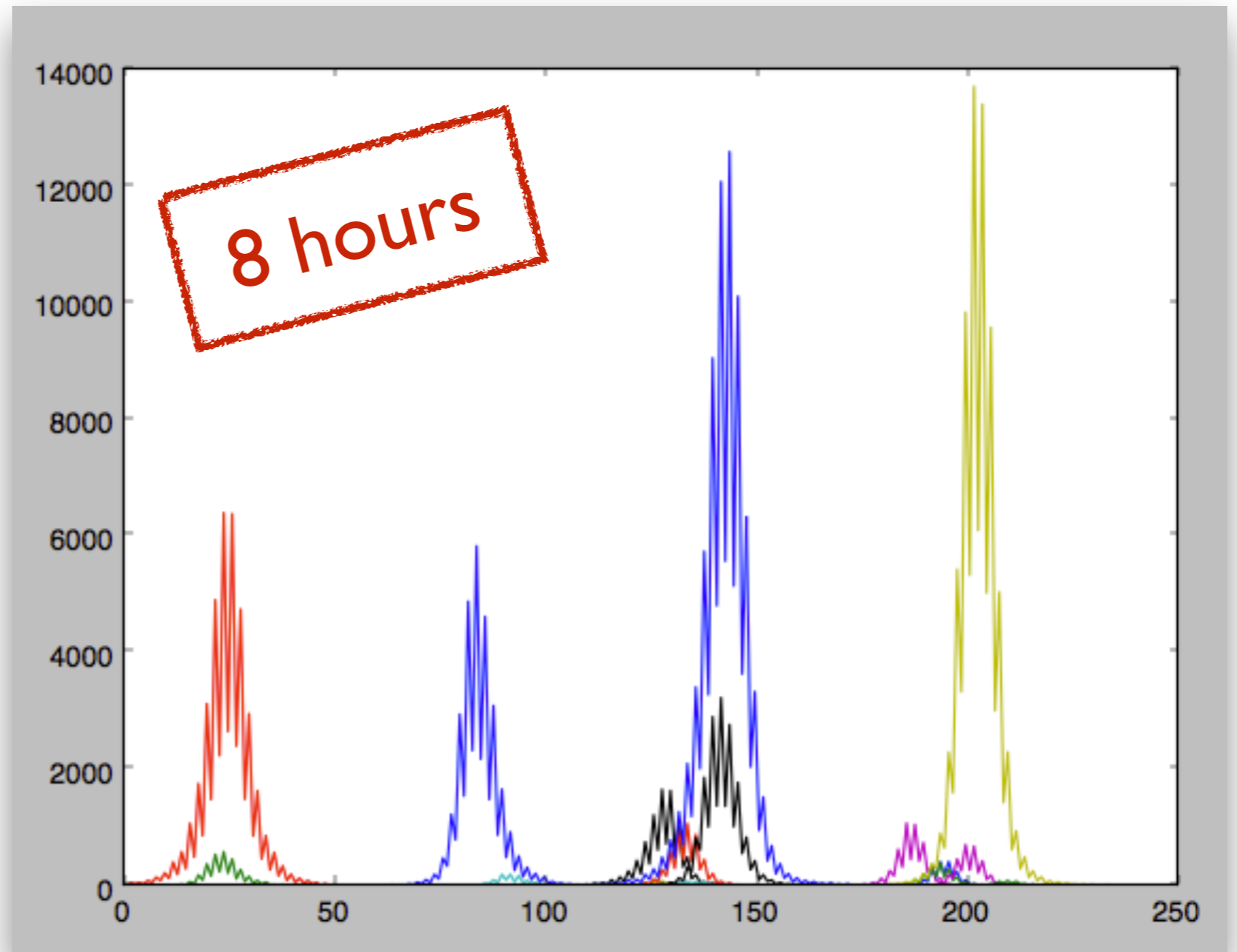
for every η_{max} & $\eta_{min} > 0$

LDL experiment and results



Design B. Gay-Para
Post-processing G. Sauder

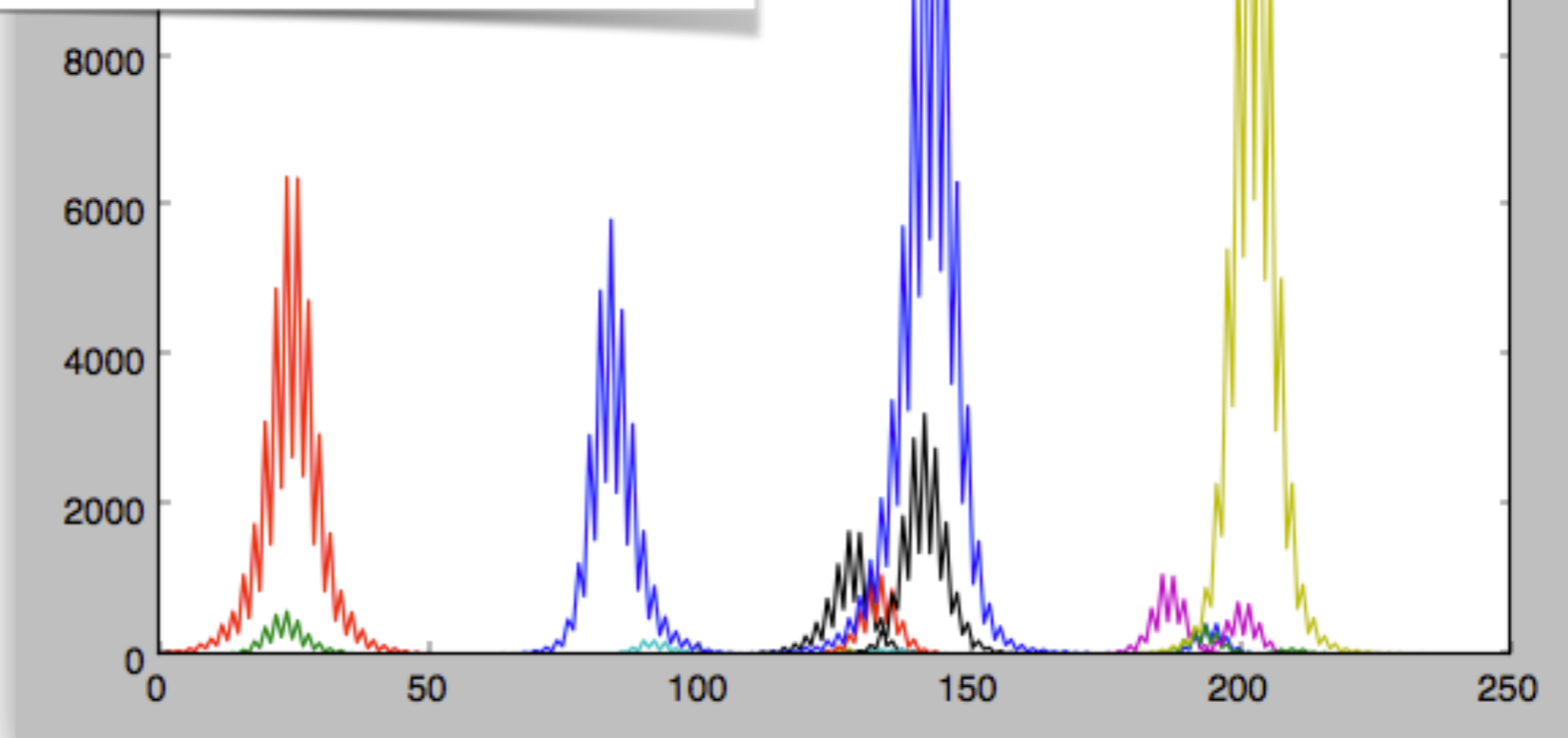
Results



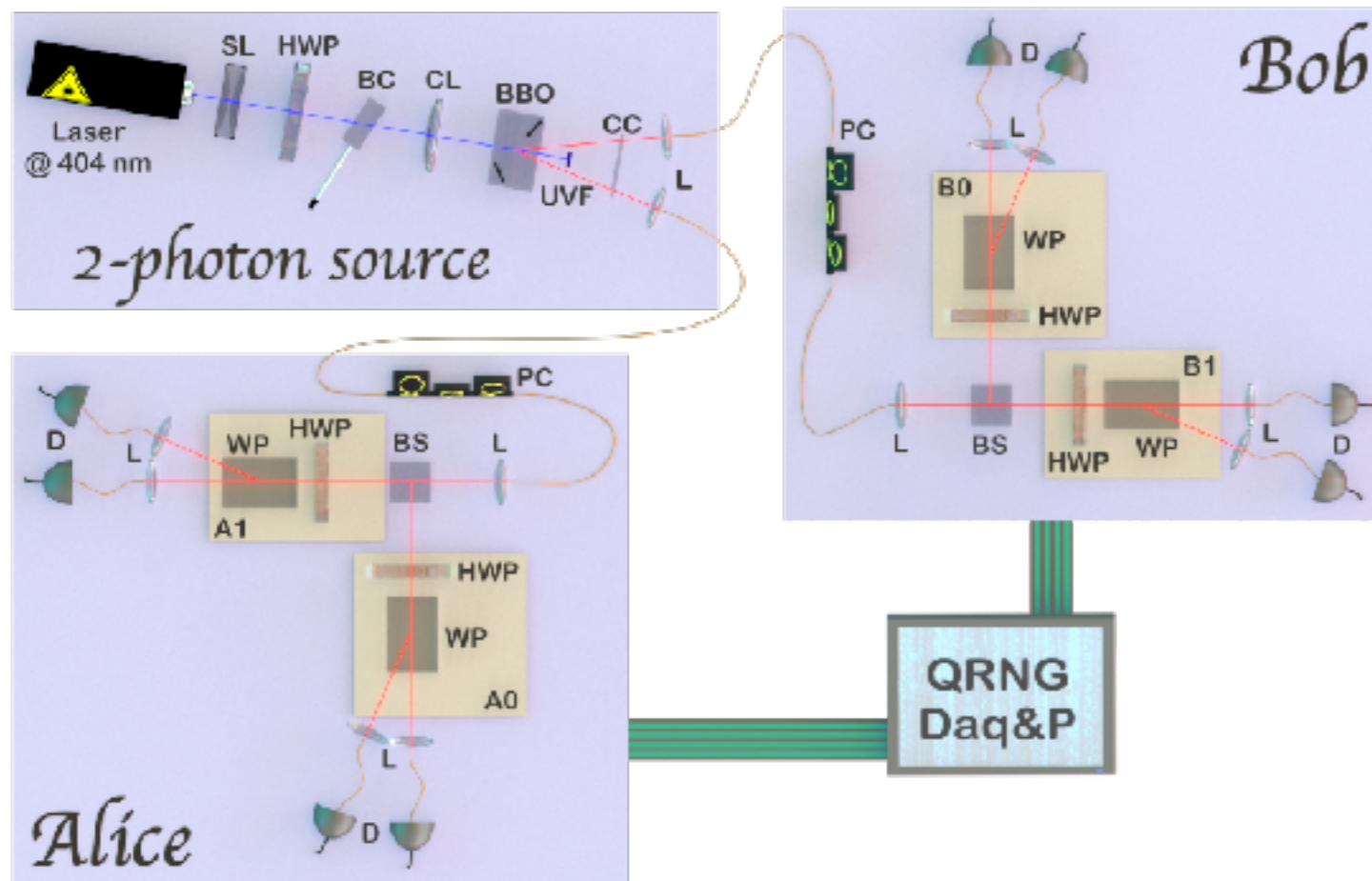
Results

Bob

Alice		Bob			
		$y = 0$		1	
x	a	$b = 0$	1	0	1
0	0	0.01977(12)	0.01003(9)	0.00435(6)	0.00112(3)
	1	0.00830(8)	0.19763(36)	0.01576(3)	0.24282(41)
1	0	0.02536(15)	0.00325(5)	0.00089(3)	0.03583(19)
	1	0.00106(3)	0.23631(41)	0.01411(12)	0.18349(39)



