

# 40 years of Bell tests

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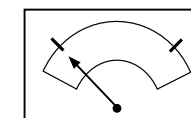
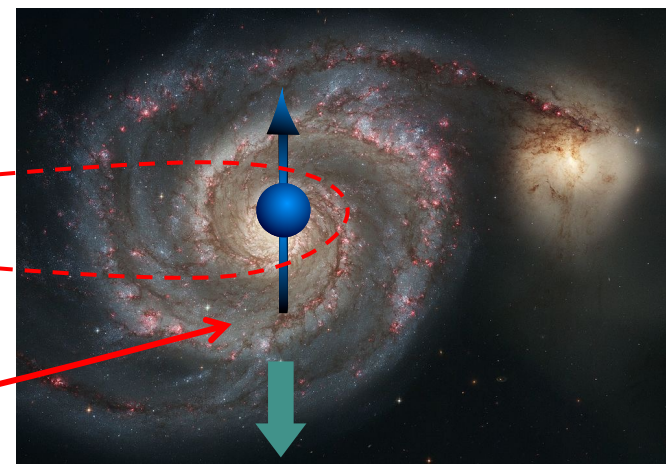
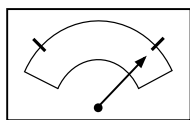
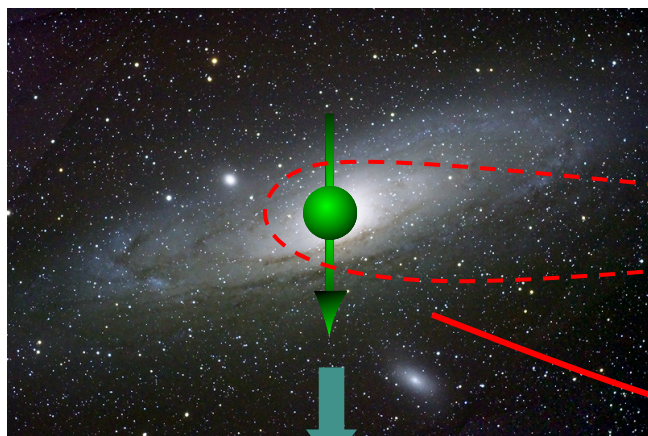


# The strangeness of entanglement

**maximally entangled state**

$$|\psi^-\rangle = \frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)$$

« the Bell singlet state »



wikiwand.com

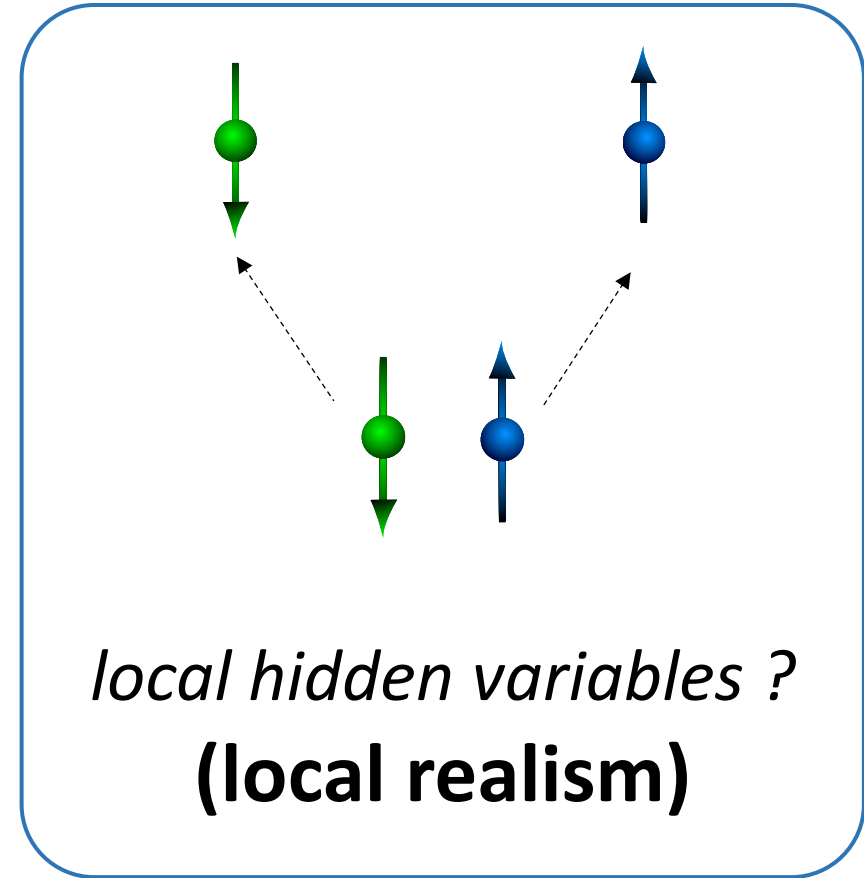
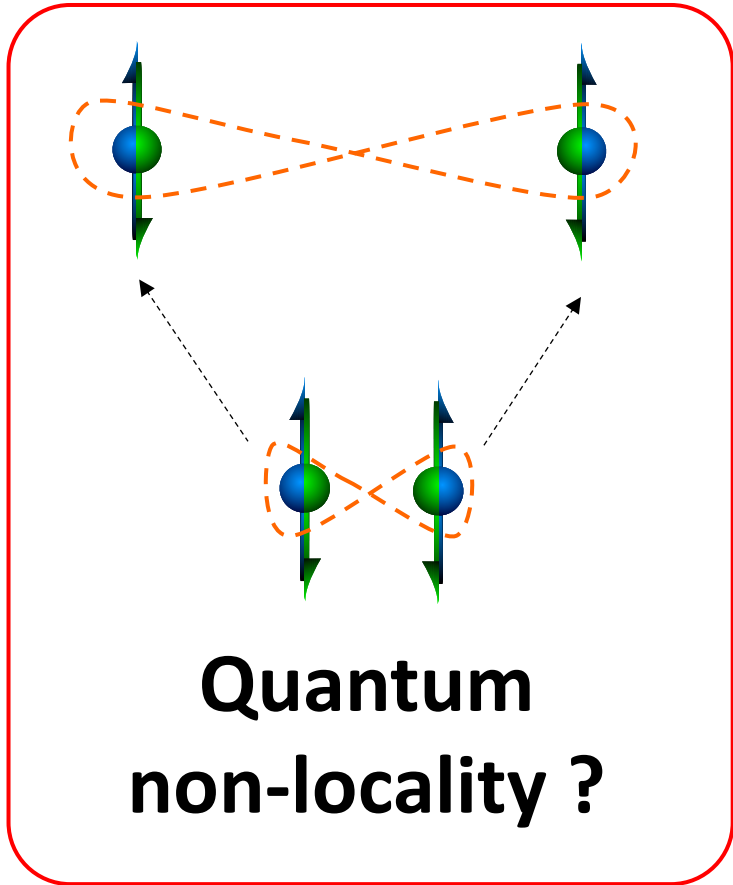
« spooky action at a distance »? (Einstein)

Quantum mechanics



measurement outcomes locally random  
but at distance perfectly correlated!

# Quantum non-locality or local realism?



# The answer: the Bell test

## Bell theorem

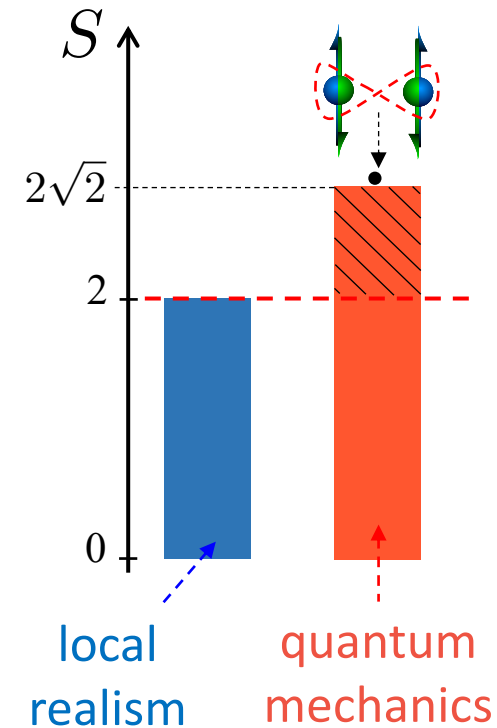
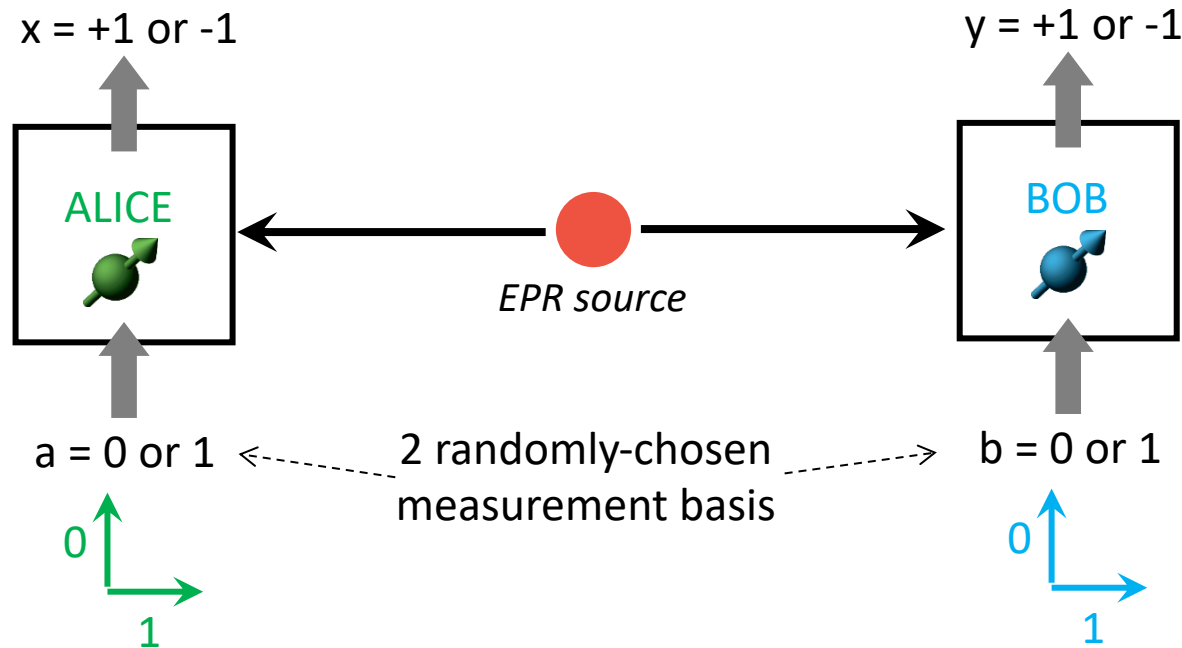
Bell, *Physics* (1964)

→ analysis of the correlations between independent measurements

## CHSH inequality :

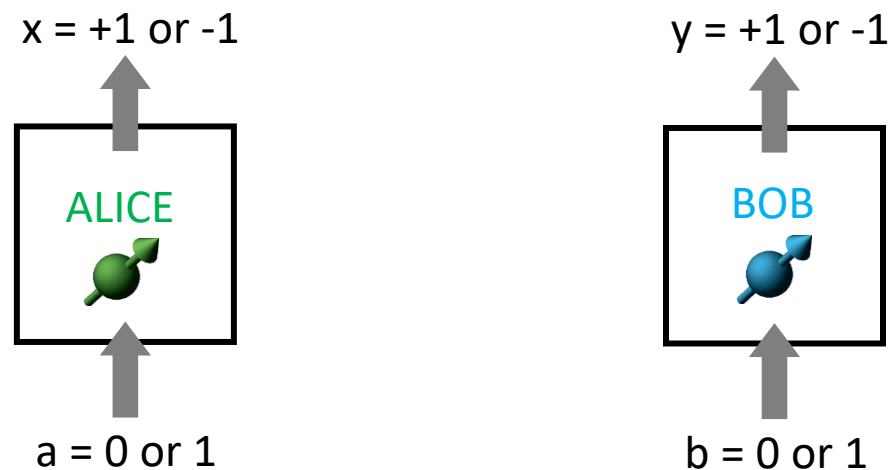
Clauser, Horne, Shimony, Holt , *PRL* **23** (1969)

$$S = | \langle x \cdot y \rangle_{(0,0)} + \langle x \cdot y \rangle_{(0,1)} + \langle x \cdot y \rangle_{(1,0)} - \langle x \cdot y \rangle_{(1,1)} |$$



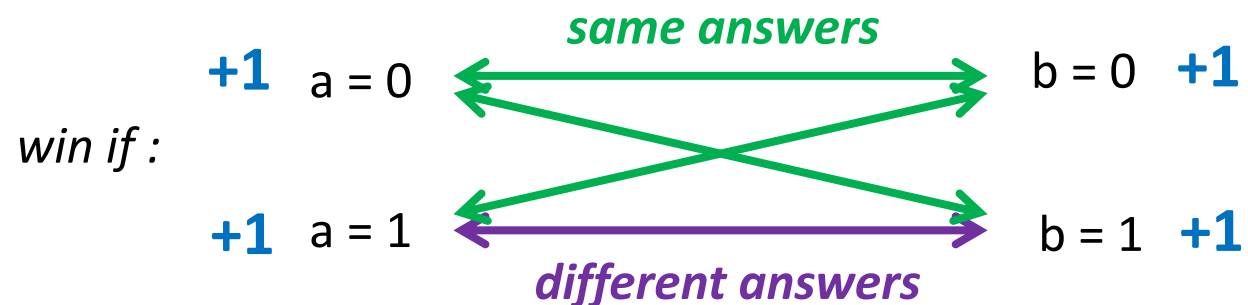
# Cheating strategies on the CHSH game

CHSH game presentation  
inspired from N. Brunner,  
@GDR IQFA, Grenoble,  
France (2012)



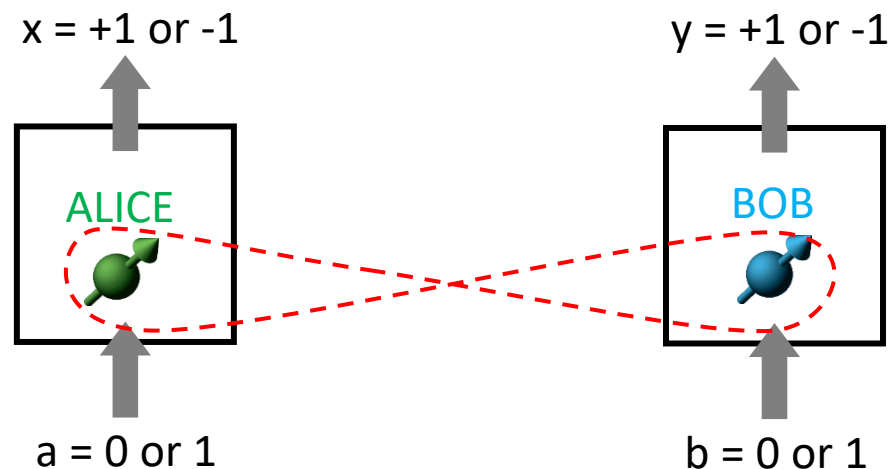
**“Classical” strategy**  
**local realism**  
 $Score \leq 3/4$

## RULES



# Cheating strategies on the CHSH game

CHSH game presentation  
inspired from N. Brunner,  
@GDR IQFA, Grenoble,  
France (2012)



**“Classical” strategy**

**local realism**

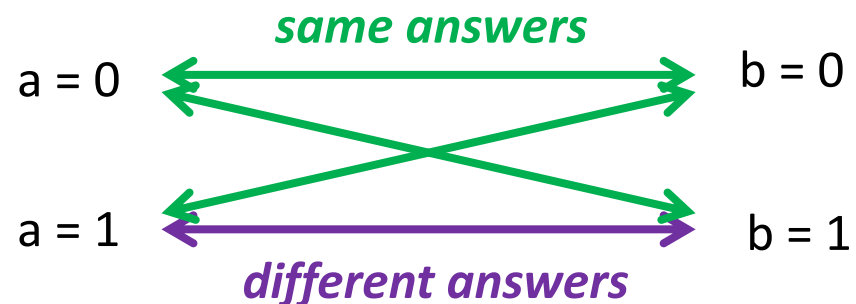
$$\text{Score} \leq 3/4$$

**Quantum strategy**  
**share entangled particles**

$$\text{Score} \leq (2 + \sqrt{2})/4 \simeq 0.85$$

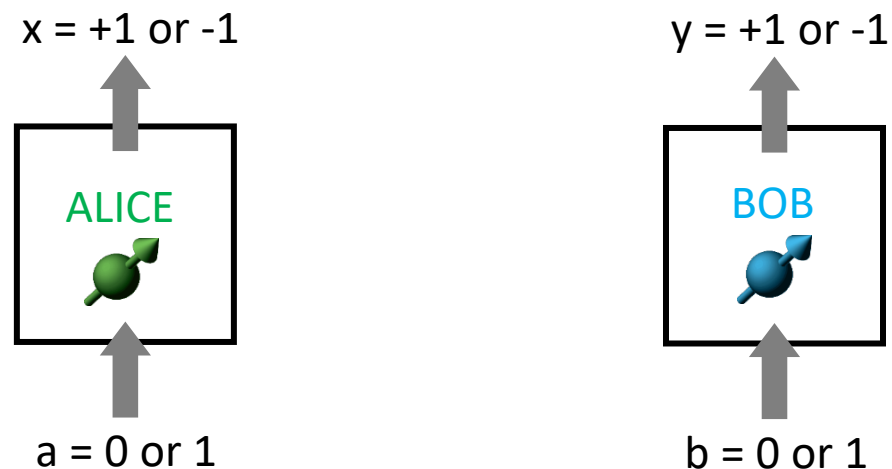
## RULES

win if :



# Cheating strategies on the CHSH game

CHSH game presentation  
inspired from N. Brunner,  
@GDR IQFA, Grenoble,  
France (2012)



**“Classical” strategy**  
**local realism**

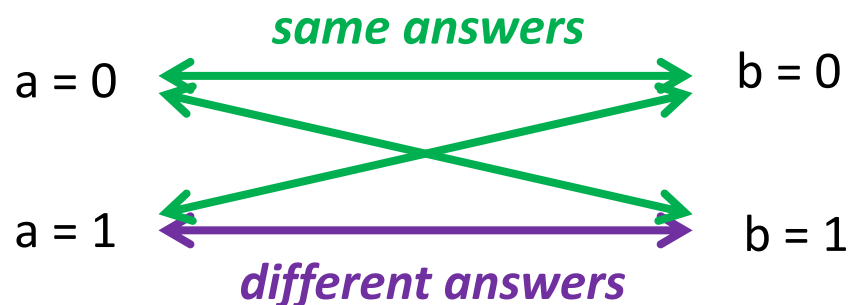
$$\text{Score} \leq 3/4$$

**Quantum strategy**  
**share entangled particles**

$$\text{Score} \leq (2 + \sqrt{2})/4 \simeq 0.85$$

## RULES

win if :



**“Cheating” strategy**

$$\text{Score} = 1$$

-> communication ---> **locality loophole**

-> choosing when to answer ---> **detection loophole**

# The answer: the Bell test **while closing loopholes!**

## Bell theorem

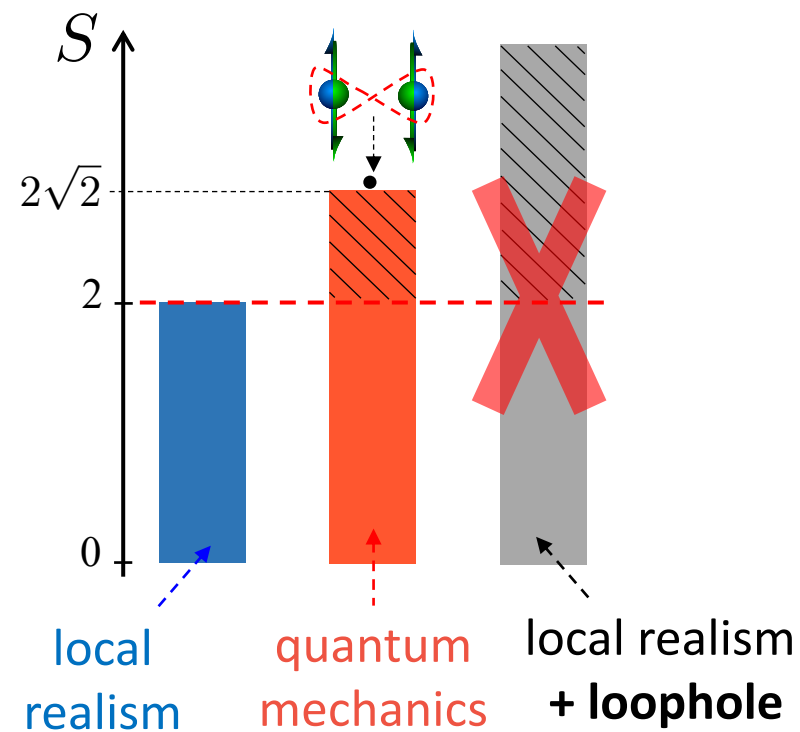
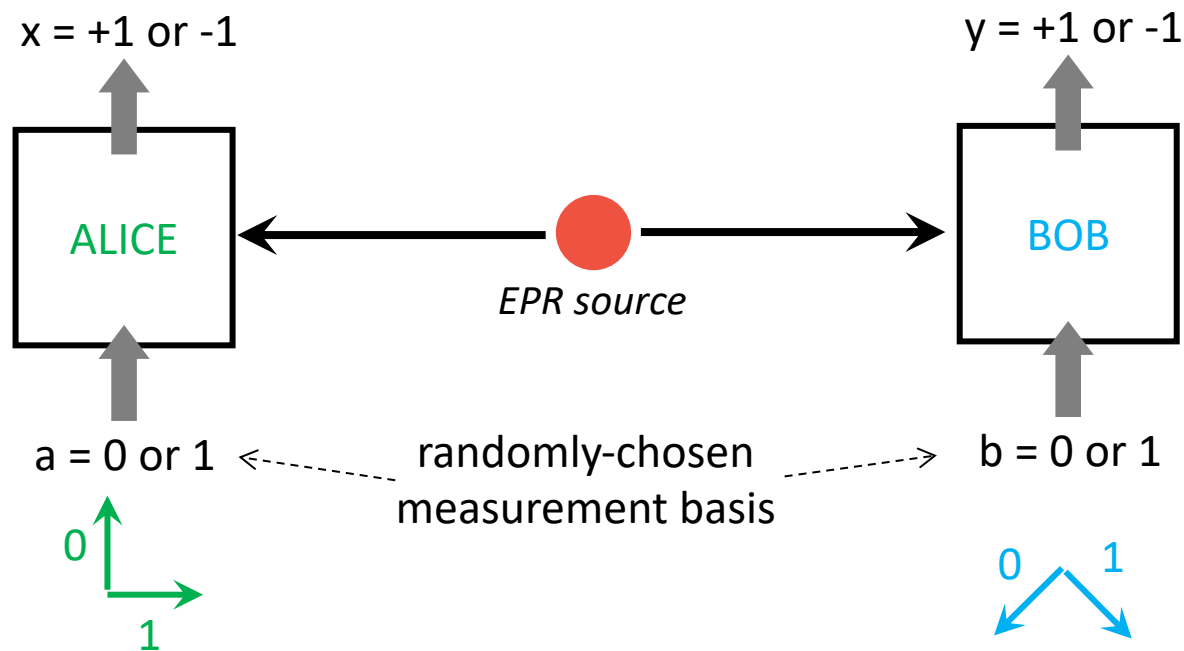
Bell, *Physics* (1964)