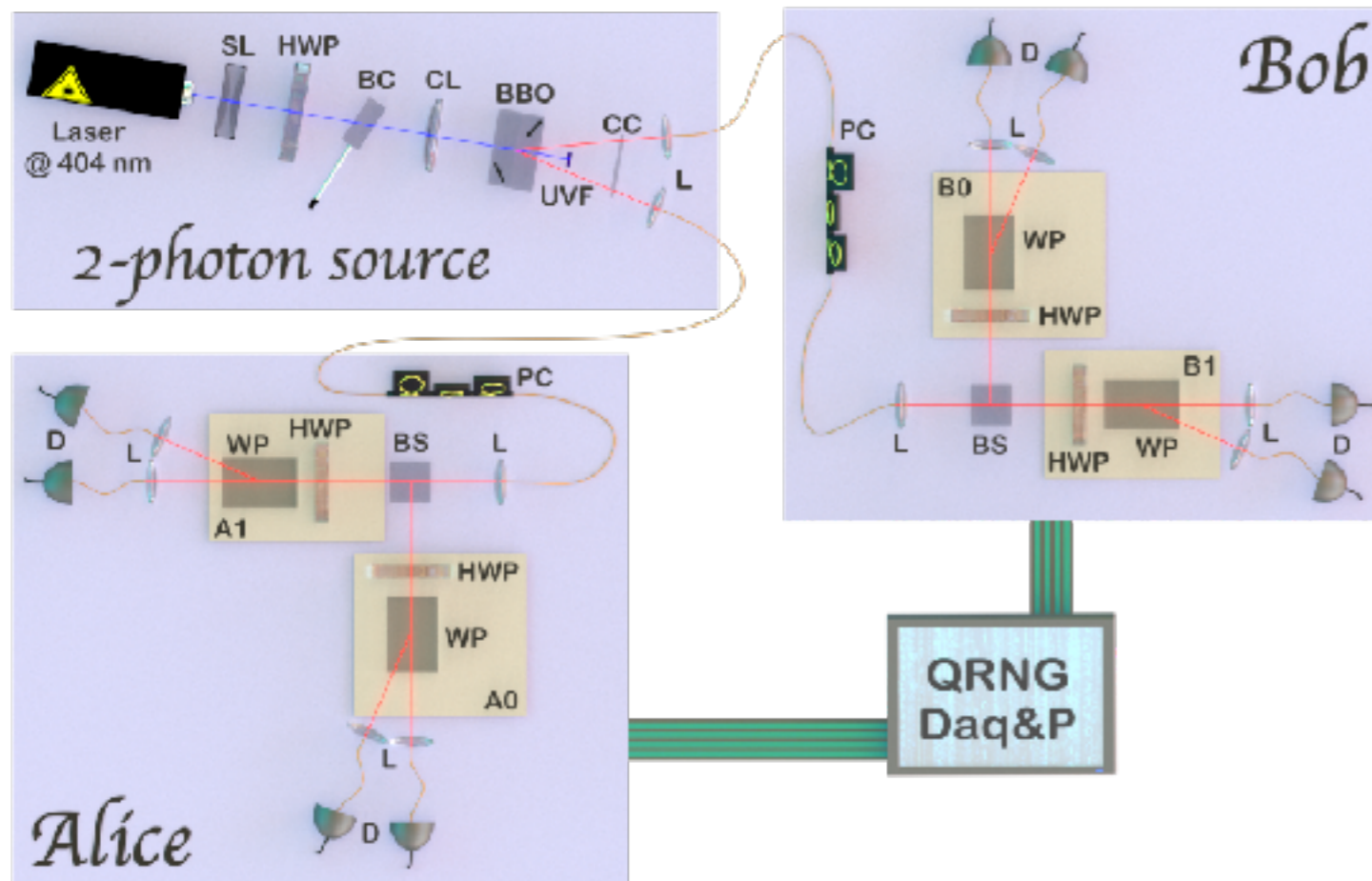
LDL experiment and results



We conclude that we can reveal non locality for :

$$\frac{\eta_{min}}{\eta_{max}} > 0.267$$

LDL experiment and results



We conclude that we can reveal non locality for :

$$\frac{\eta_{min}}{\eta_{max}} > 0.267$$



An adversary would need to lower the overall detection efficiency below 26.7% to mimic non-local correlations

OUTLINE

1. Introduction

2. Fundamental tests on nonlocality

3. Generalised Bell inequality

4. Conclusion & outlook

Conclusion

Beyond standard Bell tests:

1. How to compute new Bell like inequality.
2. (MDL) How to relax a strong original assumption of **measurement independence**.
3. (LDL) Still attest for non-locality in **lossy experiments**.