→ analysis of the correlations between **independent measurements**

CHSH inequality :

Clauser, Horne, Shimony, Holt , *PRL* **23** (1969)

$$S = | \langle x \cdot y \rangle_{(0,0)} + \langle x \cdot y \rangle_{(0,1)} + \langle x \cdot y \rangle_{(1,0)} - \langle x \cdot y \rangle_{(1,1)} |$$





# A journey through the Bell tests

**1. Closing the locality loophole**

**2. Closing the detection loophole**

**3. Loophole-free Bell tests**

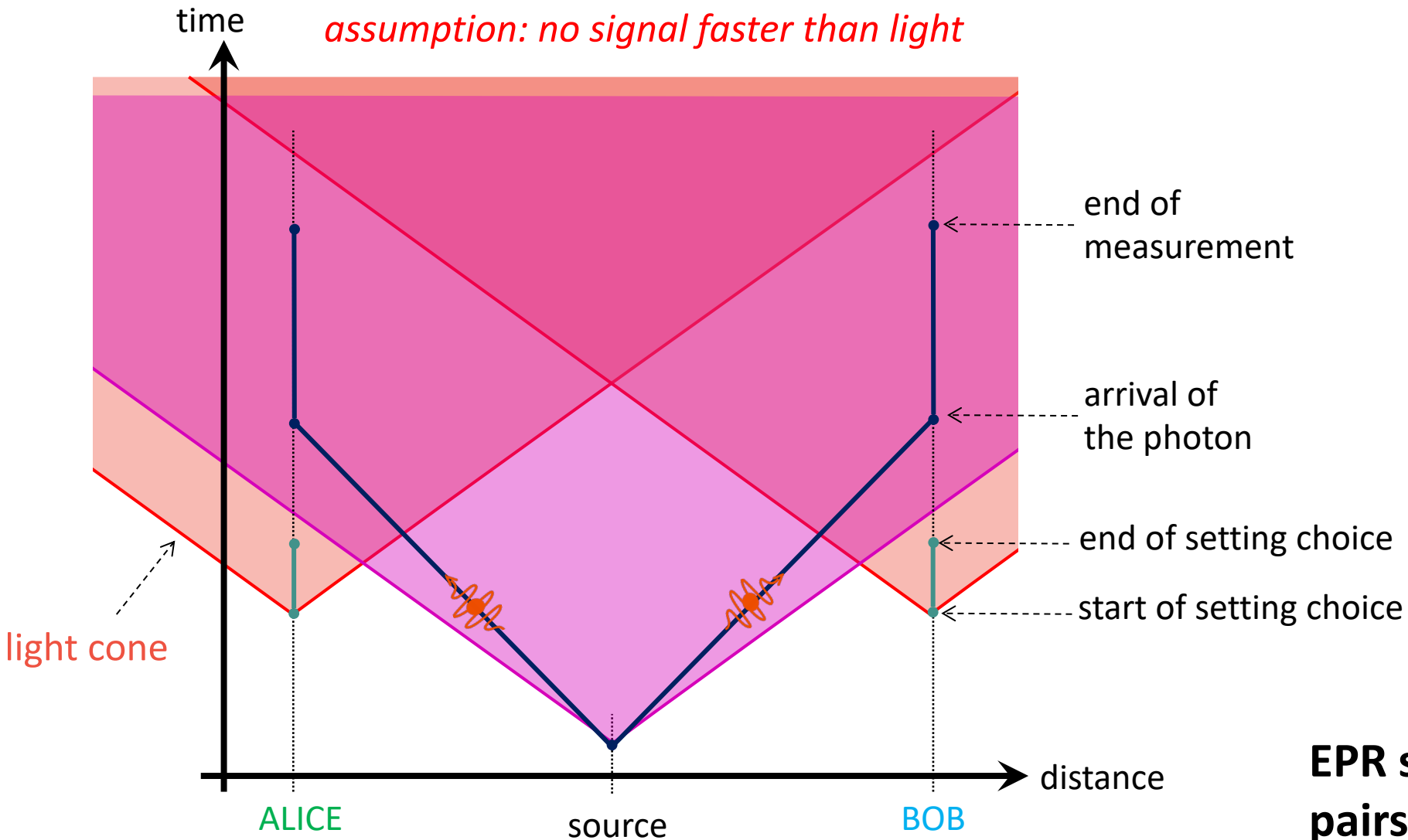
# A journey through the Bell tests

**1. Closing the locality loophole**

2. Closing the detection loophole

3. Loophole-free Bell tests

# Conditions to close the locality loophole



## locality loophole

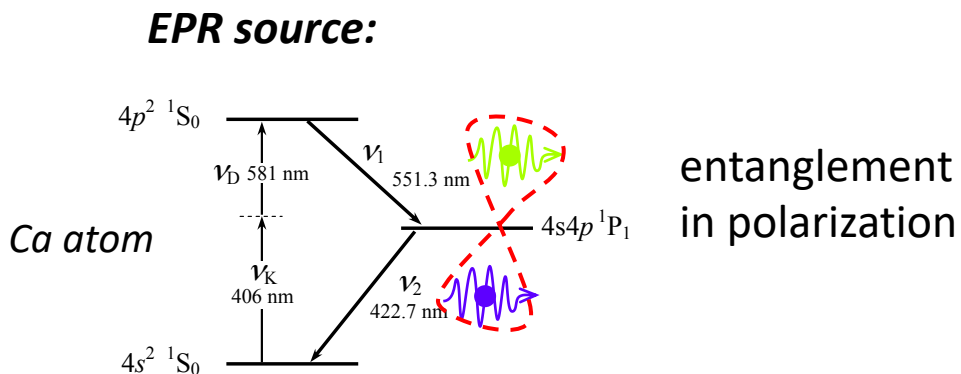


**setting choices**  
**space-like separated**  
**with the source**  
**&**  
**measurements of the**  
**particles**  
**space-like separated**

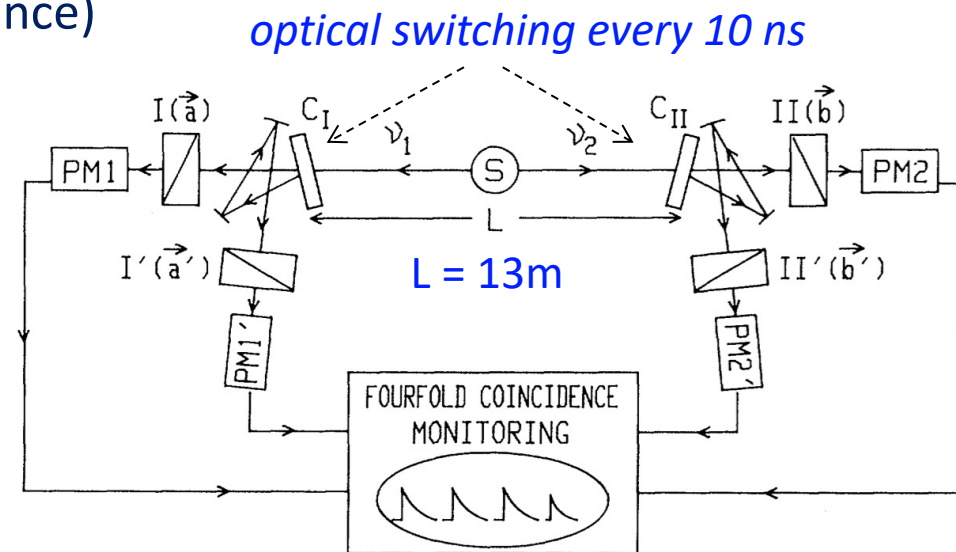
**EPR source =**   
**pairs of entangled photons**

# Changing the settings on the flight of the photons

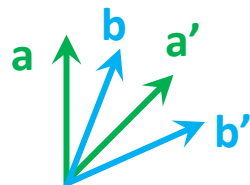
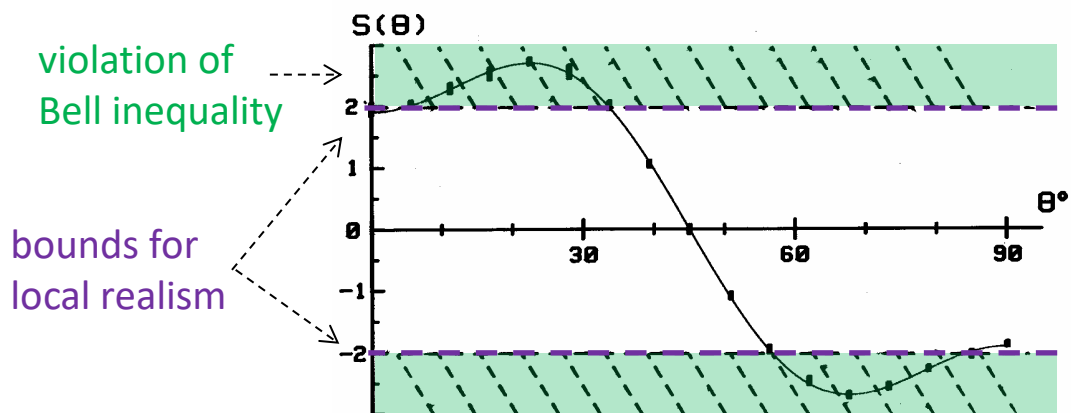
Aspect, Dalibard and Roger, *Phys. Rev. Lett.* **49** (1982) – (Orsay, France)



**! settings not random!**



**Bell parameter  $S$  vs polarizers' relative angle**



$L/c = 40\text{ns} > \text{setting switching time}$

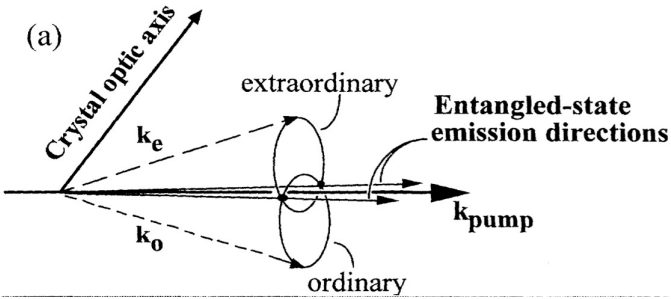
adapted CHSH inequality:  $-1 \leq S \leq 0$

**$S_{exp} = 0.101 \pm 0.020$  Bell inequality violation!**

$(S_{QM} = 0.112, t_{acq} > 3\text{h})$

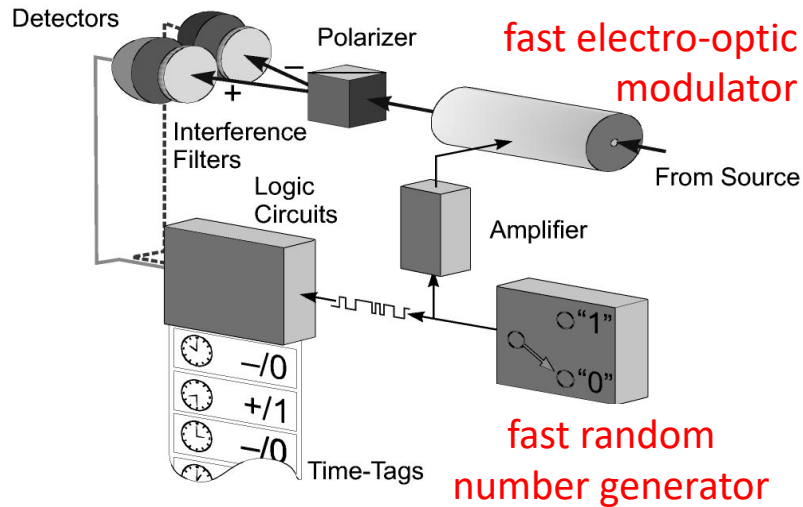
# Settings chosen by fast random number generator

Weih's *et al.*, *Phys. Rev. Lett.* **81** (1998) – (Zeilinger's group, Innsbruck, Austria)



**EPR source:**  
polarization-entangled  
photon pairs by parametric  
down-conversion

Kwiat *et al.*, *Phys. Rev. Lett.* **75** (1995)

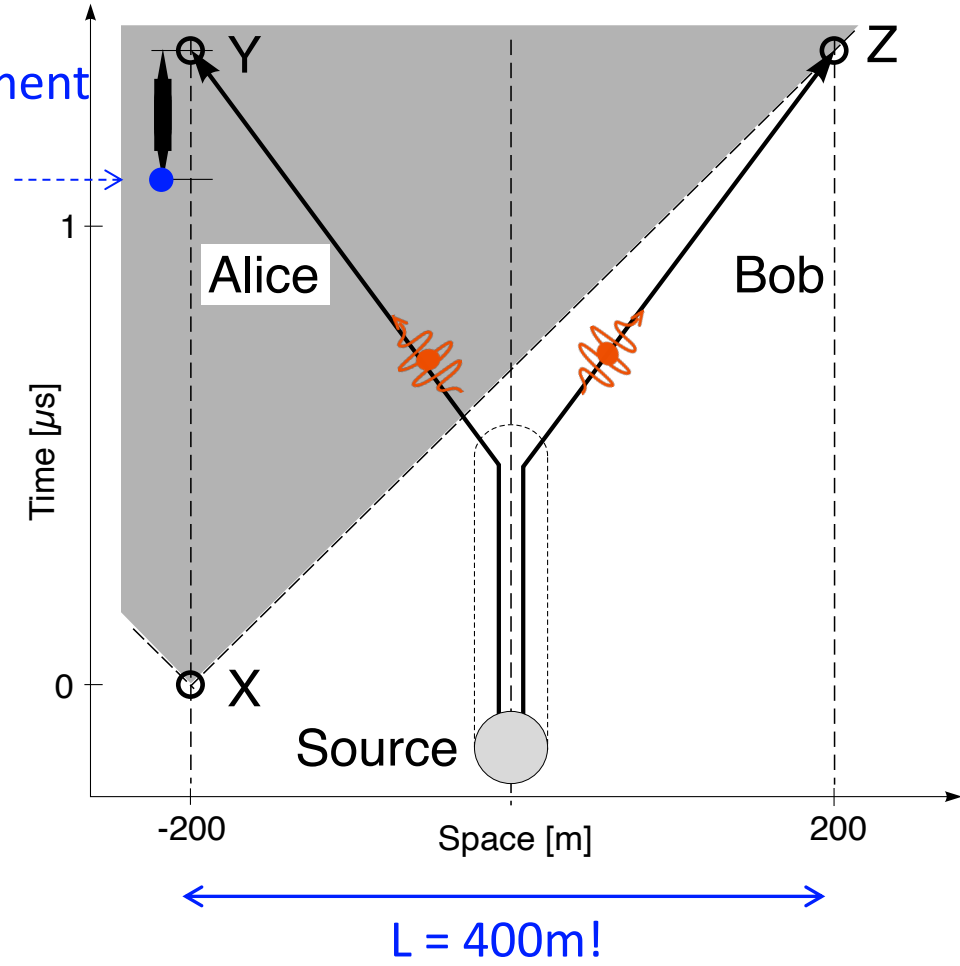


CHSH inequality:  $S \leq 2$

**$S = 2.73 \pm 0.02$**

( $t_{acq} = 10$  s)

measurement  
start



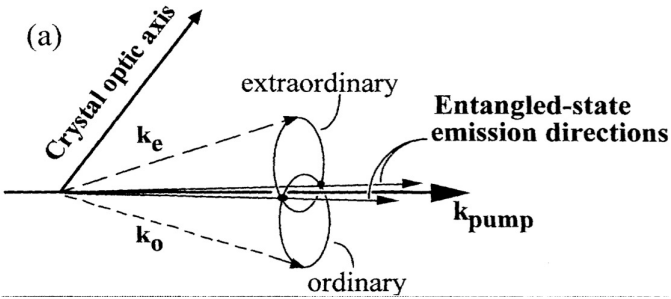
# Settings chosen by fast random number generator

1. locality loophole

Weih's *et al.*, *Phys. Rev. Lett.* **81** (1998) – (Zeilinger's group, Innsbruck, Austria)

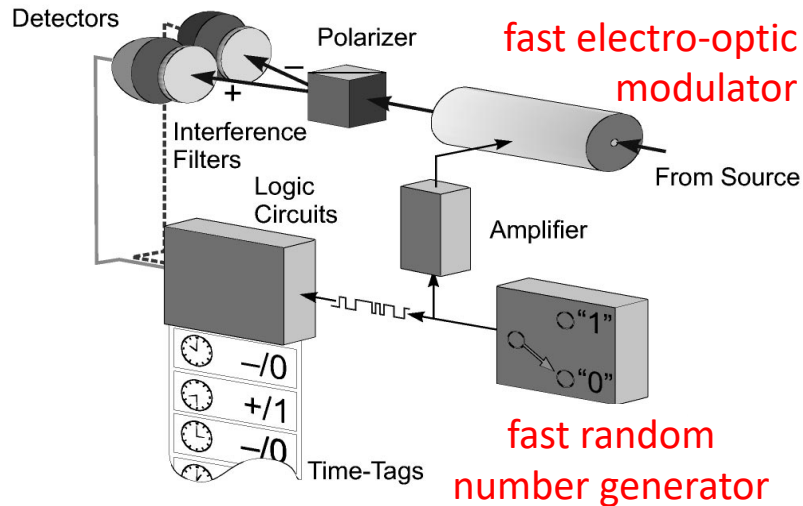


the source could influence the settings



**EPR source:**  
polarization-entangled  
photon pairs by parametric  
down-conversion

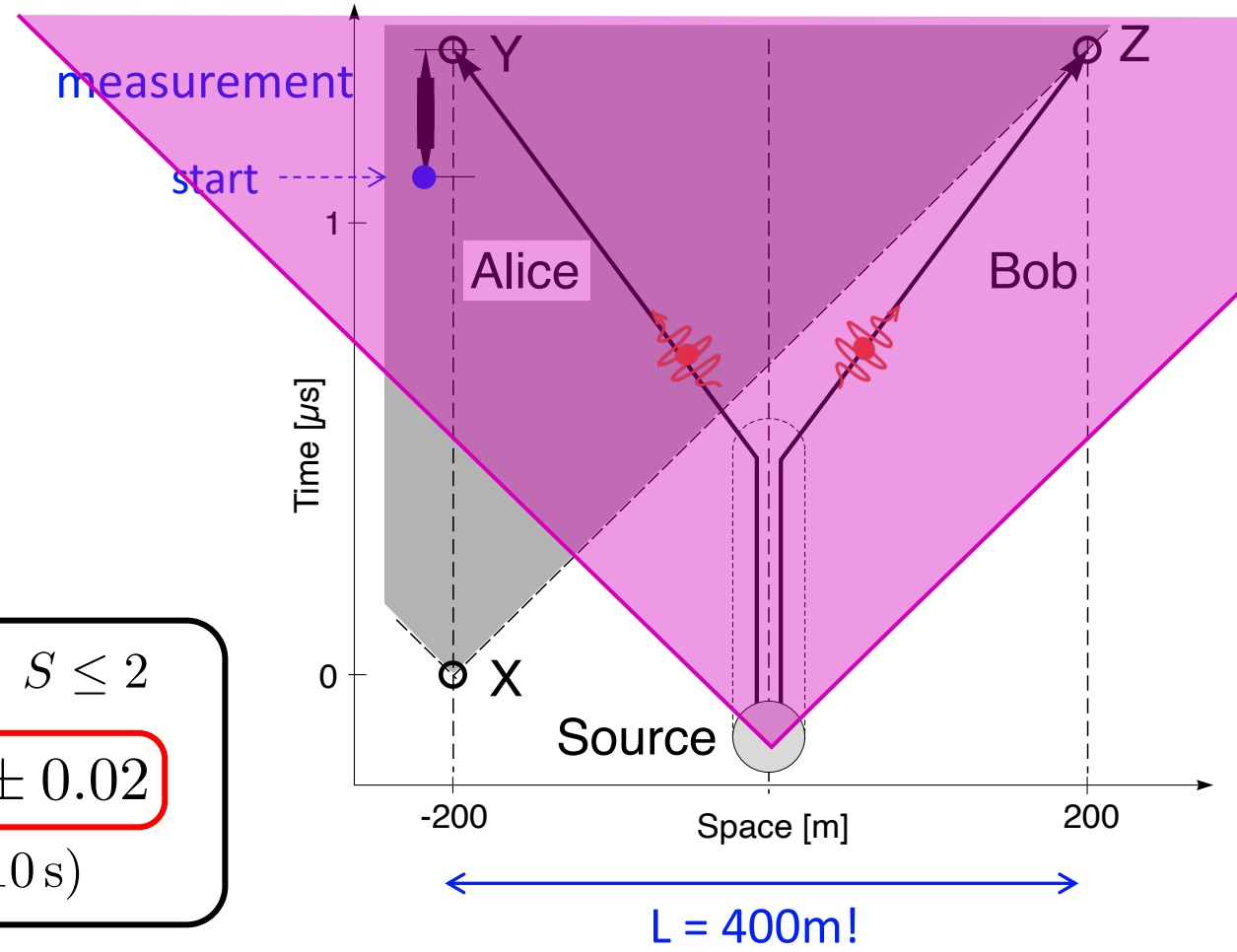
Kwiat *et al.*, *Phys. Rev. Lett.* **75** (1995)



CHSH inequality:  $S \leq 2$

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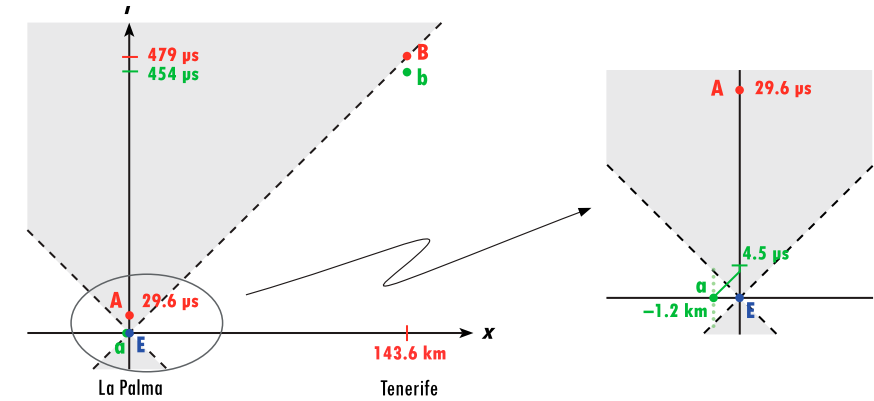
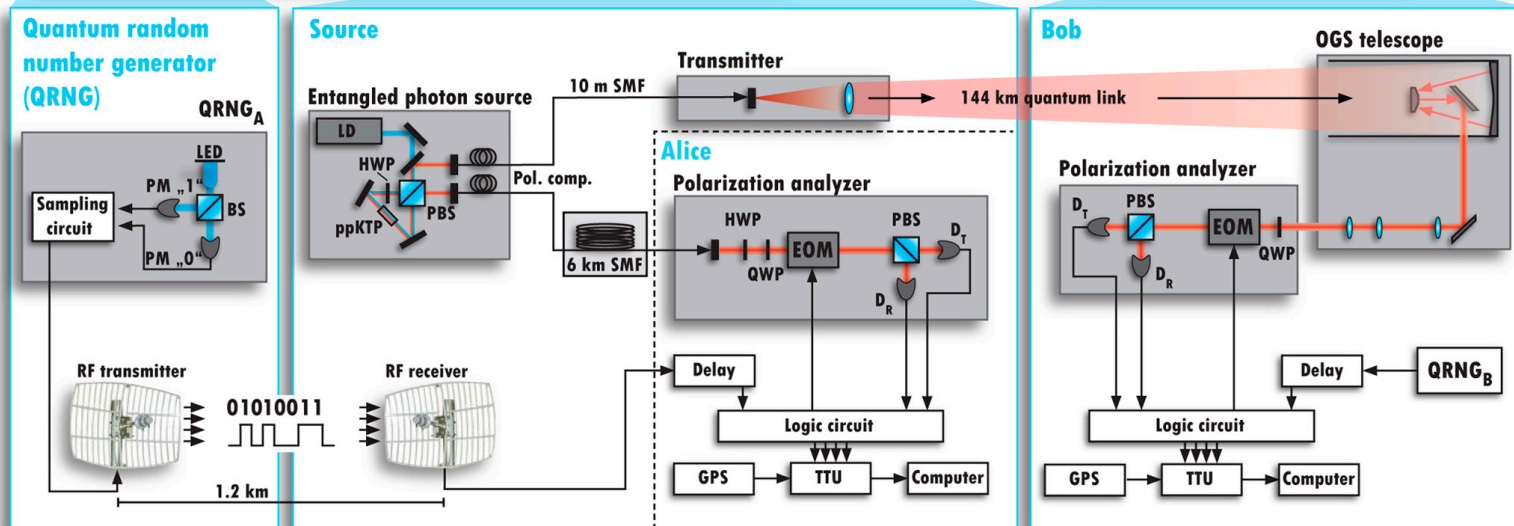
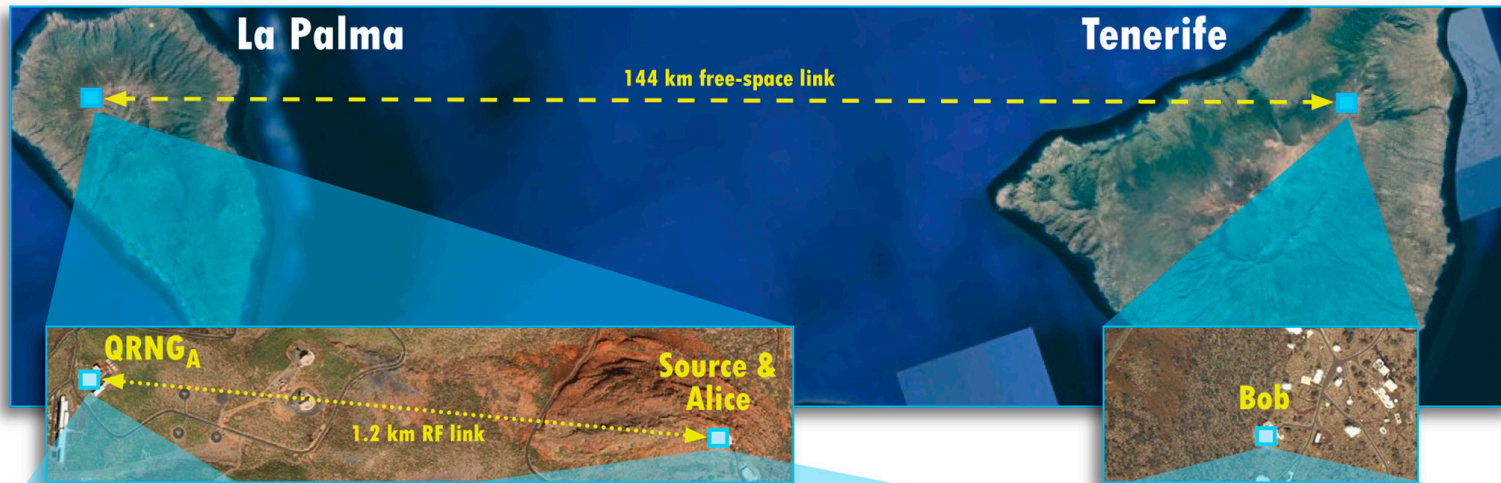
( $t_{acq} = 10$  s)



# Fully closing the locality loophole

1. locality loophole

Scheidl *et al.*, *PNAS* **107** (2010) – (Zeilinger’s group, Vienna, Austria)



locality loophole closed 

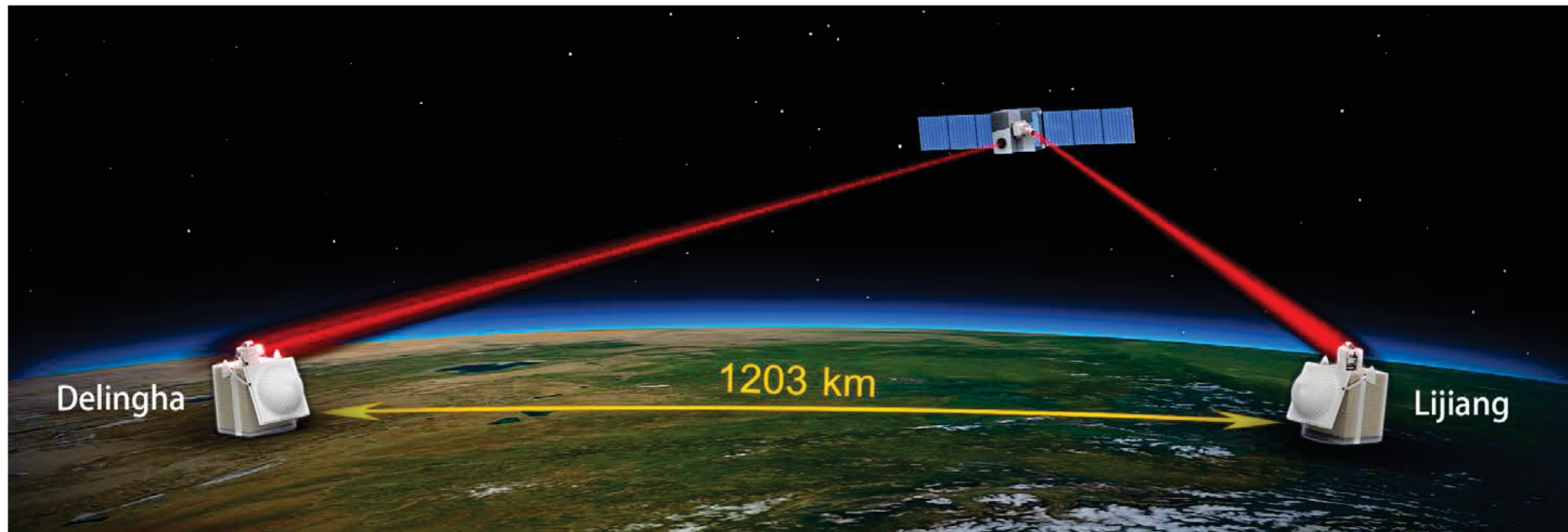
CHSH inequality:  $S \leq 2$

$S = 2.37 \pm 0.02$

( $t_{acq} = 40$  min)

# Closing the locality loophole with satellite-to-ground connections

Yin *et al.*, *Science* **356** (2017) – (Pan's group, Hefei, China)

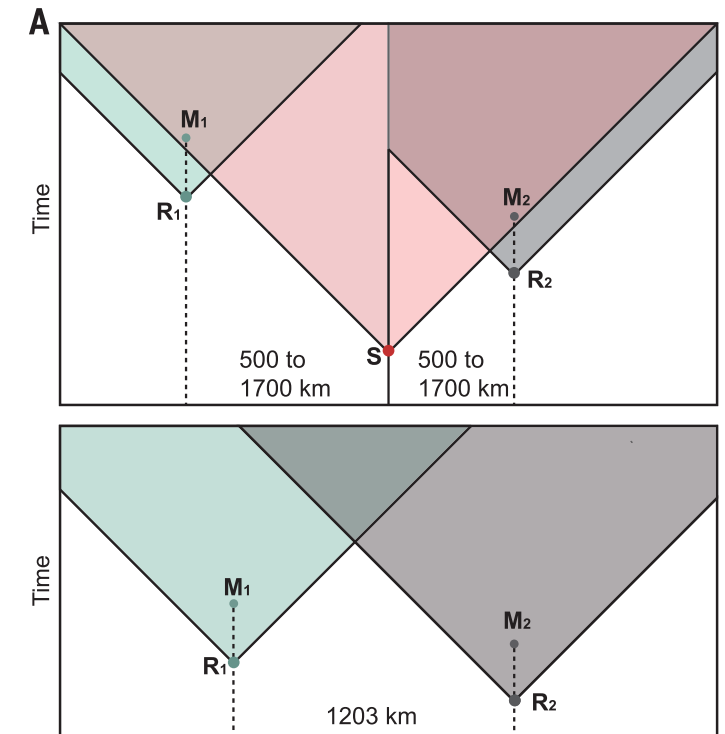


**1<sup>st</sup> Bell test with space-to-ground connection**

CHSH inequality:  $S \leq 2$

$$S = 2.37 \pm 0.09$$

( $t_{acq} \simeq 20$  min)



locality loophole closed





# A journey through the Bell tests

1. Closing the locality loophole *but fair-sampling assumption*

2. Closing the detection loophole

3. Loophole-free Bell tests

# Condition to close the detection loophole

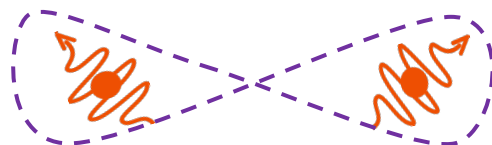
avoiding the  
fair sampling assumption



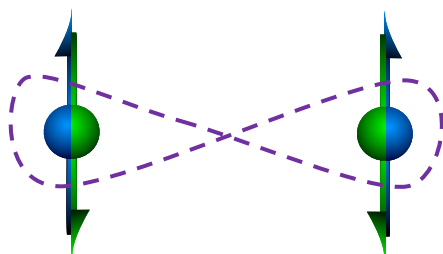
**detection loophole**



**the entangled pairs  
are detected with  
high efficiency**



very challenging  
with photons



and with atomic  
systems ?

*Ex: CHSH for singlet state,  
detection efficiency > **82.8%***

Eberhard, *PRA* **47** (1993)

# Closing the detection loophole with trapped ions

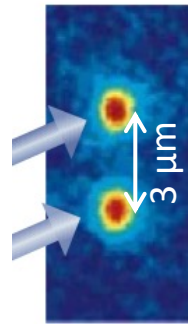
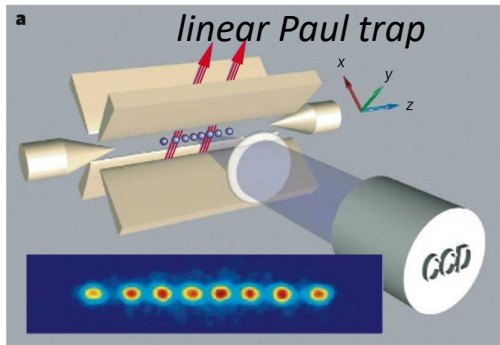
Rowe *et al.*, *Nature* **401** (2001) – (Wineland’s group, NIST, Boulder, USA)

## detection loophole

the ions are detected all the time!

**single-shot readout of the spin state:**

$|\downarrow\rangle = \text{bright state}$    $|\uparrow\rangle = \text{dark state}$  



**entangled pair:  
two  ${}^9\text{Be}^+$  trapped ions**

$$|\psi^-\rangle = \frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)$$

Blatt & Wineland, *Nature* **453** (2008)

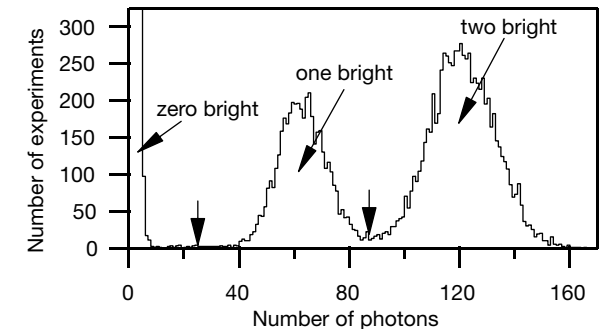
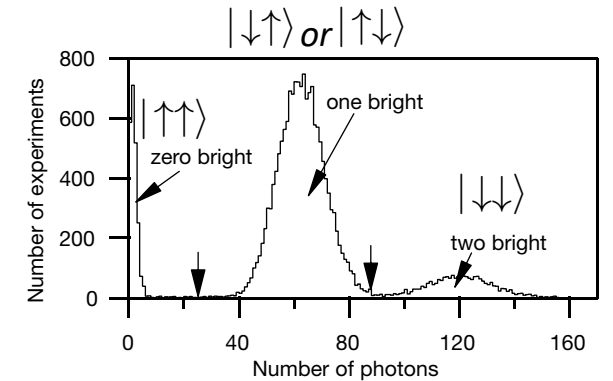
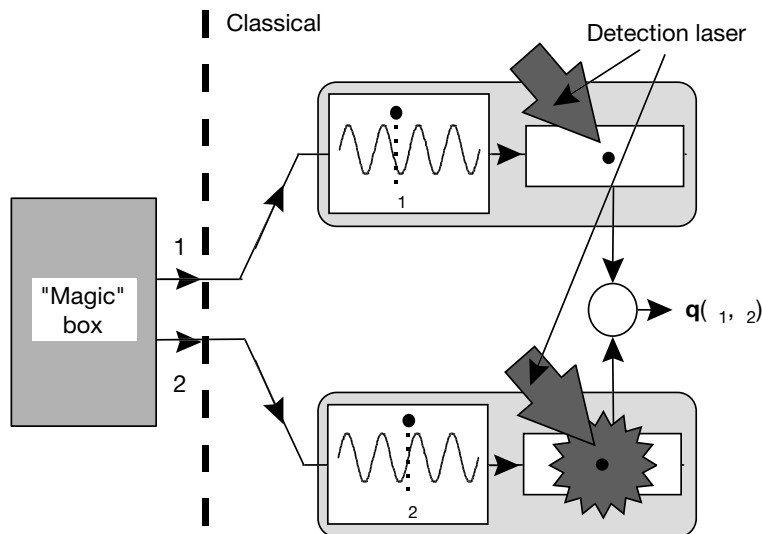
CHSH inequality:  $S \leq 2$

$S = 2.25 \pm 0.03$

increasing the distance between ions:

1m: Matsukevich *et al.*, *PRL* **100** (2008)

20m: Hofman *et al.*, *Science* **337** (2012)



# ... and with superconducting circuits

Ansmann *et al.*, *Nature* **461** (2009) – (Martinis's group, Santa Barbara, USA)

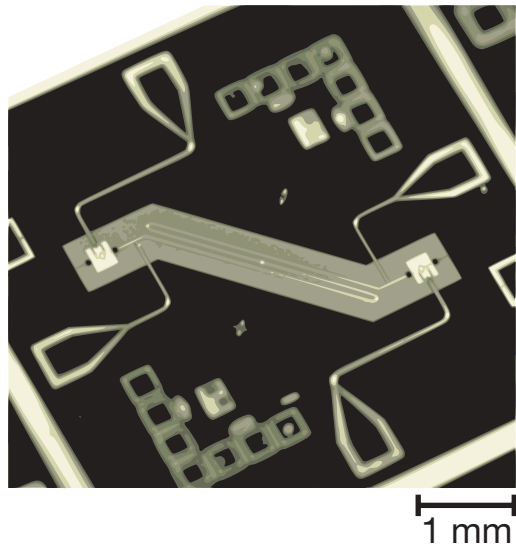


## detection loophole

the qubits are detected all the time!

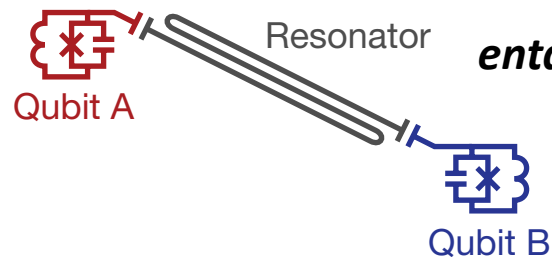
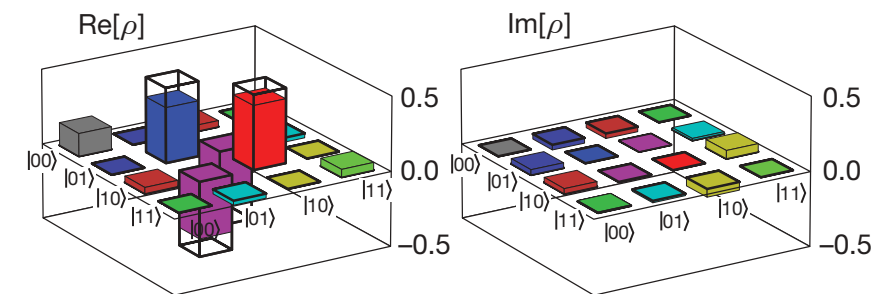
*single-shot readout of the qubit state:*

*measurement fidelities ~ 94%*



*entangled pair:  
two Josephson phase  
qubits*

*entanglement  
fidelity with the Bell  
singlet state ~ 88%*



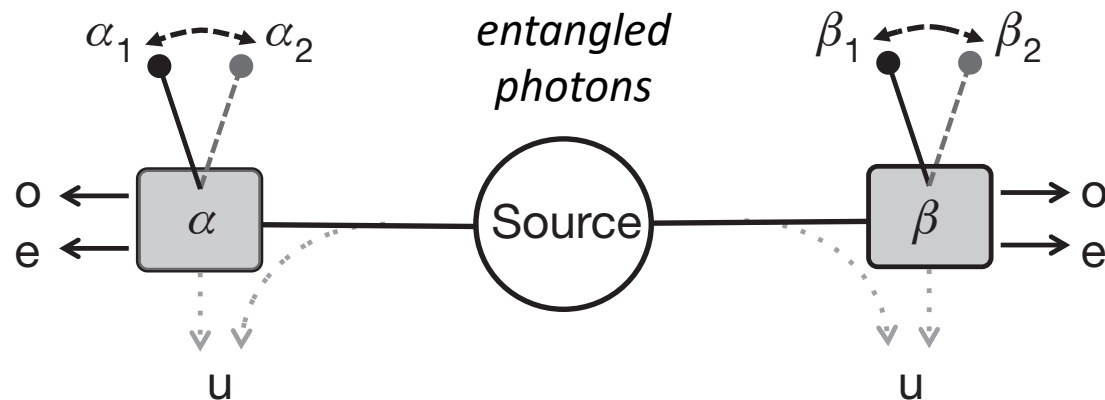
*entanglement via an  
electromagnetic  
resonator*

CHSH inequality:  $S \leq 2$

$$S = 2.0732 \pm 0.0003$$

## ... and with photons!

Guistina *et al.*, *Nature* **497** (2013) – (Zeilinger's group, Vienna, Austria)



Eberhard inequality includes  
undetected events !

$$J = -n_{oo}(\alpha_1, \beta_1) + n_{oe}(\alpha_1, \beta_2) + n_{ou}(\alpha_1, \beta_2) + n_{eo}(\alpha_2, \beta_1) + n_{uo}(\alpha_2, \beta_1) + n_{oo}(\alpha_2, \beta_2) \geq 0$$

 **detection loophole**

**closed if with efficiency >2/3**

*Alice's arm efficiency ~ 74%*

*Bob's arm efficiency ~ 79%*

*(detector: superconducting TESs)*

*Eberhard inequality:  $J \geq 0$*

$$J = -126,715 \pm 1,837$$

# A journey through the Bell tests

1. Closing the locality loophole *but fair-sampling assumption*

2. Closing the detection loophole  
*but no-signaling assumption*

**3. Loophole-free Bell tests**

# Loophole-free Bell tests



locality loophole & detection loophole both closed!