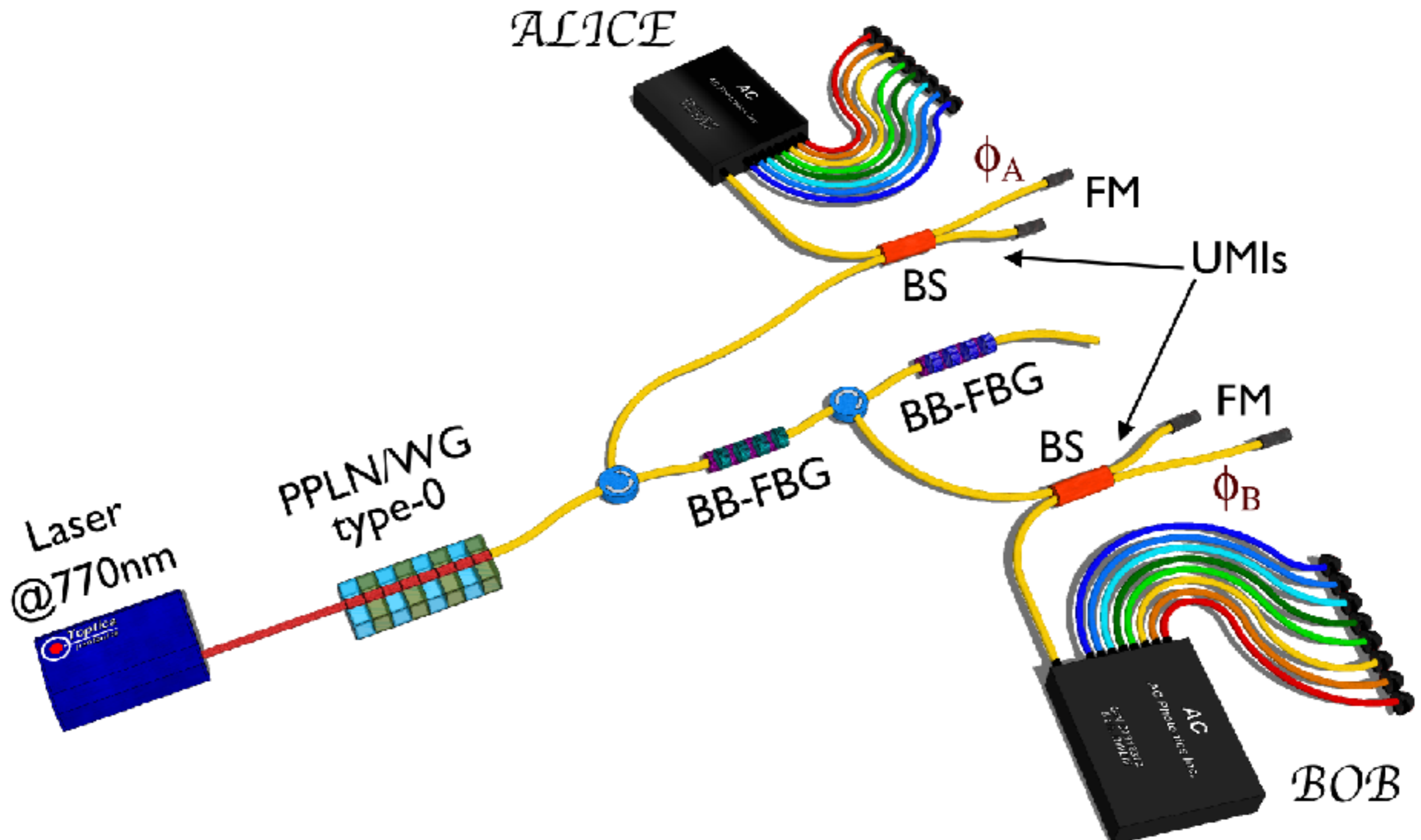
Which may be useful for (semi-)DIQIP applications

Thanks for your attention !

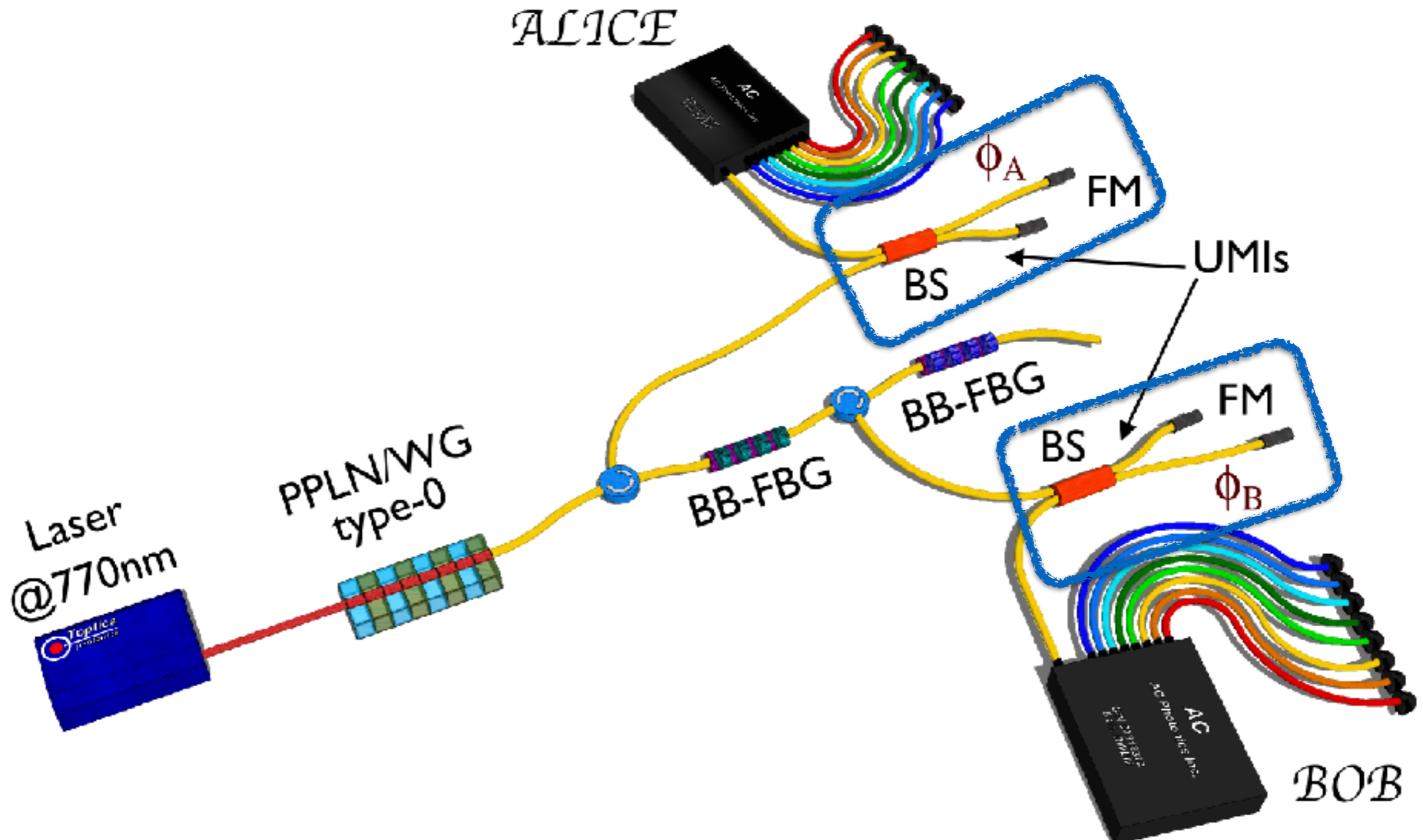
The QILM team @ CNRS LPMC Nice (& GAP Geneva) sebastien.tanzilli@unice.fr