## Entangled spins in diamond

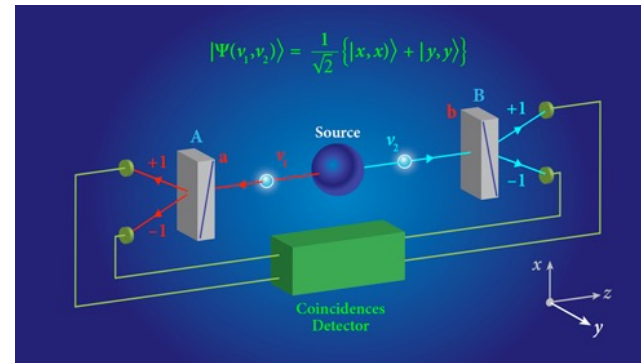
Hanson's group (Delft) 2015



Hensen *et al.*, *Nature* **119** (2015)

## Entangled photons

Zeilinger's group (Vienna) 2015  
NIST (Boulder) 2015

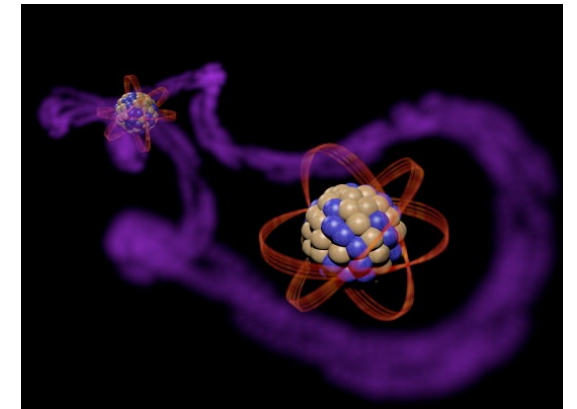


APS/Alan Stonebraker

Giustina *et al.*, *PRL* **115** (2015)  
Shalm *et al.*, *PRL* **115** (2015)

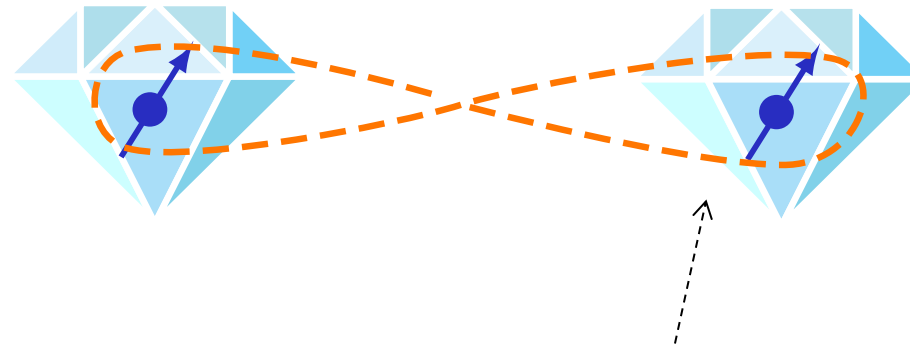
## Entangled atoms

Weinfurter's group (Munich) 2016



Rosenfeld *et al.*, *PRL* **119** (2017)

### a loophole-free Bell test with spins in diamond



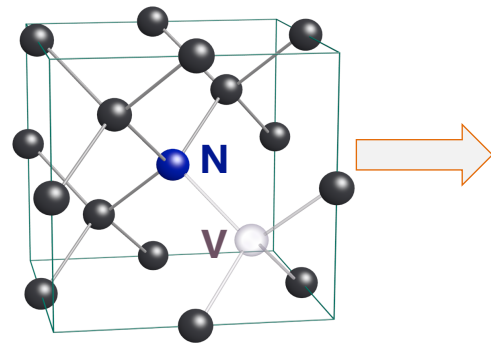
!!! solid-state artificial atom !!!

#### Requirements

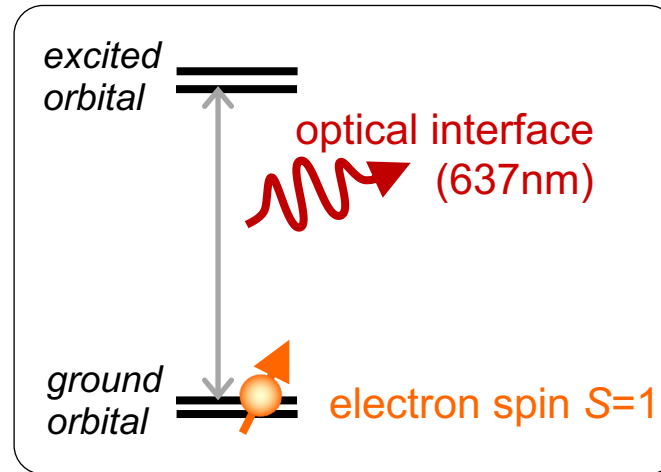
- close both the detection and locality loopholes
- quantum correlations must exceed the local realist bound
  - > high entanglement fidelity, fast manipulation with high fidelity, fast readout with high fidelity...



# The NV center: a « trapped-atom » in diamond

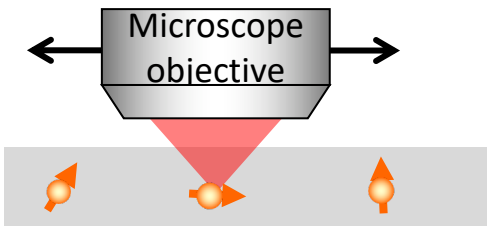


@ 3 K

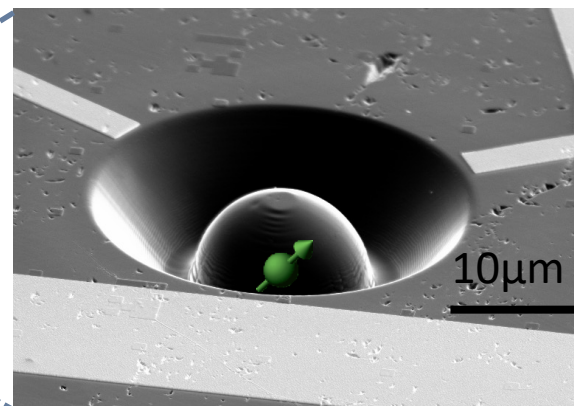
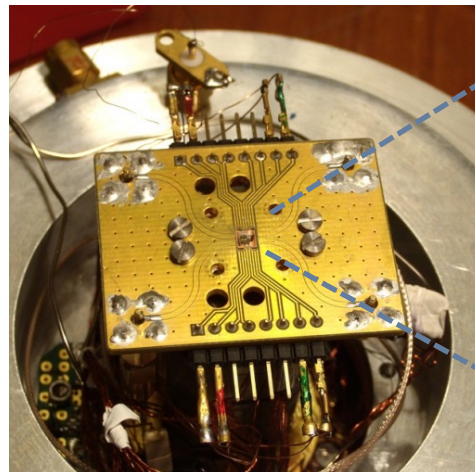
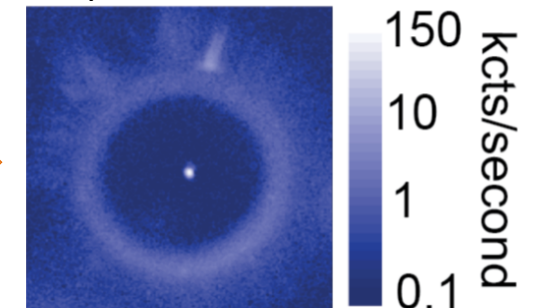


-> quantum sensing  
(talk tomorrow)

detection of **single NVs**  
through  
confocal microscopy

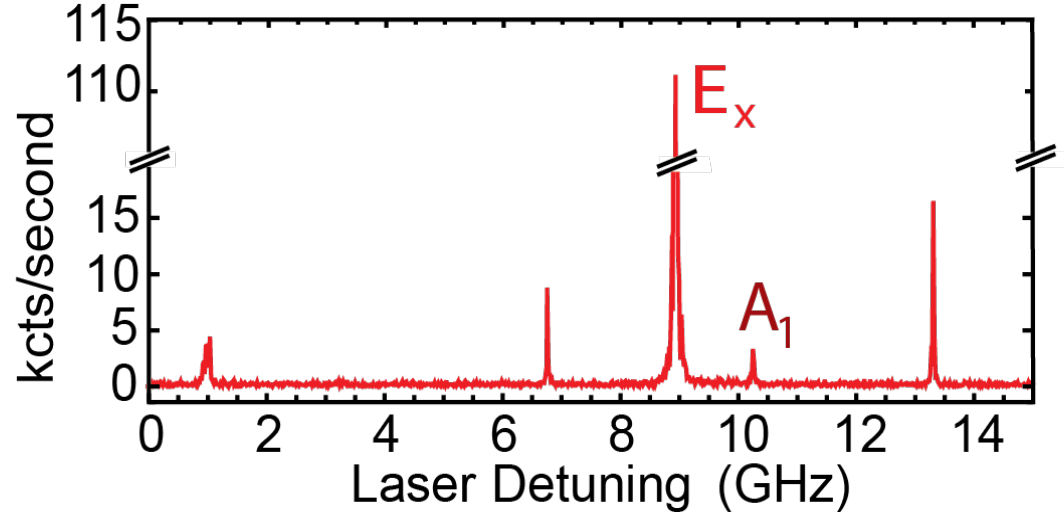


optical scan

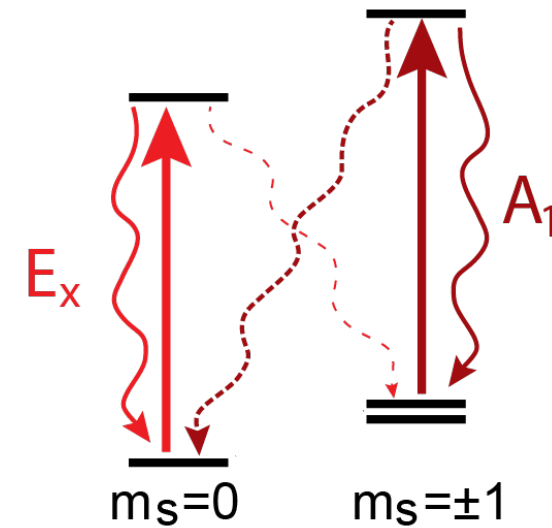


### Resonant excitation

*Photoluminescence excitation spectrum*



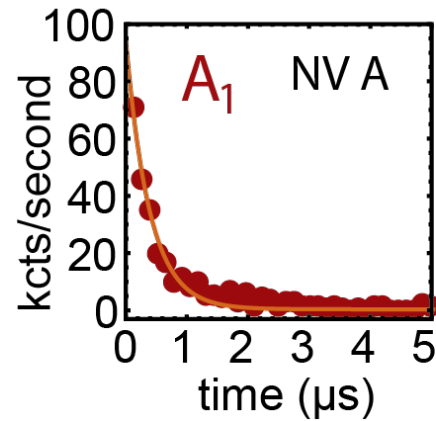
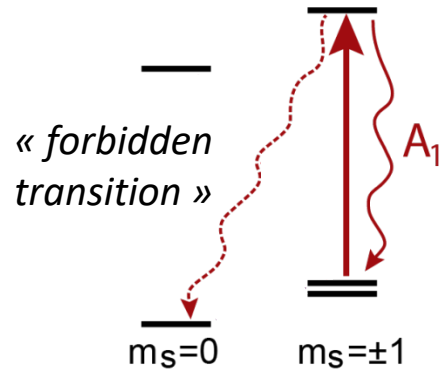
### Spin-conserving transition



*ground state =  
electronic spin triplet ( $S = 1$ )*

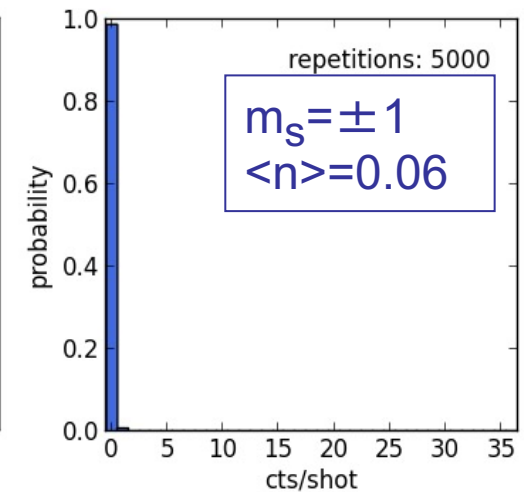
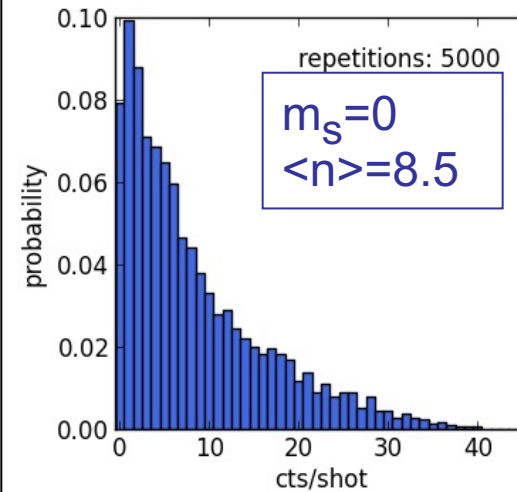
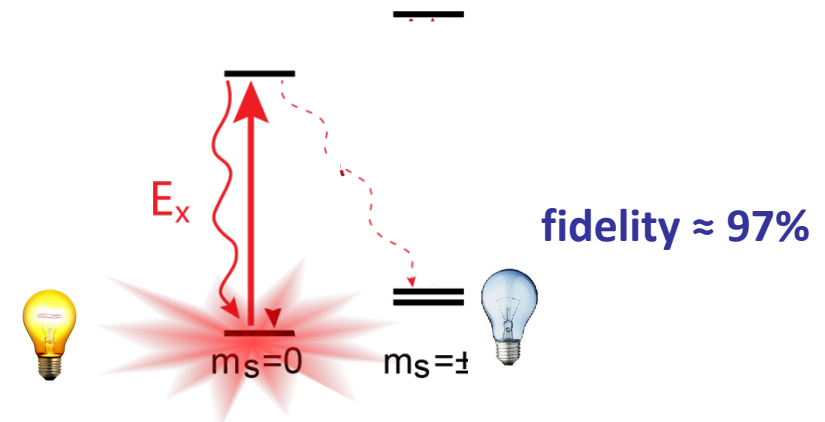
# Initialization and readout by resonant excitation

## Initialization



fidelity > 99.7%

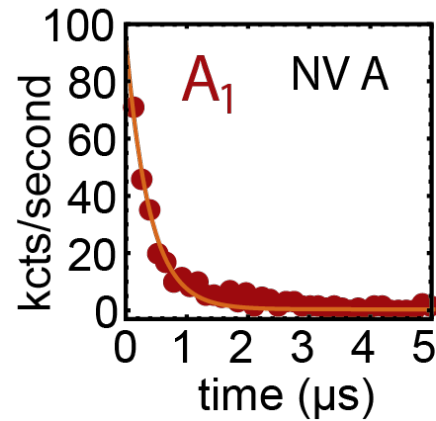
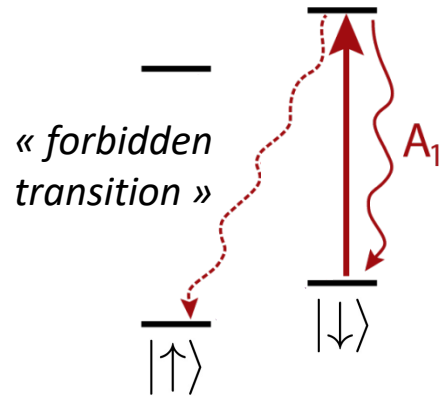
## Single-Shot Readout



Robledo *et al.*, *Nature* **477** (2011)

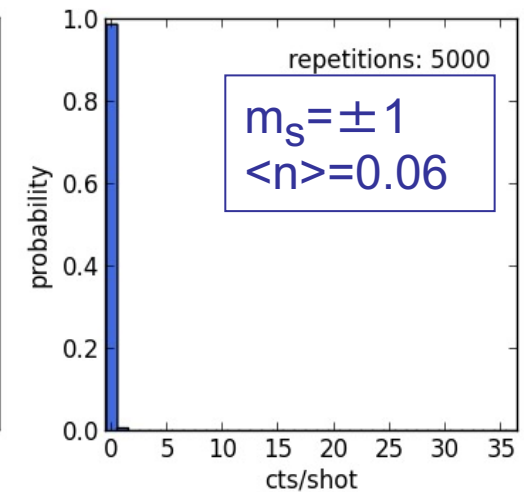
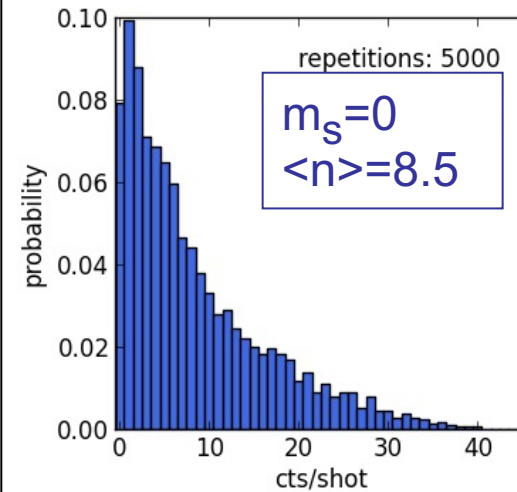
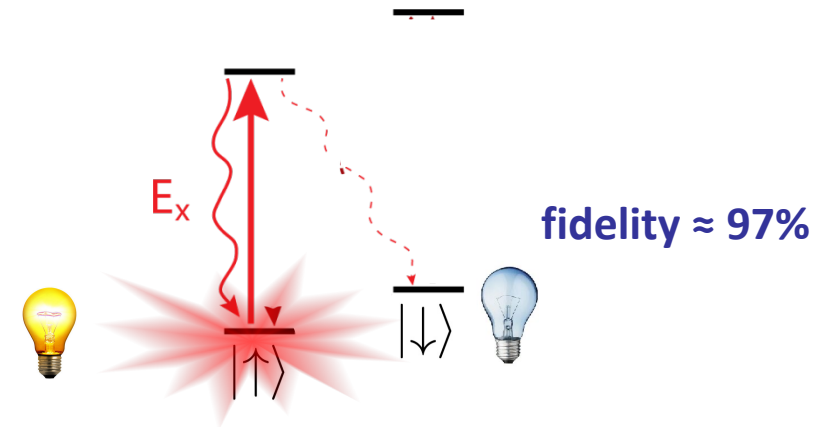
# Initialization and readout by resonant excitation

## Initialization

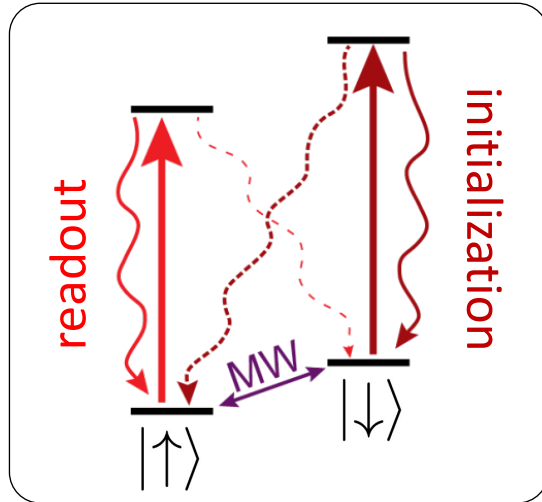


fidelity > 99.7%

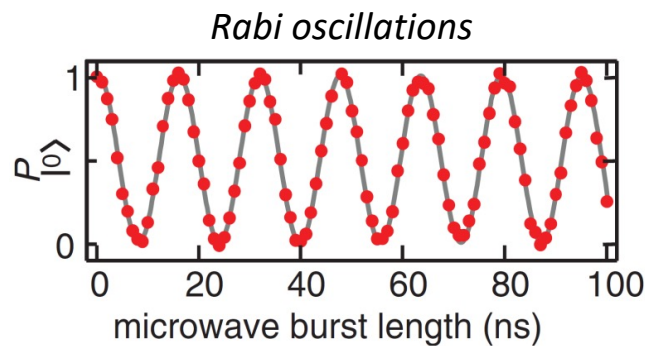
## Single-Shot Readout



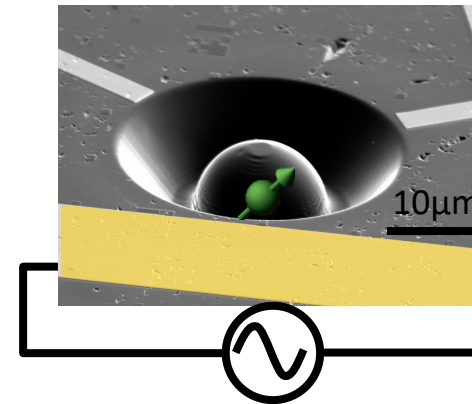
Robledo *et al.*, *Nature* **477** (2011)



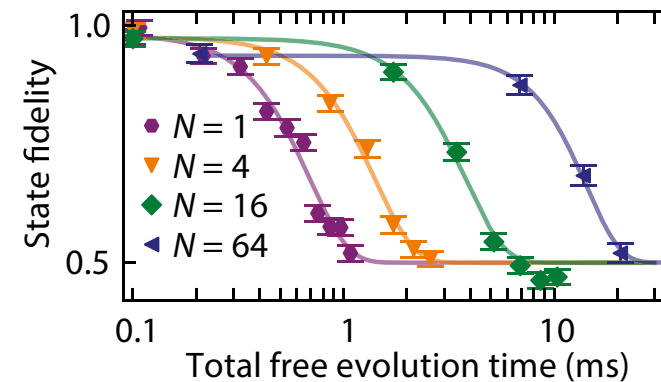
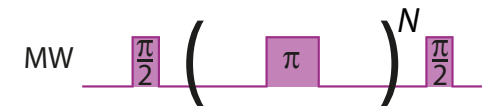
**High fidelity  
spin manipulation**



De Lange *et al.*, *Science* **23** (2010)

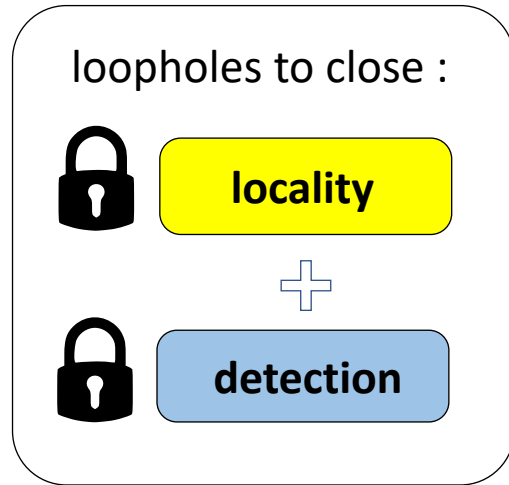


**Long coherence time**



Many related works by Stuttgart, Harvard, Chicago, Ulm,...