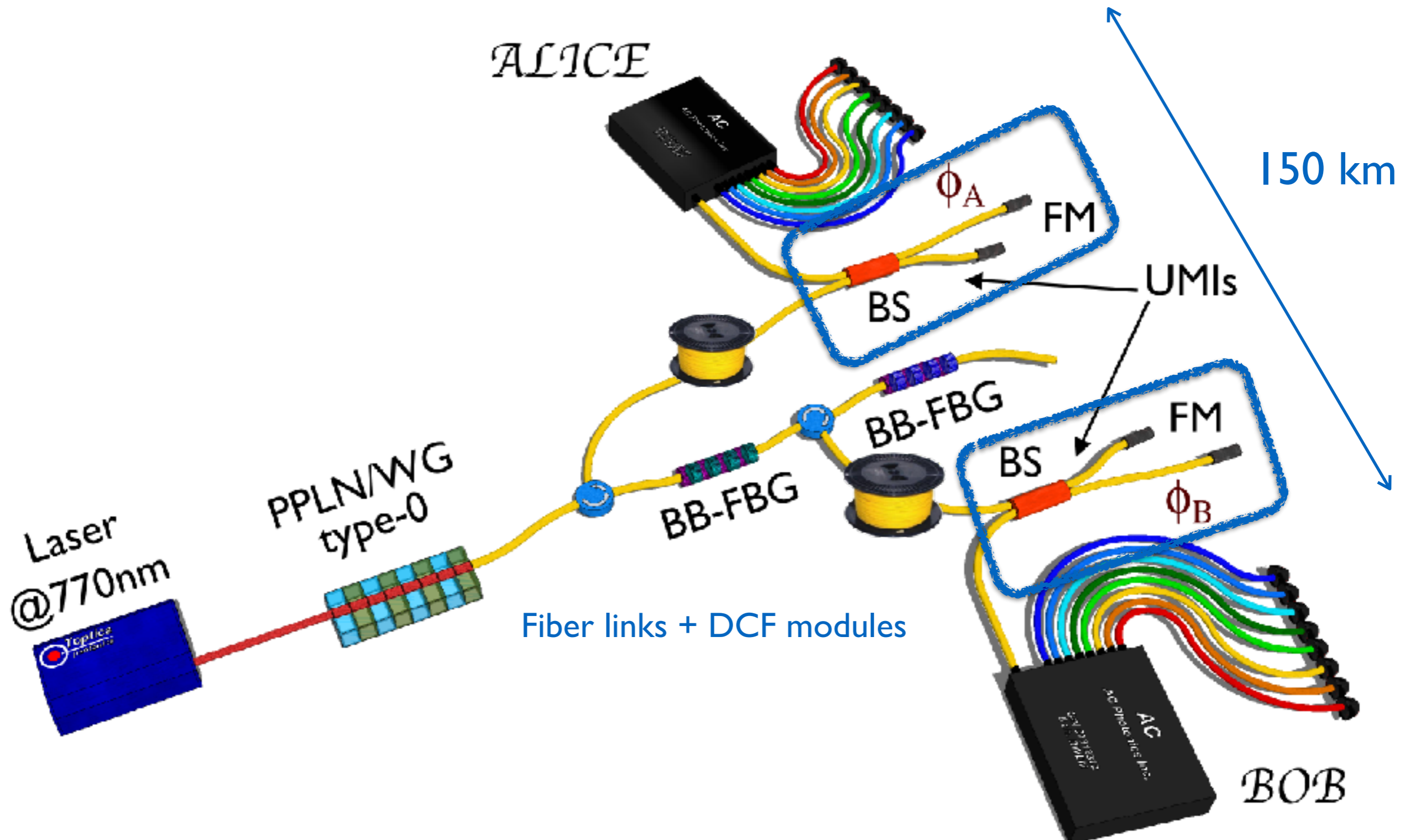
Distributing ET-entanglement



Distributing ET-entanglement



Distributing ET-entanglement

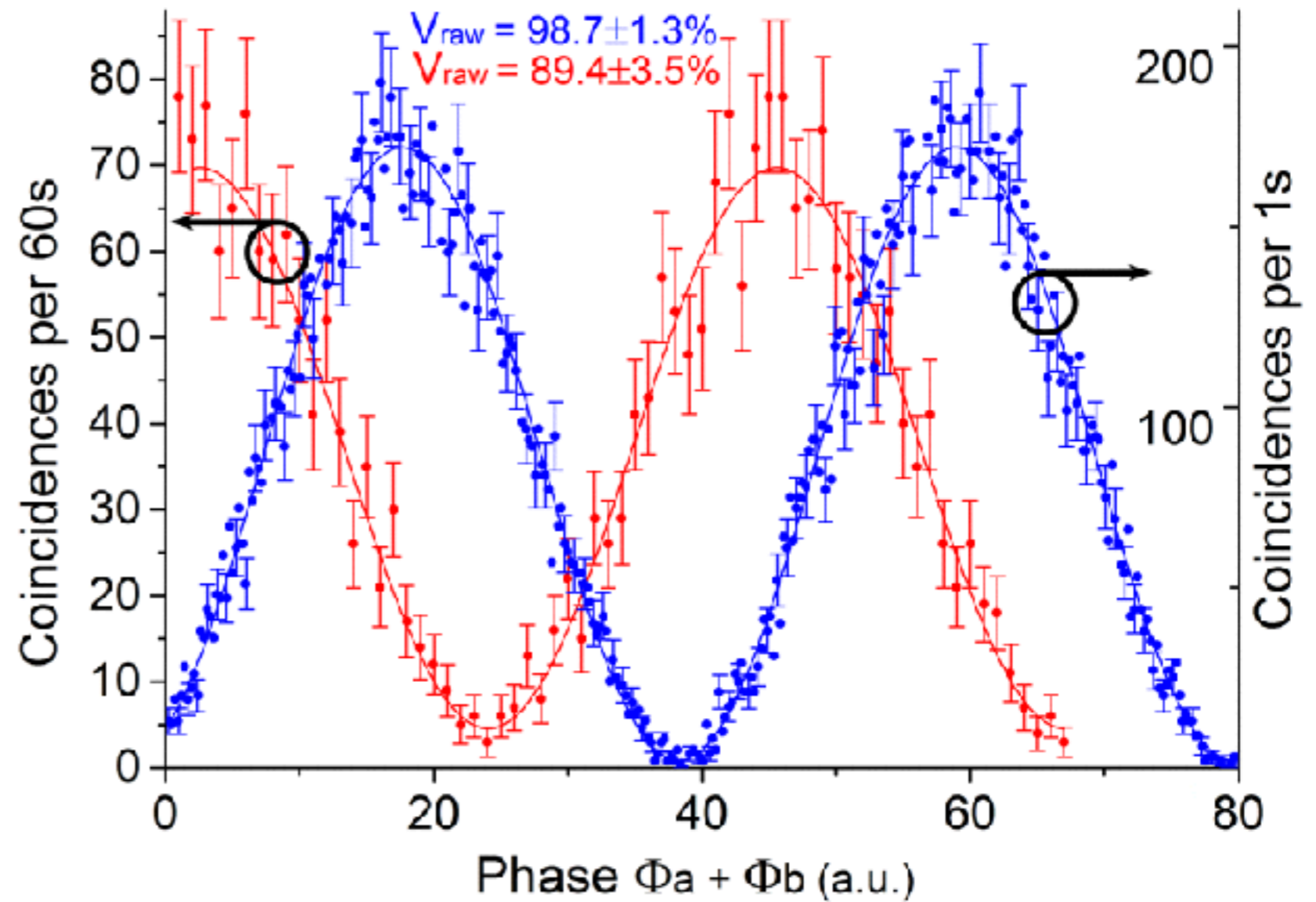


Results - using DWDMs



Aktas *et al*, LPR **10**, p451 (2016)