# Recap: requirements for a loophole-free Bell test

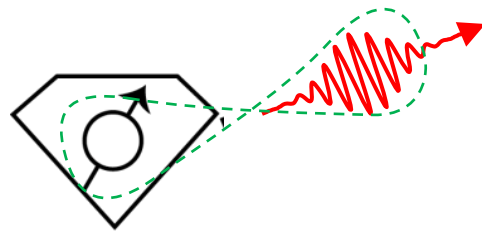


→ generation of a long-distance entangled pair of spins with high fidelity

→ fast single-shot readout of the spins with high fidelity

→ **Generating long-distance entanglement**

Bernien et al., *Nature* **497** (2013)



# Recap: requirements for a loophole-free Bell test

loopholes to close :



locality



detection

→ generation of a long-distance entangled pair of spins with high fidelity

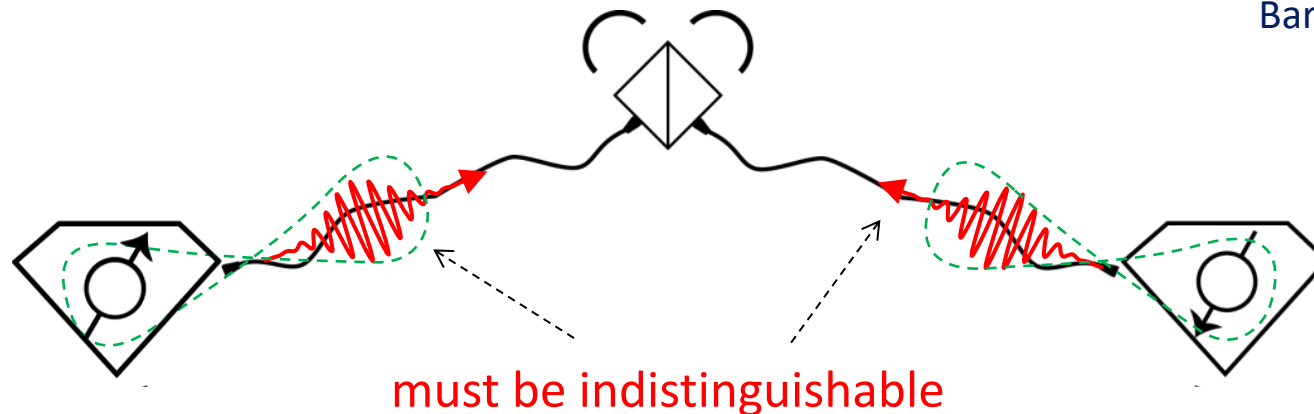
→ fast single-shot readout of the spins with high fidelity



**Generating long-distance entanglement**

Bernien et al., *Nature* **497** (2013)

Barrett & Kok, *PRA* **71** (2005)



# Recap: requirements for a loophole-free Bell test

loopholes to close :



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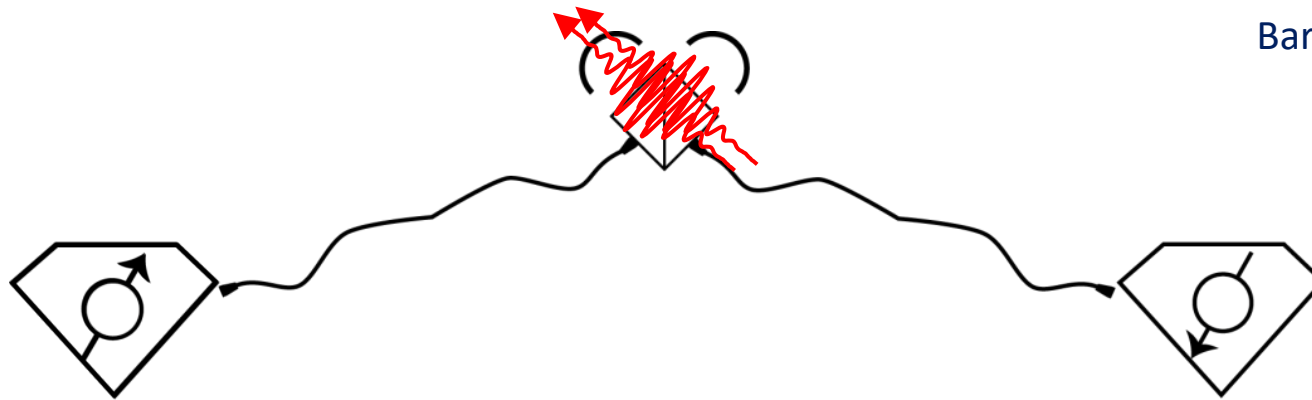
→ fast single-shot readout of the spins with high fidelity



**Generating long-distance entanglement**

Bernien et al., *Nature* **497** (2013)

Barrett & Kok, *PRA* **71** (2005)





# Recap: requirements for a loophole-free Bell test

loopholes to close :



locality



detection

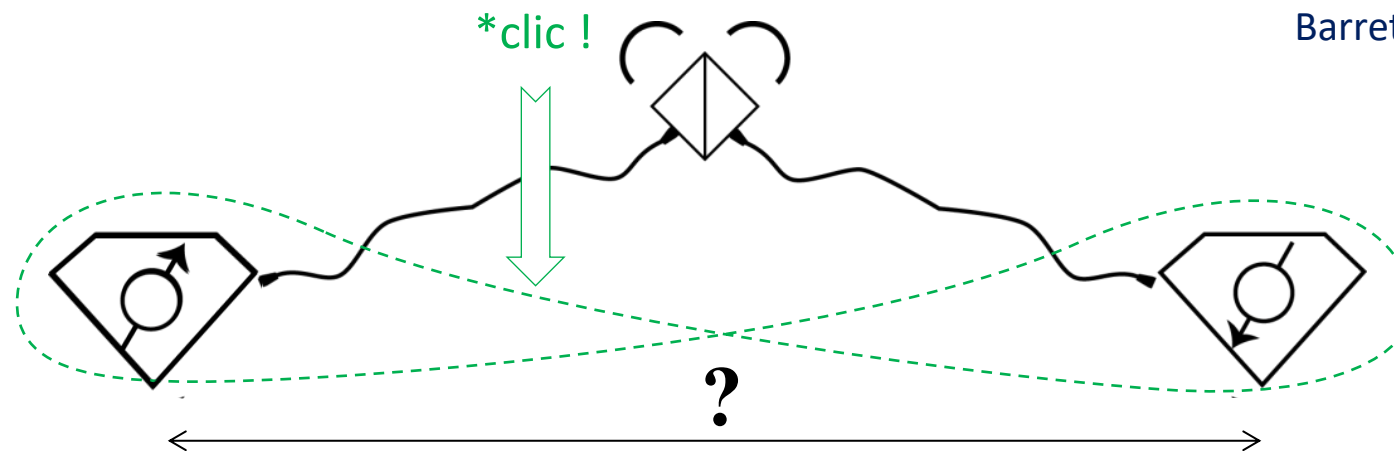
→ generation of a long-distance entangled pair of spins with high fidelity

→ fast single-shot readout of the spins with high fidelity

→ **Generating long-distance entanglement**

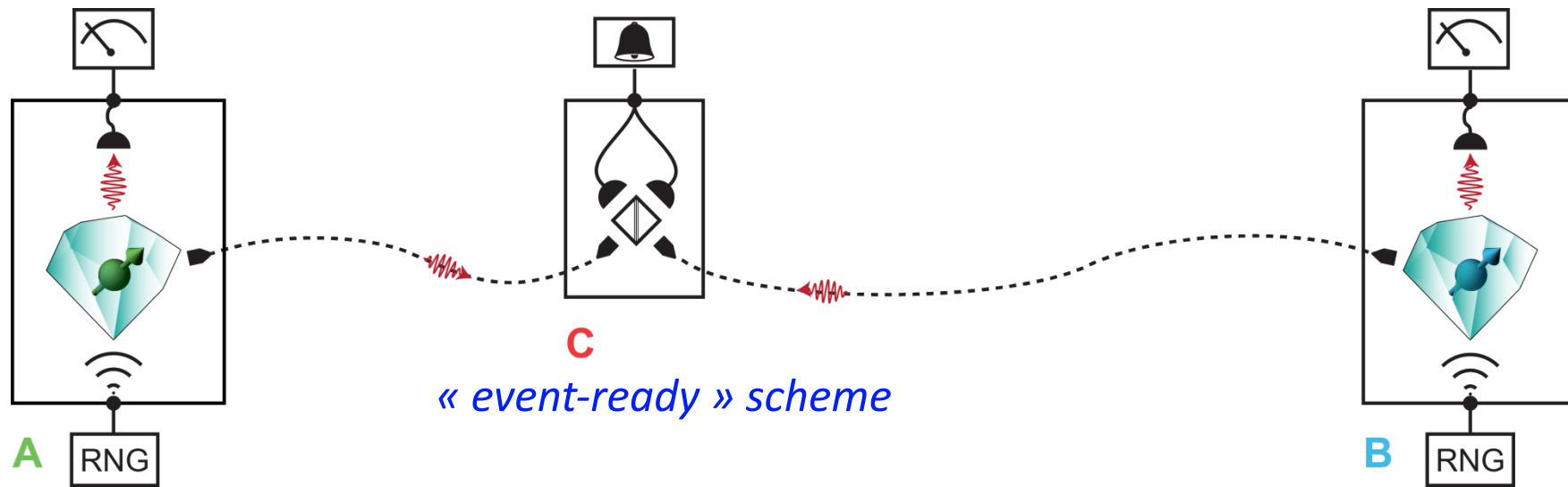
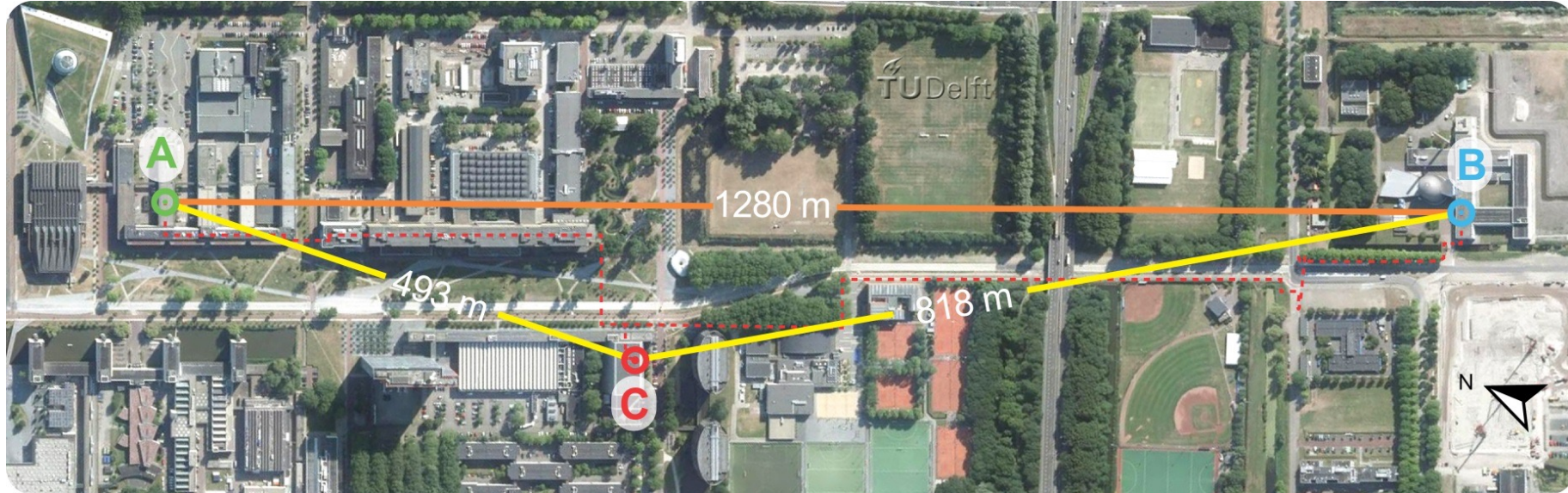
Bernien et al., *Nature* **497** (2013)

Barrett & Kok, *PRA* **71** (2005)

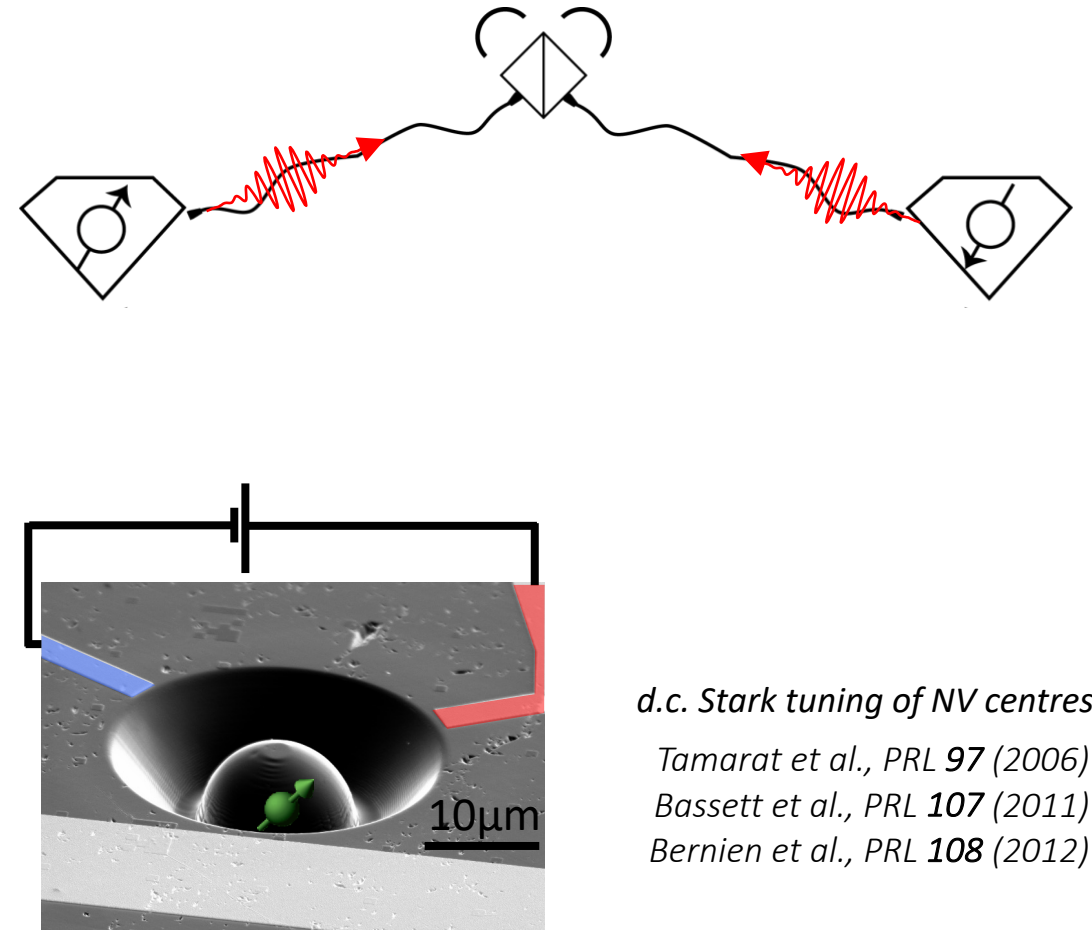
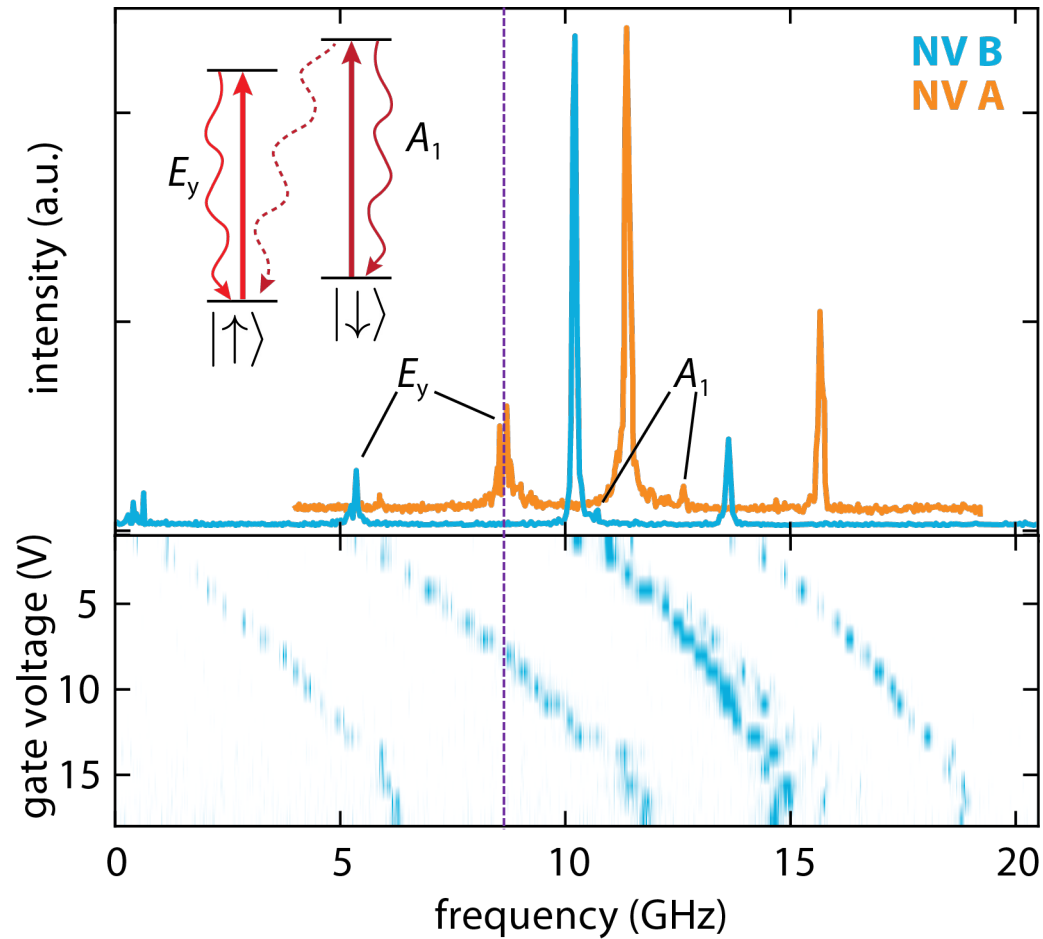


entanglement  
swapping

# A loophole-free Bell test in Delft



# Challenge: getting indistinguishable photons



*d.c. Stark tuning of NV centres:*

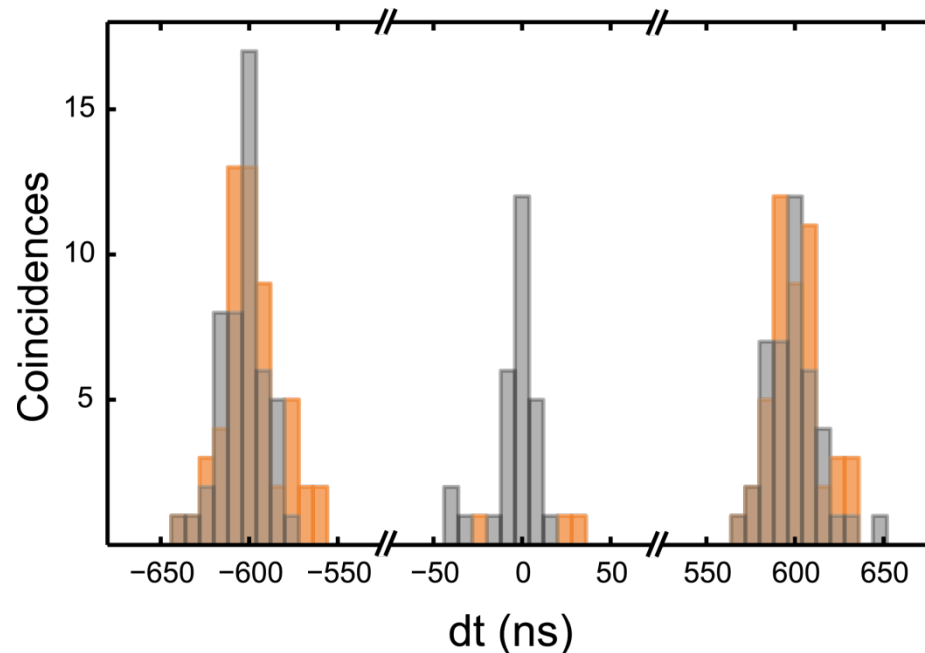
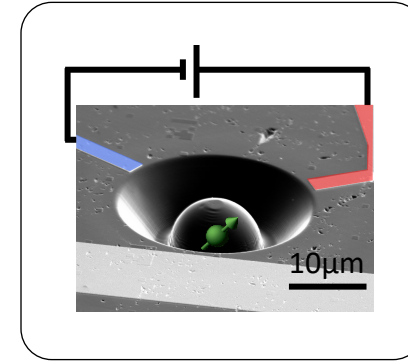
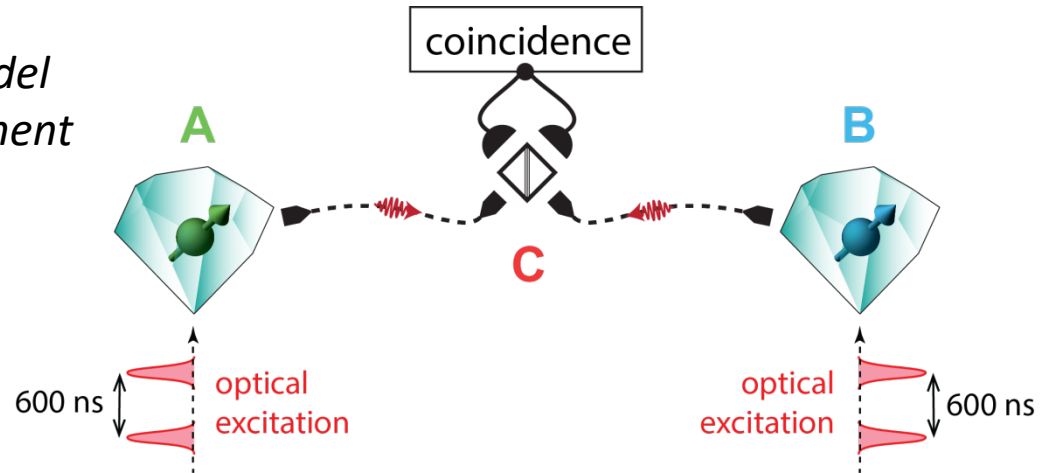
*Tamarat et al., PRL 97 (2006)*