0 km & 150 km

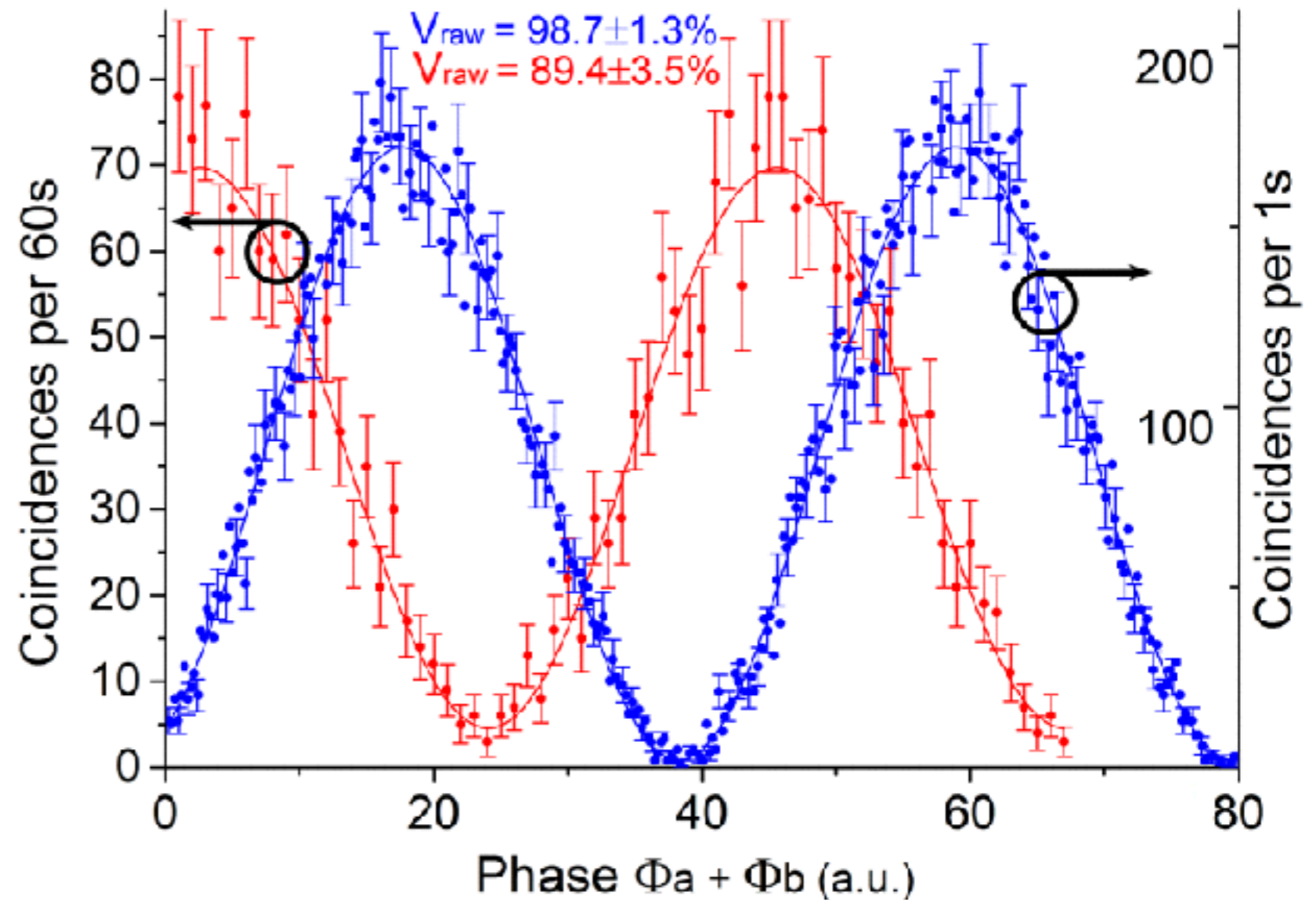


Results - using DWDMs



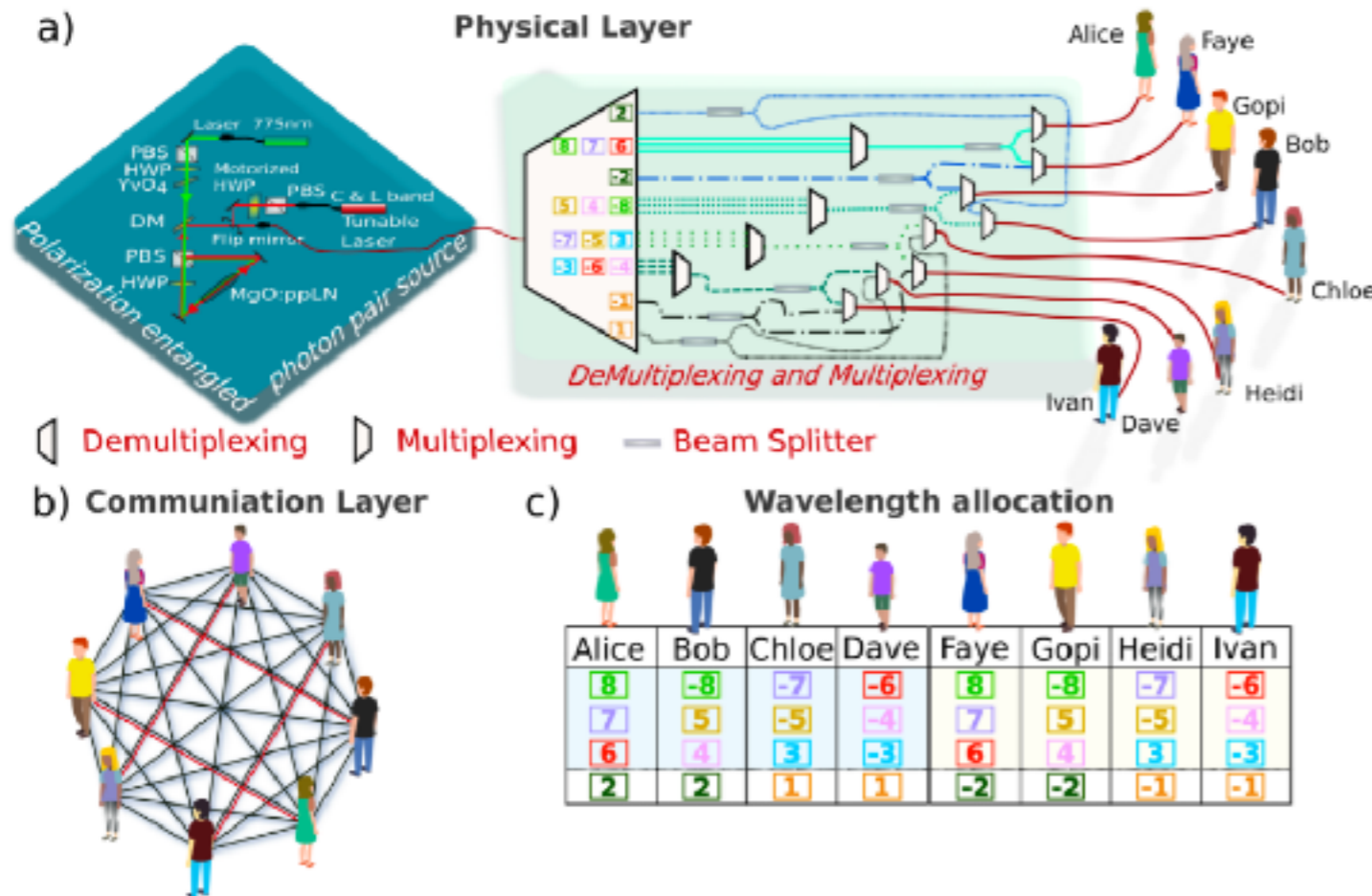
Aktas et al, LPR **10**, p451 (2016)

0 km & 150 km



Similar patterns and V 's are obtained for all pairs of channels @ 150 km
Key rate scales with the number of exploited paired channels...