*Bassett et al., PRL 107 (2011)*

*Bernien et al., PRL 108 (2012)*

# Challenge: getting indistinguishable photons

Hong-Ou-Mandel  
(HOM) experiment



■ Distinguishable  
■ Undistinguishable



Interference visibility

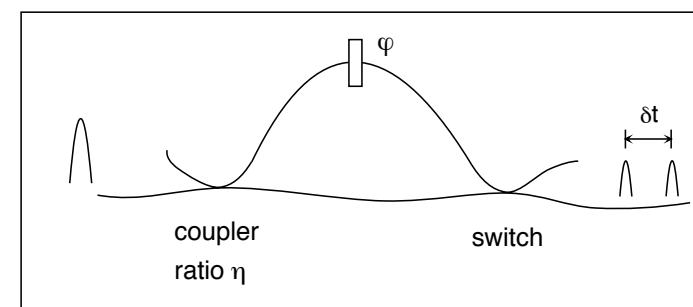
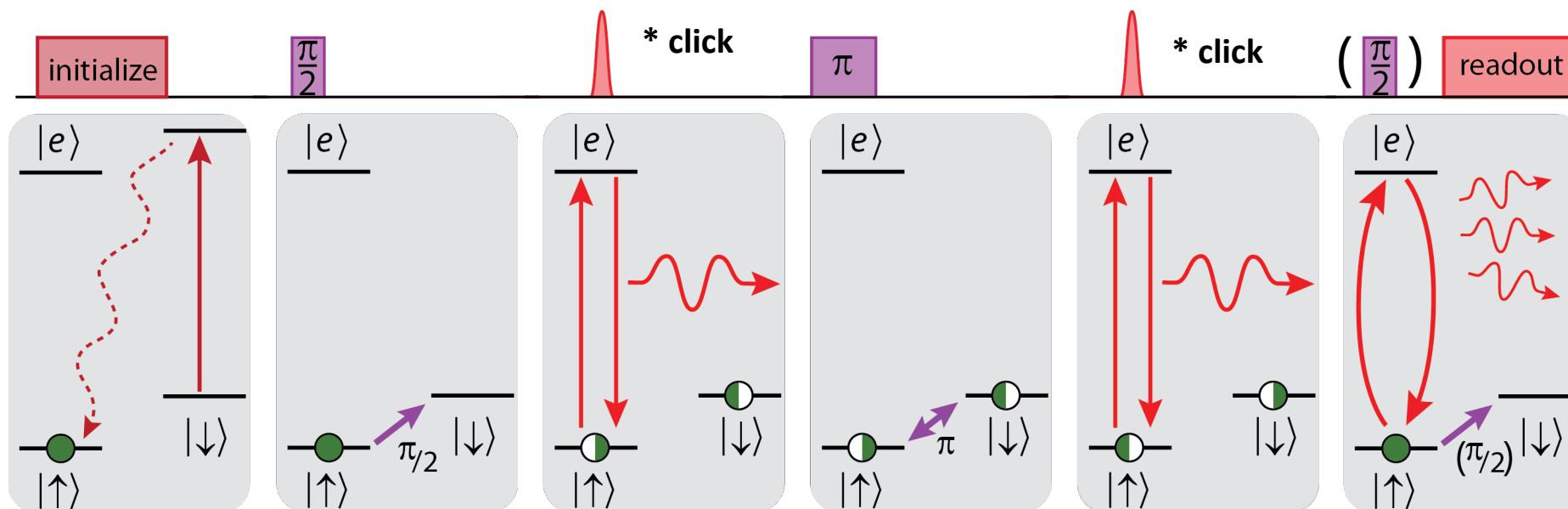
$$V = 90 \pm 6$$

bound on entanglement fidelity :

$$\mathcal{F} < (1 + V)/2$$

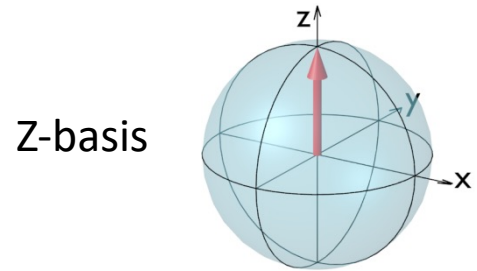
# The long-distance entanglement protocol

Barrett and Kok, PRA 71 (2005)



Brendel et al., PRL 82 (1999)

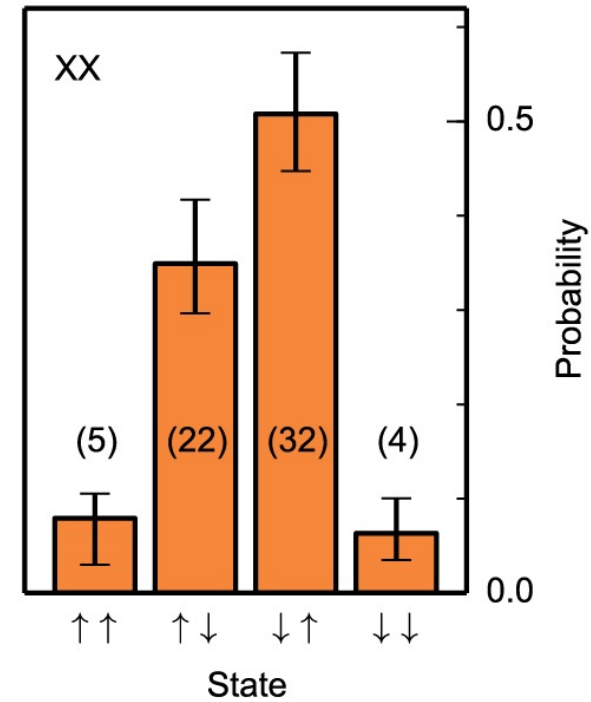
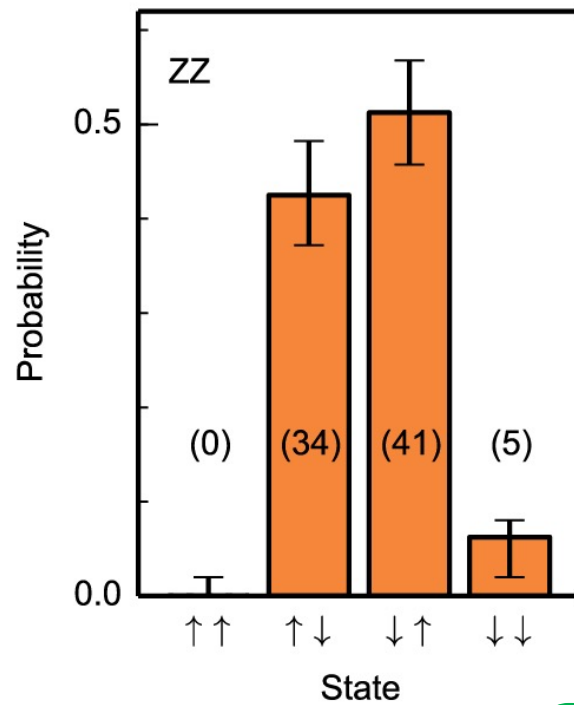
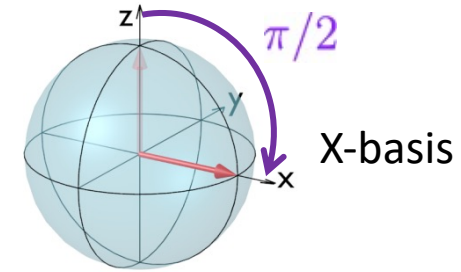
# Entanglement check: correlation measurements



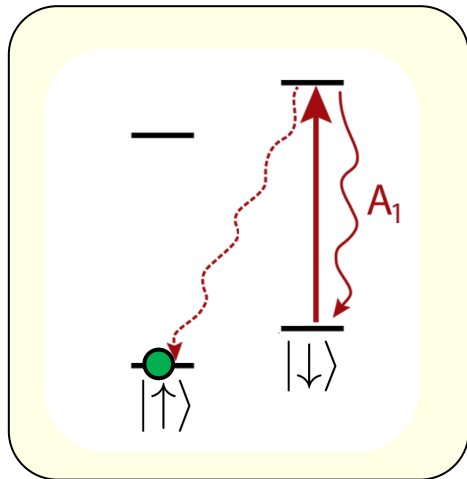
Perfect target state:

$$|\psi_{-}\rangle = \frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)$$

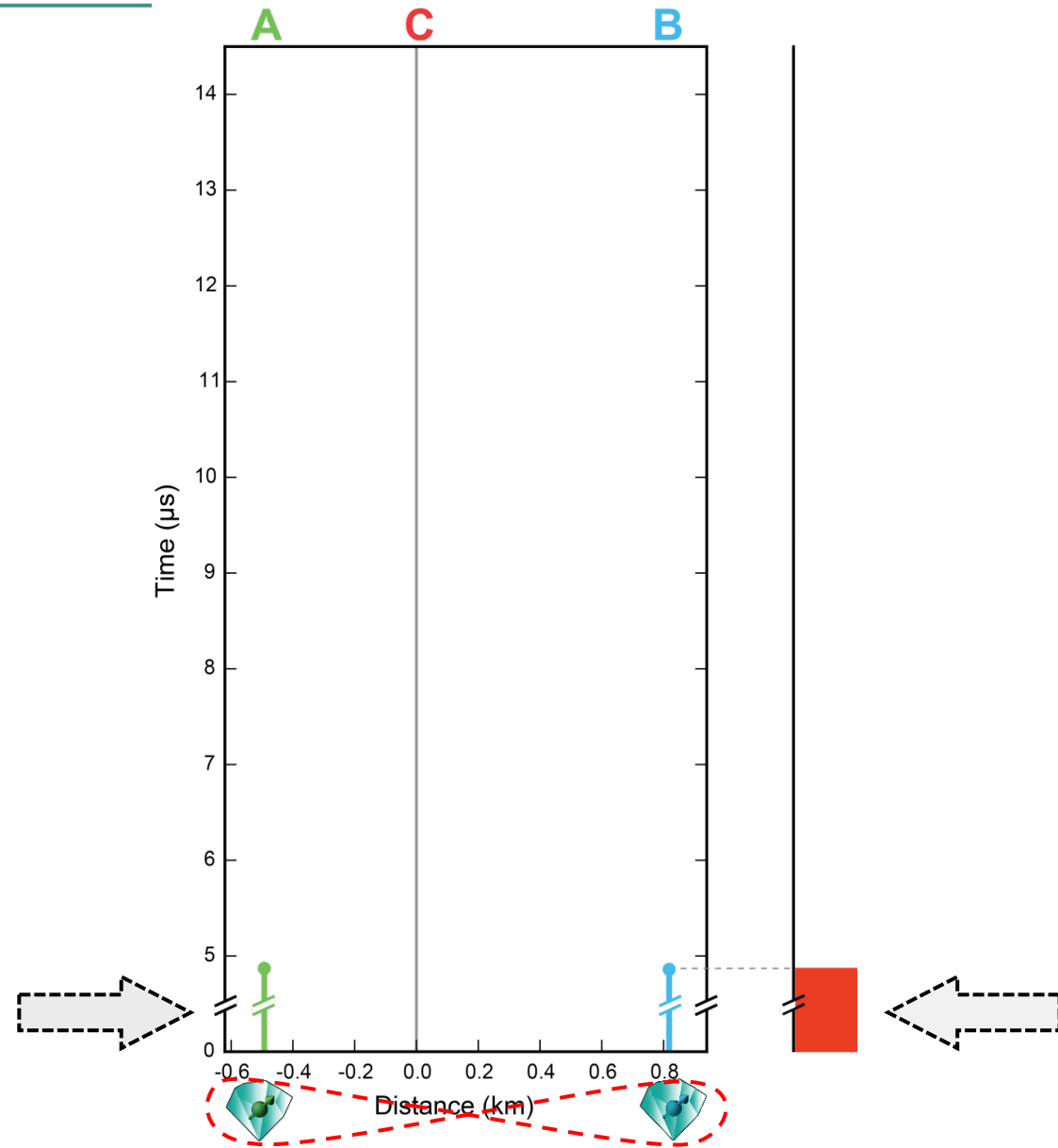
“Bell singlet state”



**Entanglement fidelity =  $92 \pm 3\%$**

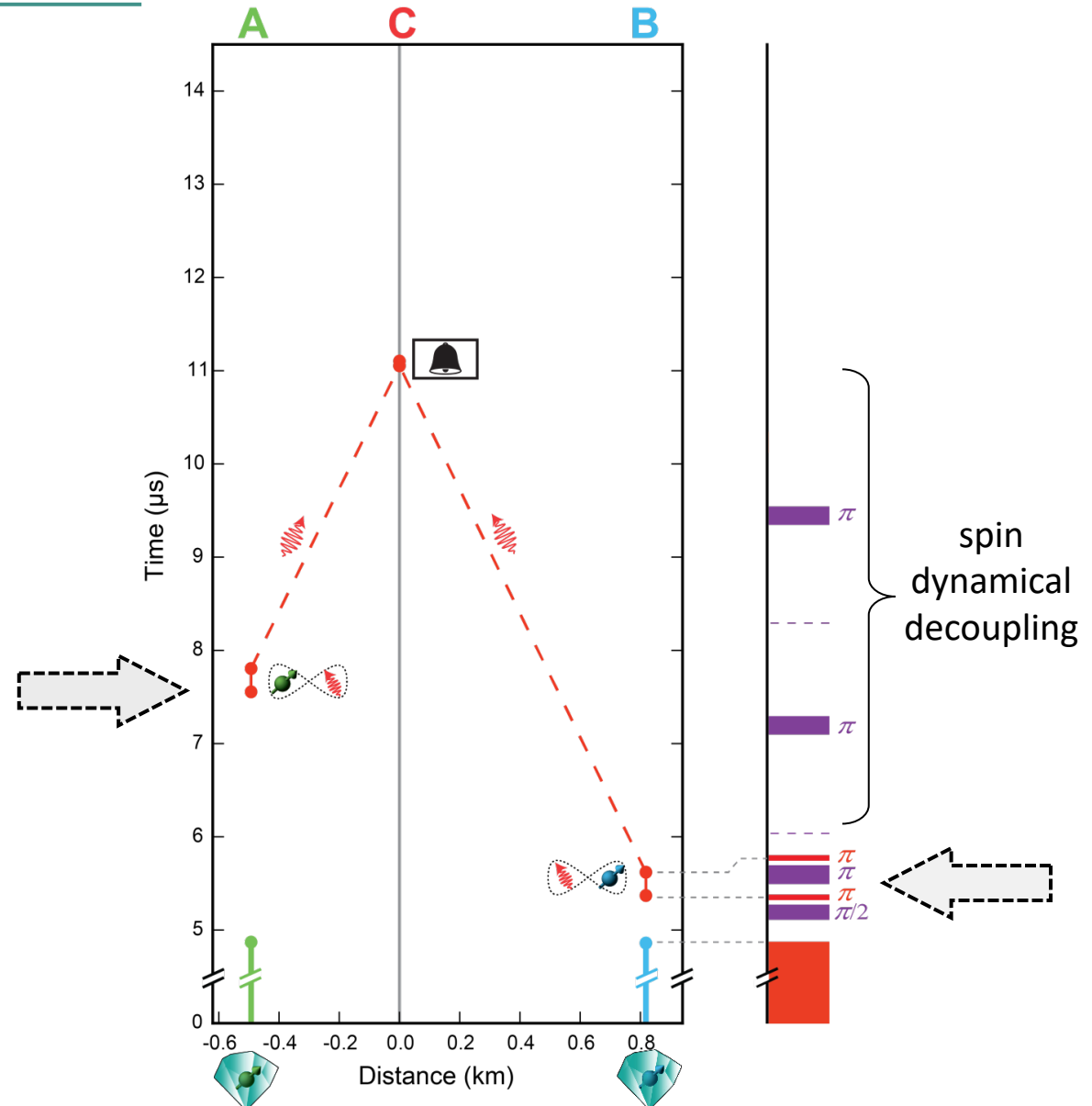


1 Initialization/Reset



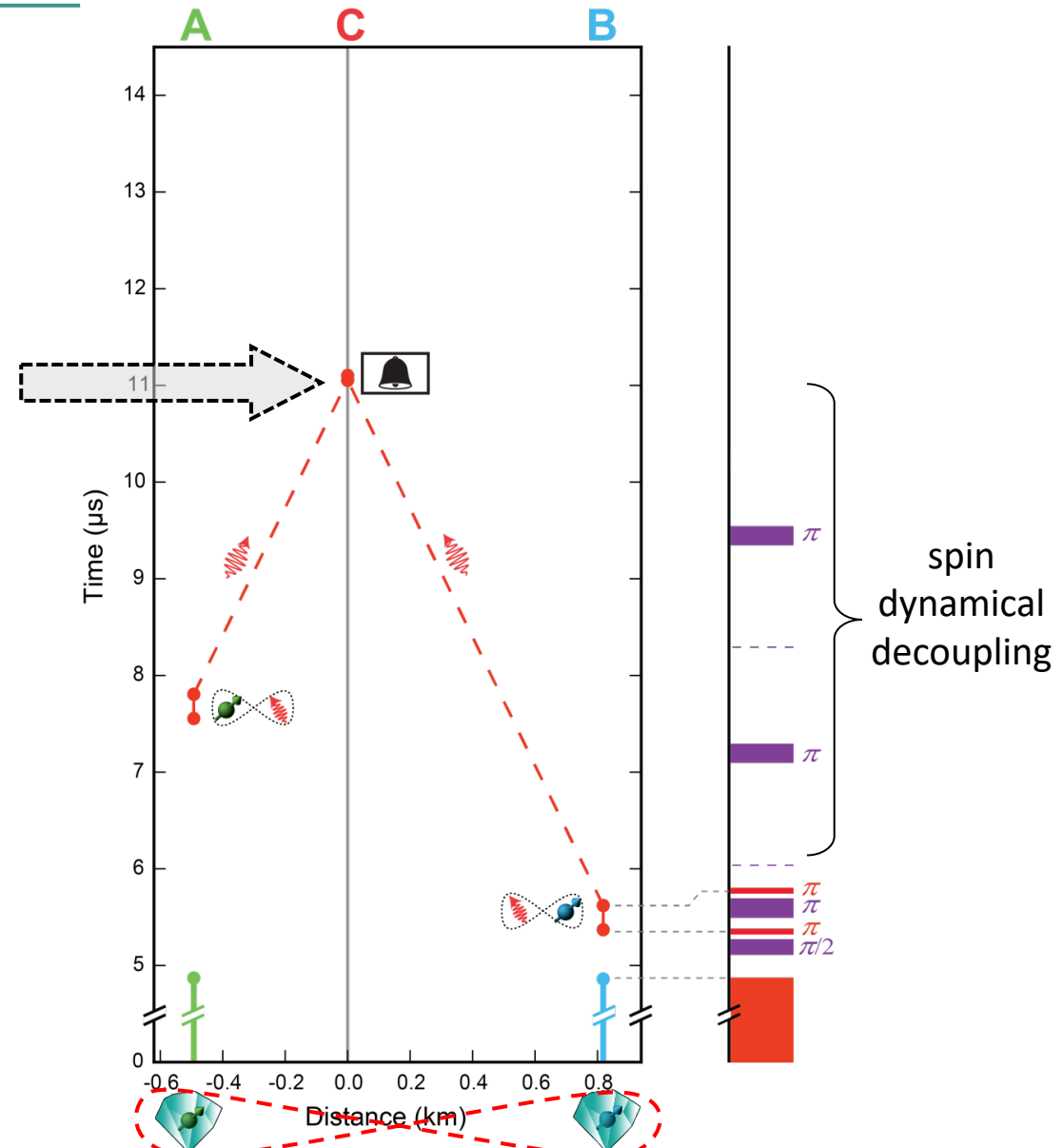
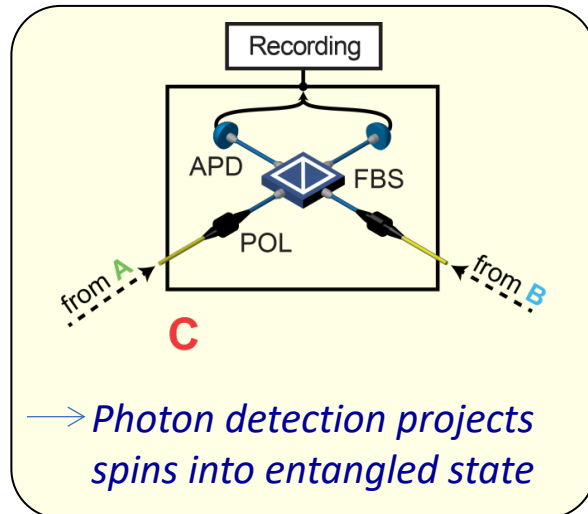
# Experimental scheme (2)

2 Spin-photon entanglement





### 3 Entanglement generation



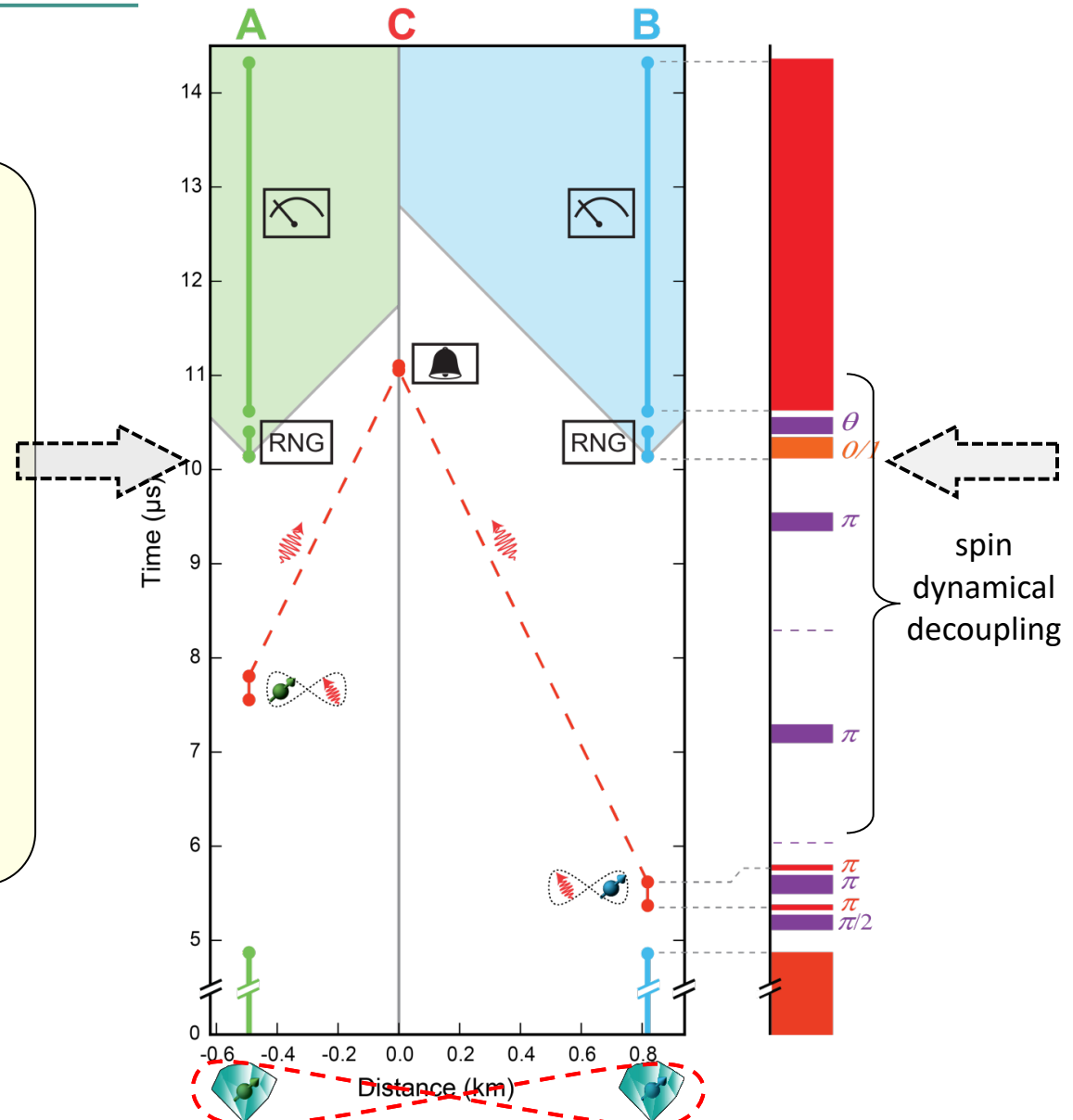
### 4 Random basis choice

1!

Fresh Quantum  
Random Number  
Generator **ICFO**

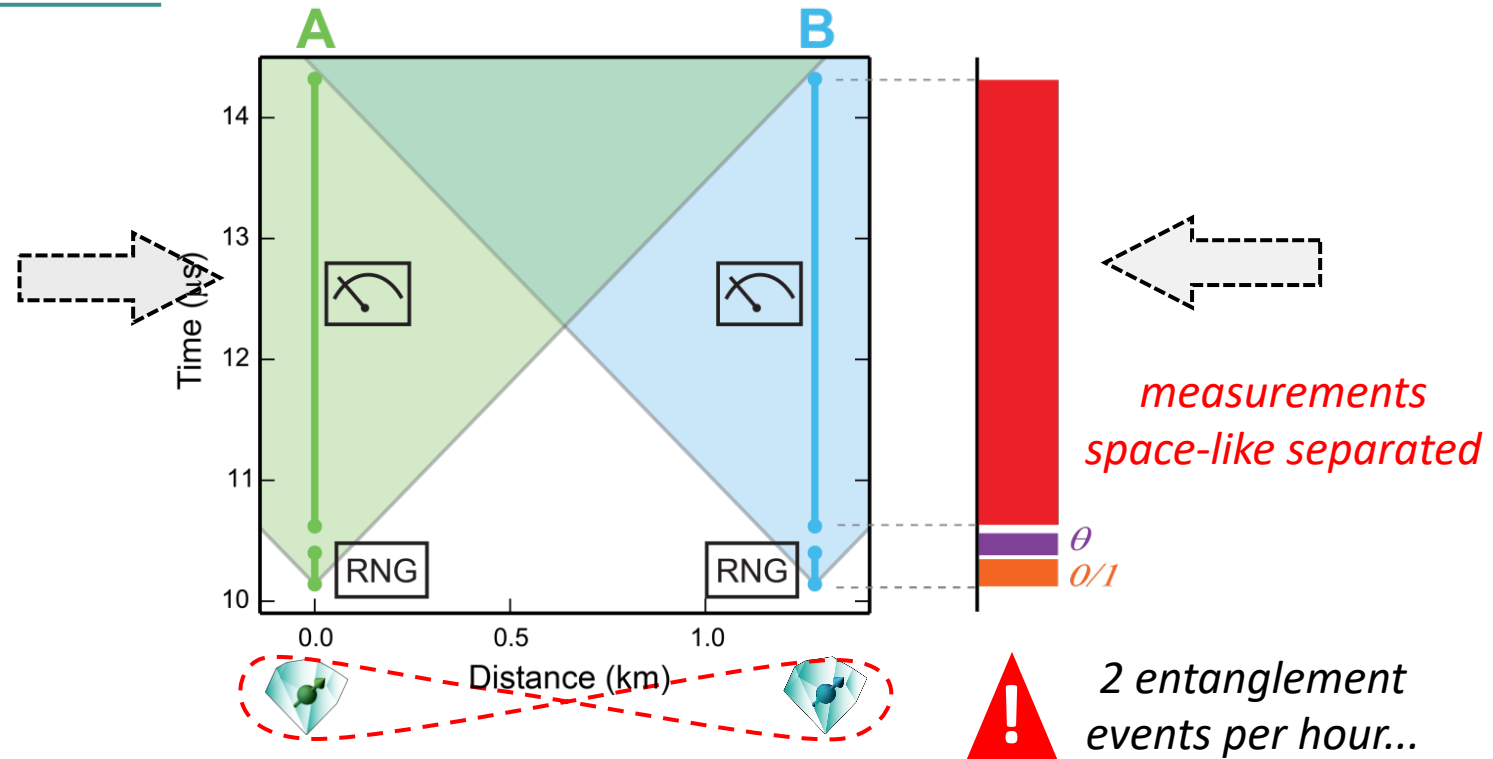
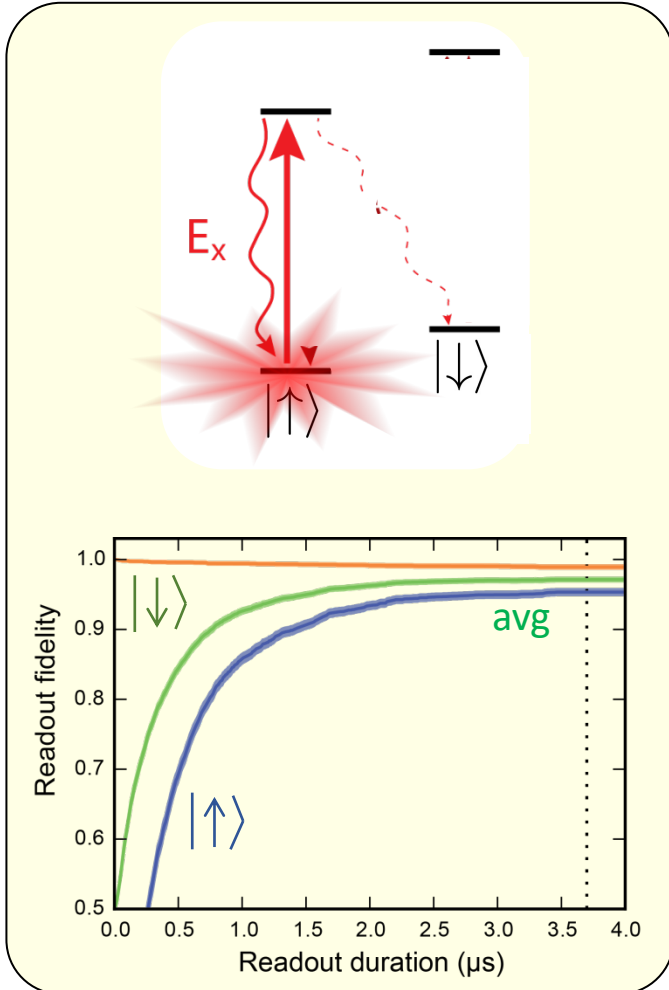
Abellán *et al.*, PRL **115** (2015)  
(Mitchell's group, Barcelona, Spain)

*Event-ready signal space-like  
separated from random  
number generation (RNG)*



5

### Basis rotation and readout



A, B & C space-like separated  $\Rightarrow$  **locality loophole**

single-shot readout  $\Rightarrow$  **detection loophole**

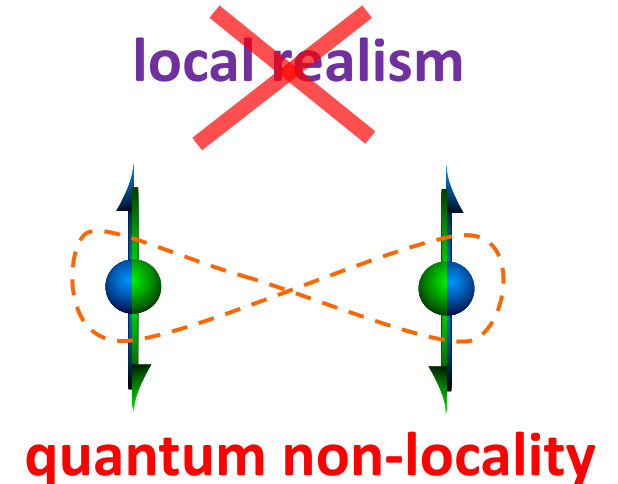
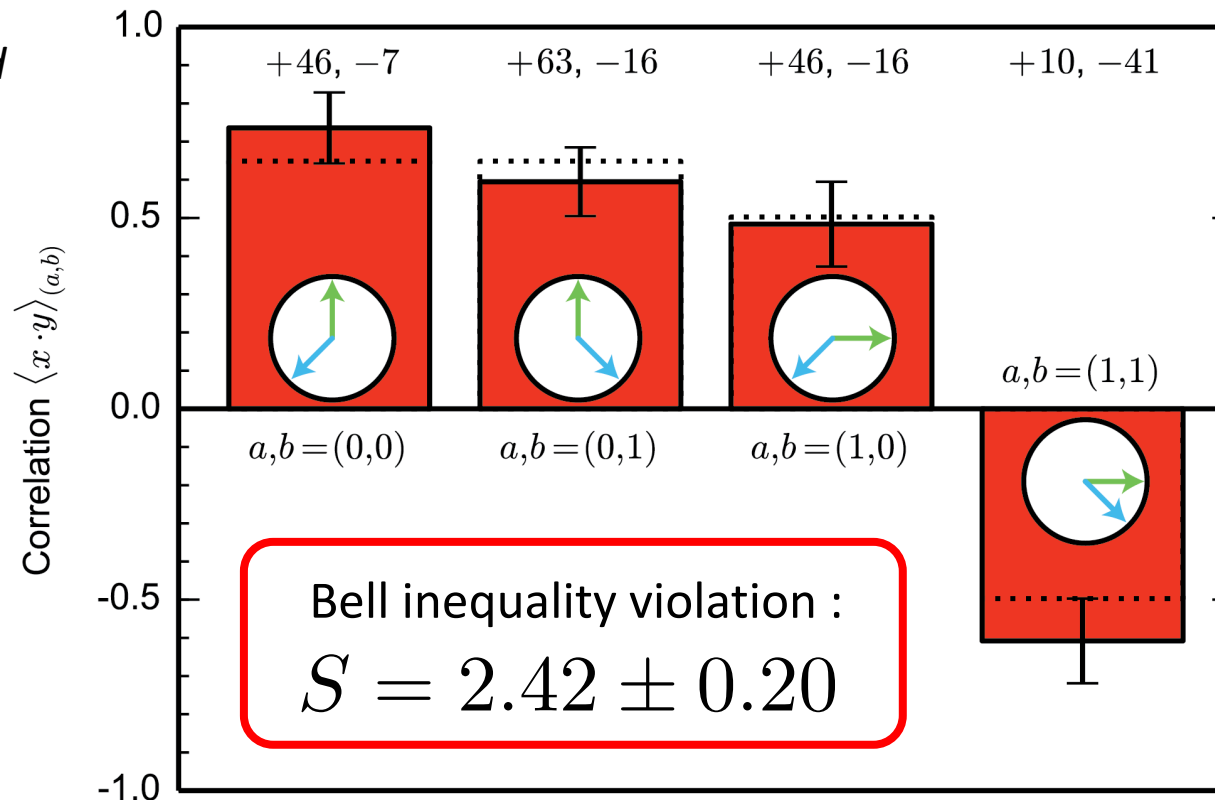
# Experimental scheme (5)

CHSH inequality

$$S = | \langle x \cdot y \rangle_{(0,0)} + \langle x \cdot y \rangle_{(0,1)} + \langle x \cdot y \rangle_{(1,0)} - \langle x \cdot y \rangle_{(1,1)} | \leq 2$$

local realist bound

... 18 days and nights later:

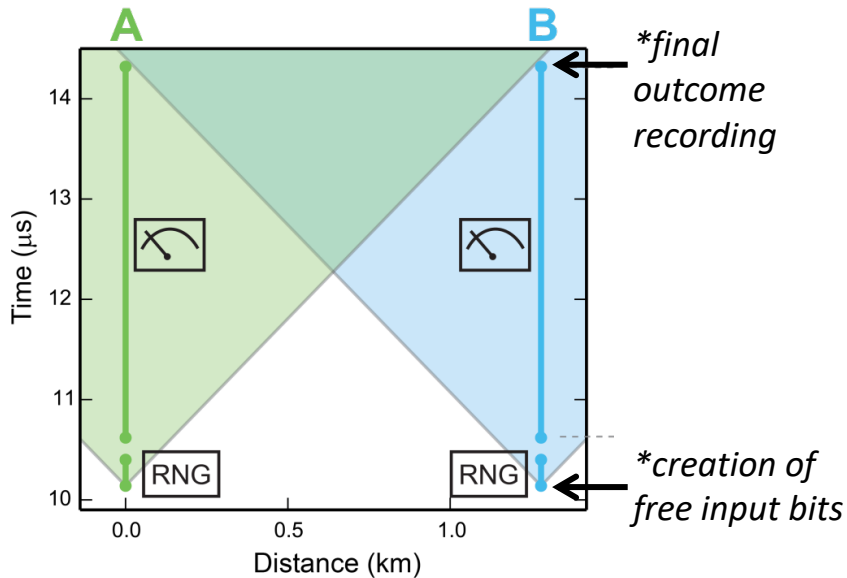


B. Hensen *et al.*, *Nature* **526** (2015)

probability that any local realist model  
could have produce the data :

**p-value = 0.039**

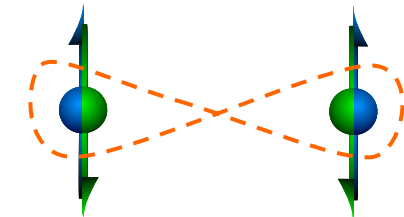
# Level of significance?



probability that the observed data (or more extreme) would result under the assumption that our experiment is ruled by a local realist model\* ?  
= **p-value**

2 scenarios :

	Conventional	Complete
Memory ?	No	Yes
Outcome distribution	Gaussian	Arbitrary
Random bit unpredictability	Perfect	Partial
p-value	$S = 2.42 \pm 0.20$ $p = 0.019$	<b><math>p = 0.039</math></b>



**New realization: Dec. 2015**  
+ 300 events (210h over 22 days)

$$S_{\text{tot}} = 2.380 \pm 0.136$$

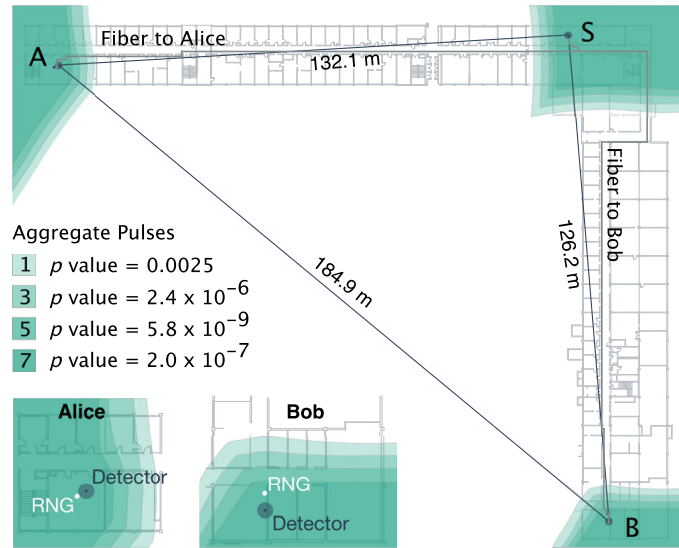
$p = 0.017$  (independent runs)

$p = 0.008$  (single run)