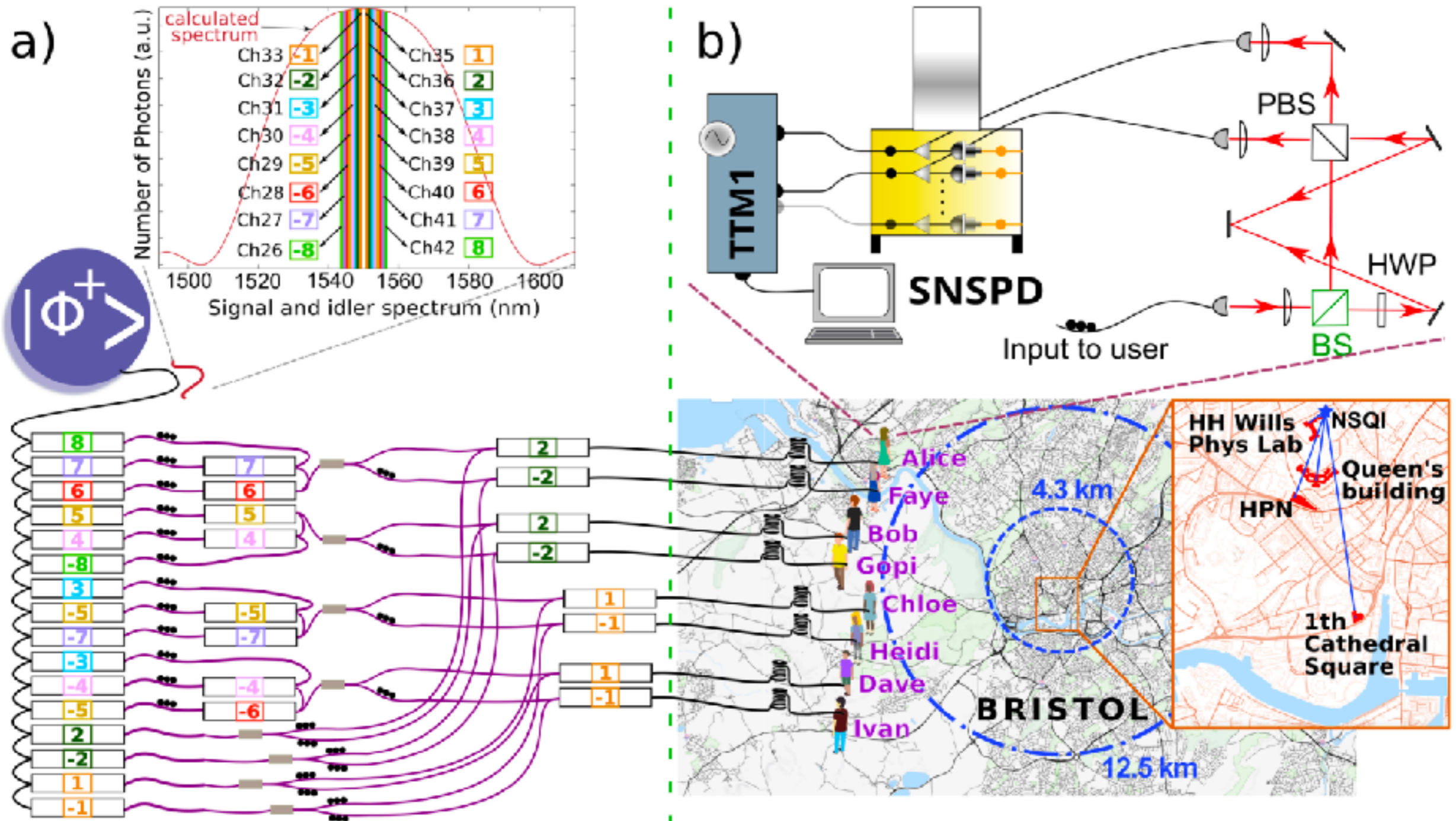
Consequences



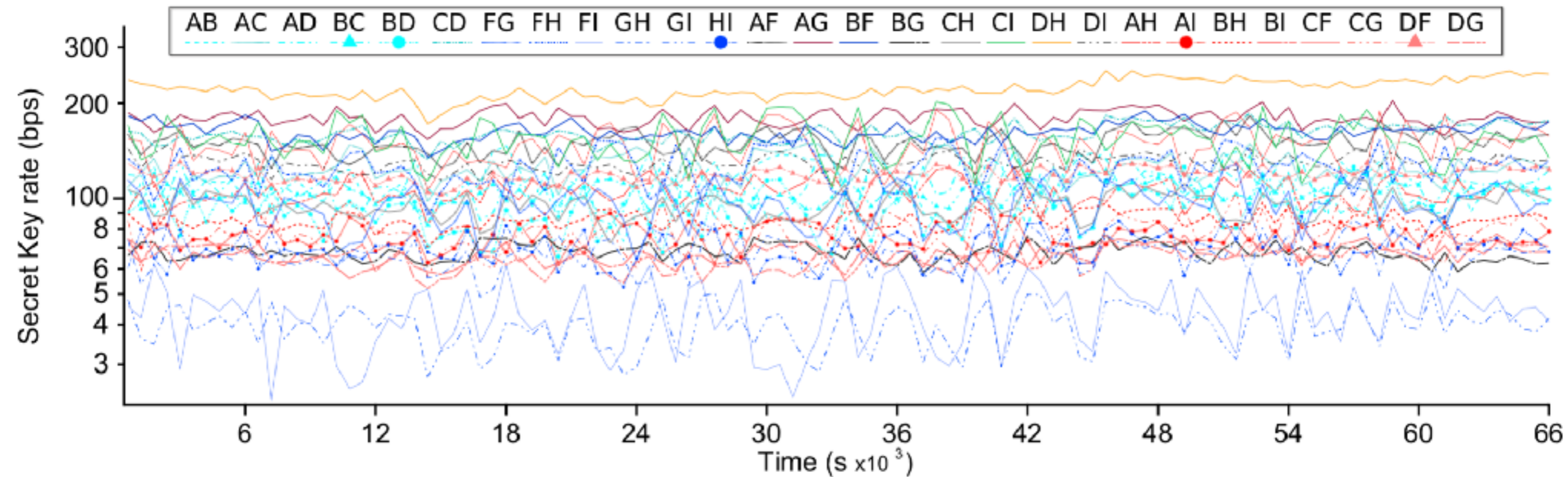
Extended Data Table II. **Total secure key (Mega bits) for the laboratory demonstration** as measured continuously over 18.45 hours after accounting for all finite key size effects.

	Alice	Bob	Chloe	Dave	Faye	Gopi	Heidi	Ivan
Alice		8.67	9.54	8.48	11.22	14.08	8.14	6.15
Bob			8.04	7.33	13.28	5.26	6.62	5.21
Chloe				12.27	11.10	6.14	7.12	12.10
Dave					9.17	5.25	17.01	9.95
Faye						7.27	9.59	3.65
Gopi							5.25	3.26
Heidi								5.49

Consequences



Consequences



- Fully connected Network (28 links)
- All positive SKR for ~18h (SNSPDs cycle time)