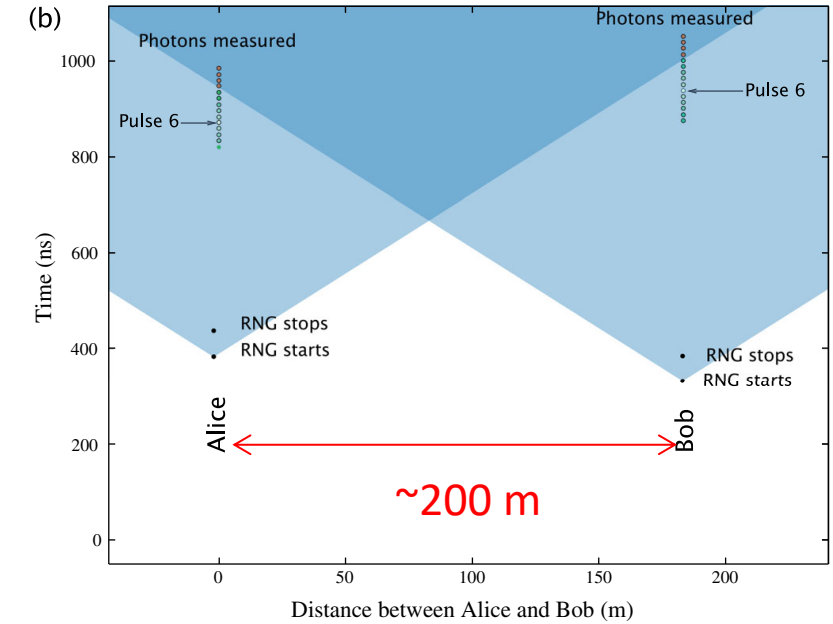
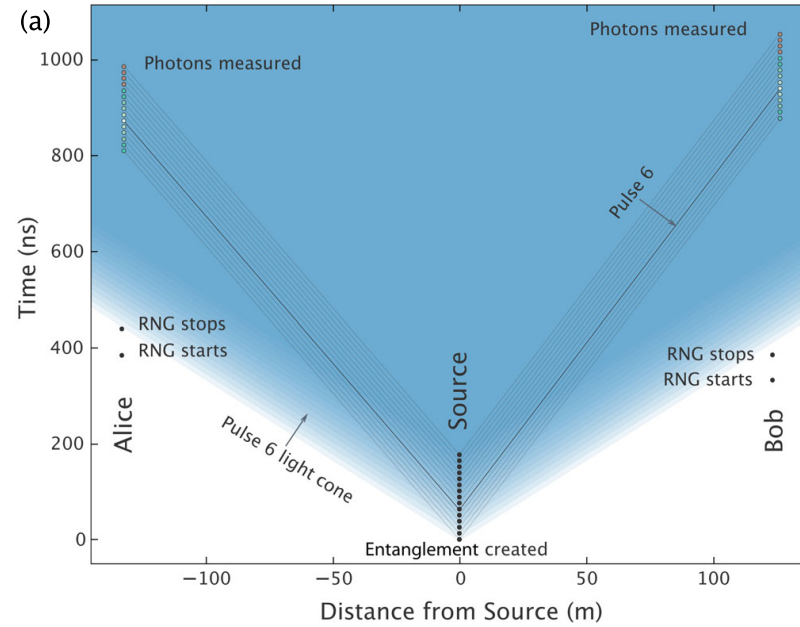
Hensen *et al.*, *Scient. Rep.* **6** (2016)

# The Boulder loophole-free Bell test with photons

Shalm et al., *PRL* **115** (2015) (NIST)



+ detection efficiency > 72.5%



*Clauser-Horne type inequality:*

$$P(++|ab) \leq P(+0|ab') + P(0+|a'b) + P(++|a'b')$$

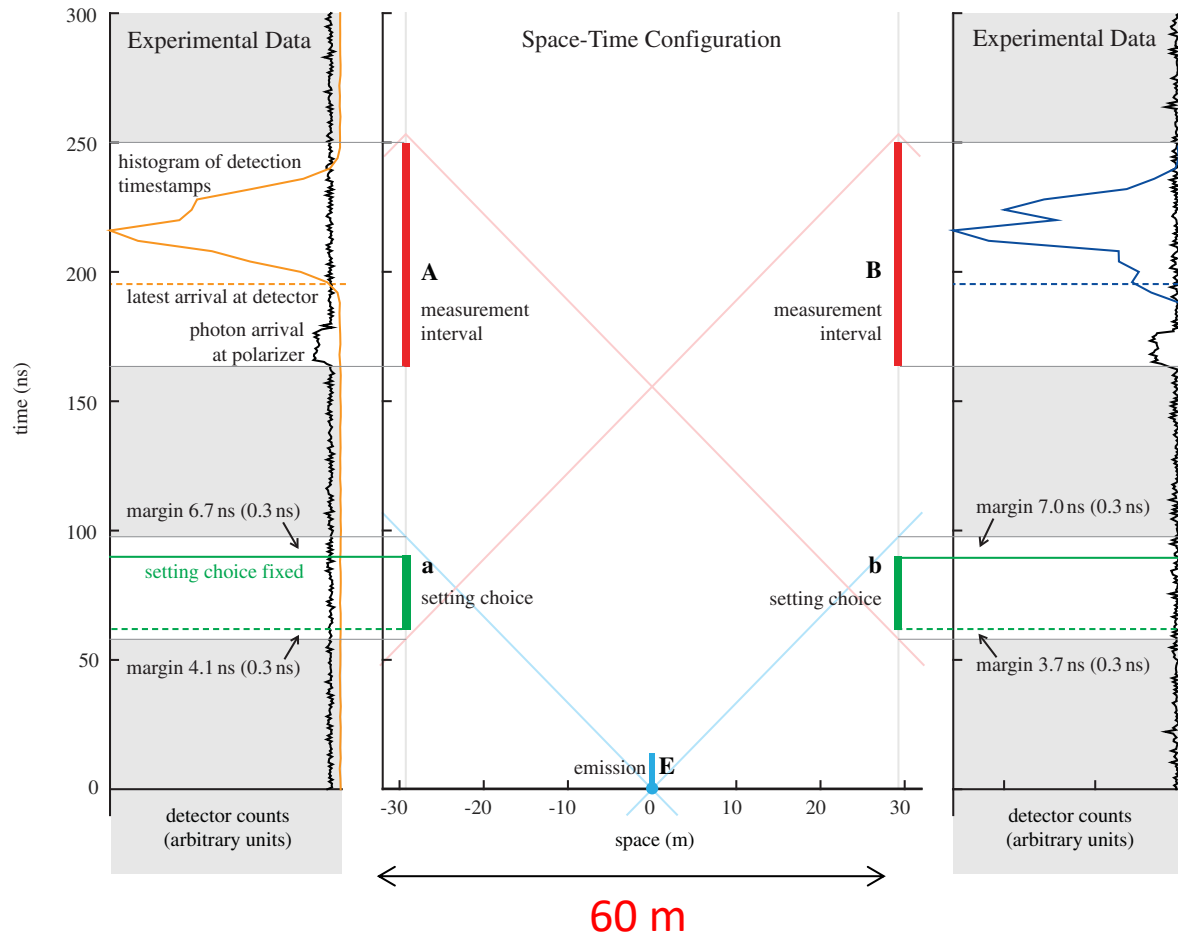
30 min per run,  
180 millions entangled pairs detected!

*maximum propability that experiment statistics produced by local realim:*

$$p = 2.3 \cdot 10^{-7}$$

# The Vienna loophole-free Bell test with photons

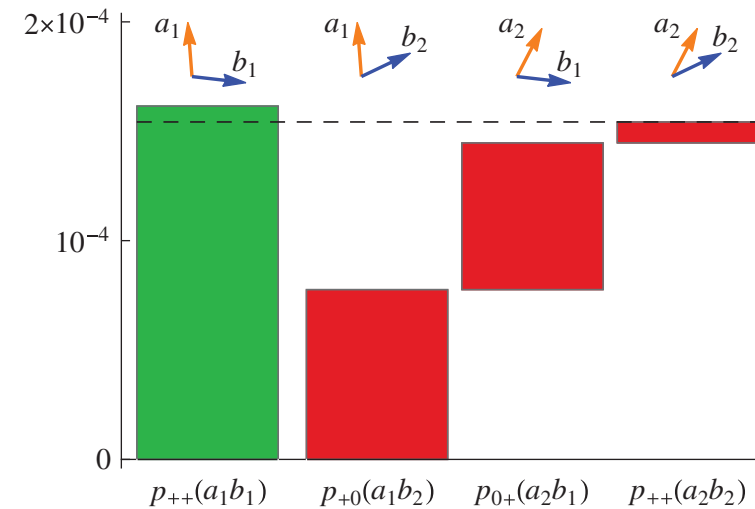
Giustina *et al.*, *PRL* **115** (2015) – (Zeilinger's group)



+ detection efficiencies > 2/3

CH-Eberhard type-inequality:

$$J \equiv p_{++}(a_1 b_1) - p_{+0}(a_1 b_2) - p_{0+}(a_2 b_1) - p_{++}(a_2 b_2) \leq 0.$$



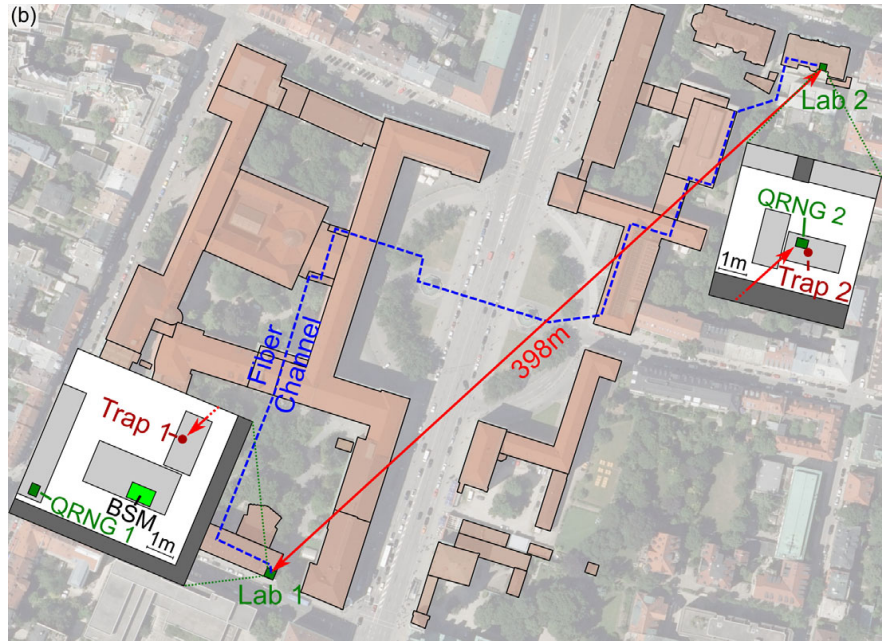
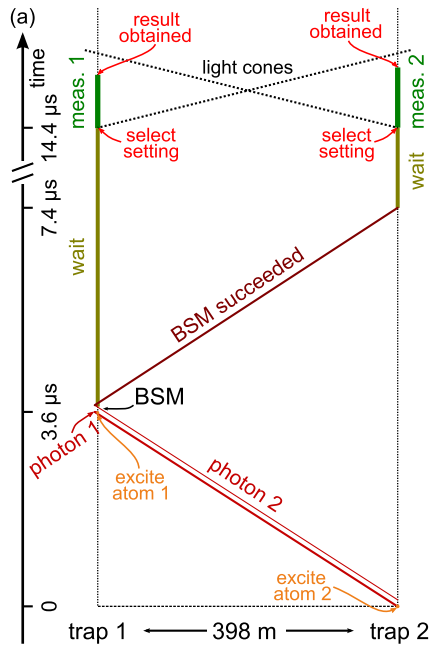
$$J = 7.27 \cdot 10^{-6}$$

$$p = 3.7 \cdot 10^{-31}$$

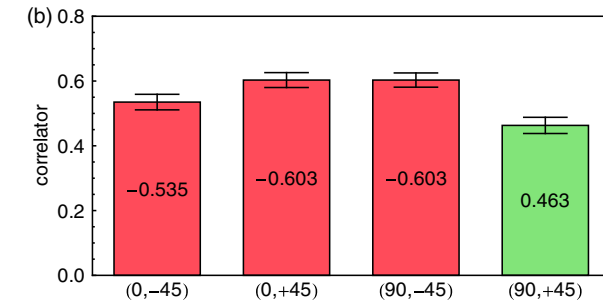
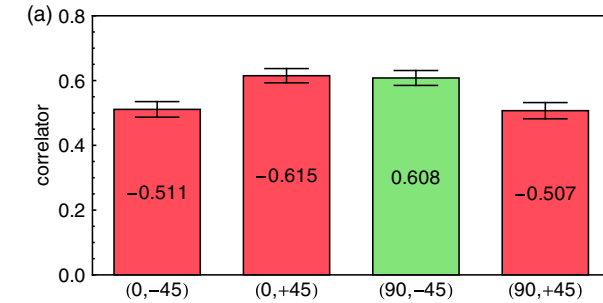
3500 entangled pairs per second!

# The Munich loophole-free Bell test with atoms

Rosenfeld *et al.*, *PRL* **119** (2017) – (Weinfurter's group)



entanglement between 2 single  $^{87}\text{Rb}$  atoms  
in an optical dipole trap



CHSH inequality:  $S \leq 2$

$$S = 2.221 \pm 0.033$$

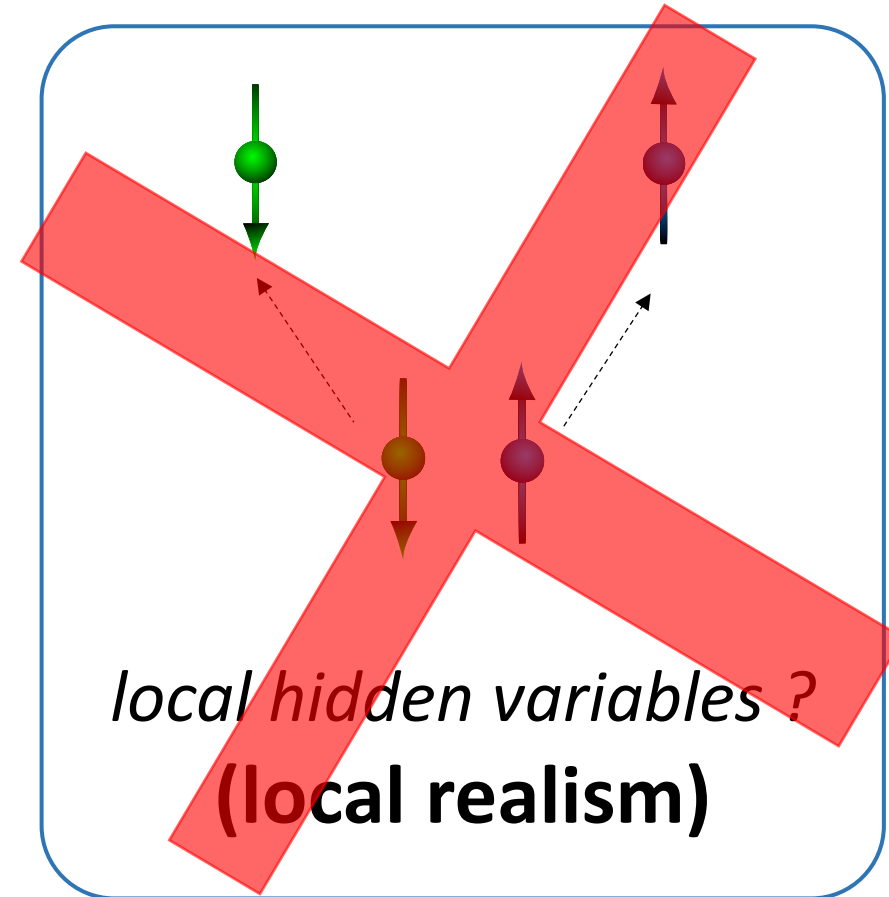
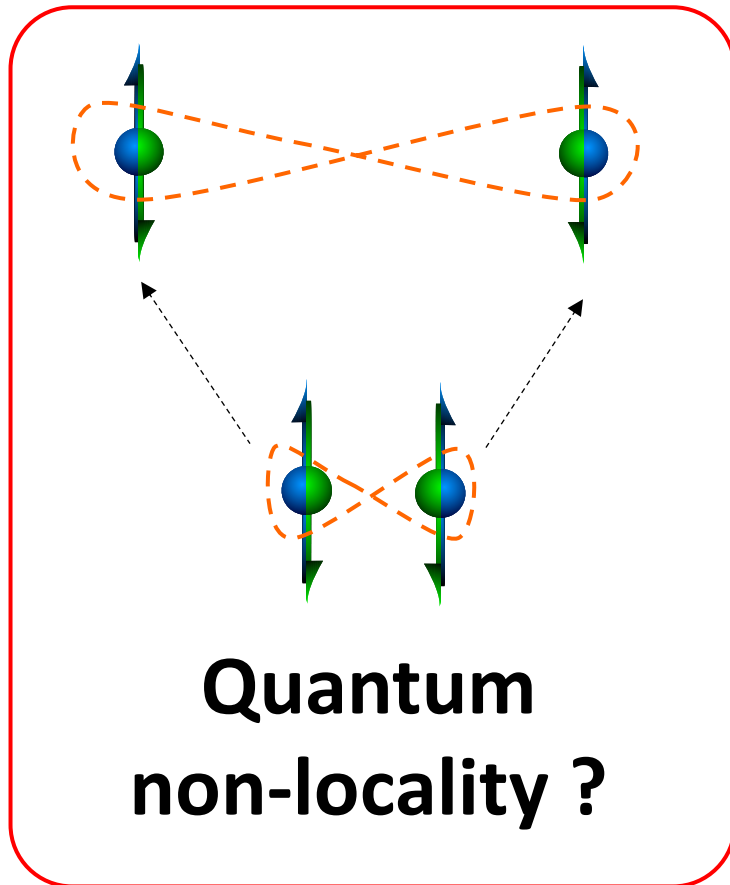
$$p < 2.57 \cdot 10^{-9}$$

5000 events during 4 days!



# « Death by experiment of local realism »

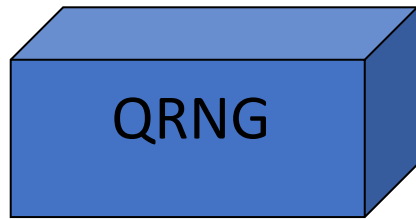
Wiseman, *Nature* **526** (2015)



**What is next?**

# True randomness? True free-will?

measurements settings chosen by



quantum random number generator ?

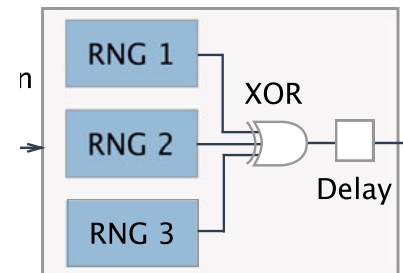


or human choice?

or combine them?

the NIST loophole-free Bell test

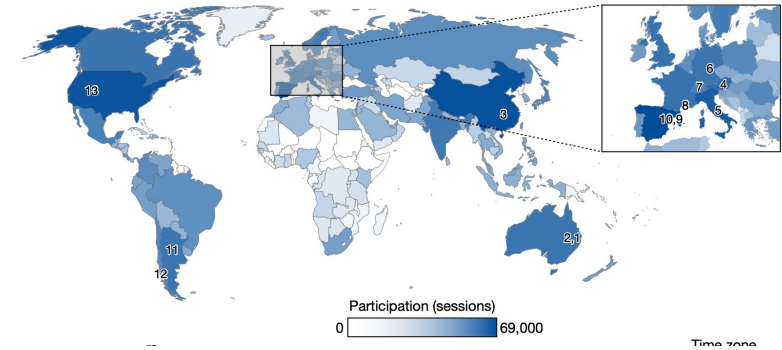
Shalm et al., *PRL* **115** (2015)



settings chosen by humans with online game

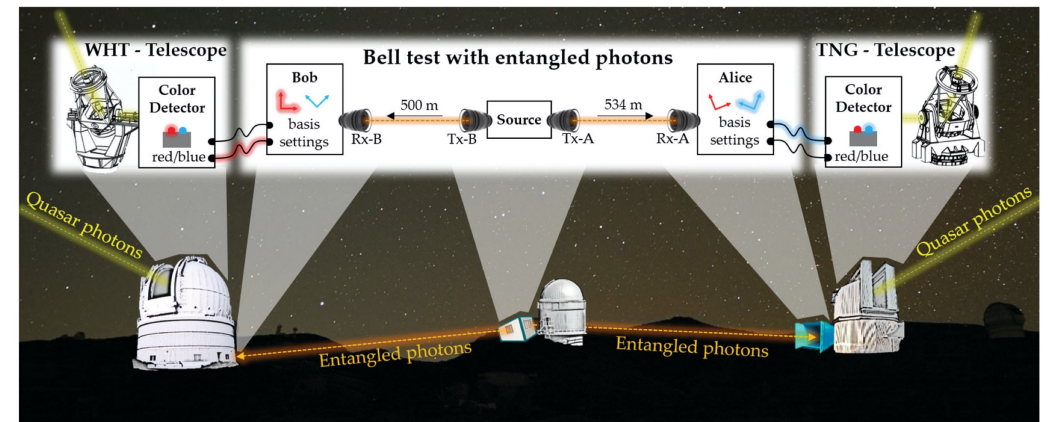
the Big Bell test

Abellan et al., *Nature* **557** (2018)



settings chosen by quasars billions of years ago

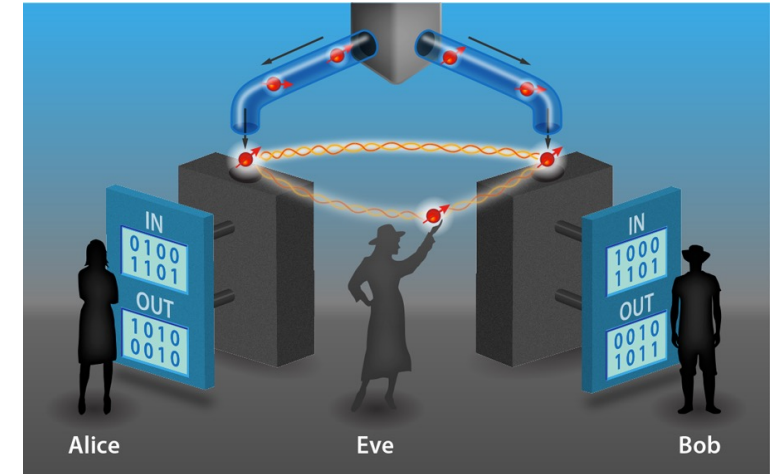
Rauch et al., *PRL* **121** (2018) – (Zeilinger's group)



→ Applications : device-independent protocols

→ randomness expansion *Pironio et al., Nature 464 (2010)*

→ quantum key distribution *Acín et al., PRL 98 (2007)*



Colbeck, *Physics* 7 (2014)

→ towards quantum networks...

