NON-LOCALITY AND CONTEXTUALITY

THEMATIC SCHOOL

"FROM THE FIRST TO THE SECOND QUANTUM REVOLUTION: THEORY AND APPLICATIONS".

PEYRESQ

20 SEPTEMBER 2021

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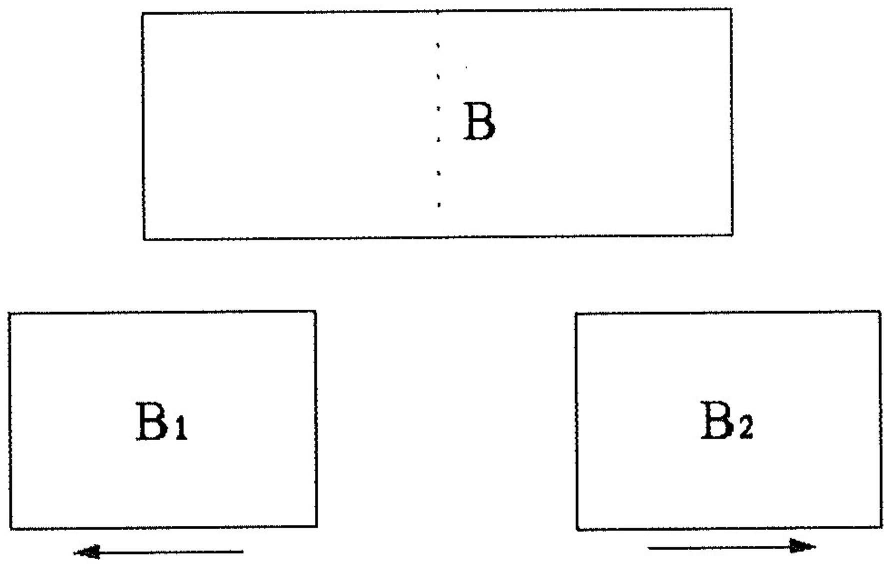
EINSTEIN'S BOXES



A single particle is in Box B. One cuts the box in two half-boxes,

$| \text{ state} > = | \; B >$

The state becomes

$$\longrightarrow \frac{1}{\sqrt{2}}(|B_1 > + |B_2 >)$$

where $|B_i > =$ particle "is" in box B_i, $i = 1, 2$.

The two half-boxes B_1 and B_2 are then separated and sent as far apart as one wants.

If one opens one of the boxes (say B_1) and that one does *not* find the particle, one *knows* that it is in B_2. Therefore, the state "collapses" instantaneously and in a non local way.

One opens box $B_1 \longrightarrow$ nothing

This is a "measurement", therefore state $\longrightarrow$ $|B_2>$

(and, if one opens the box B_2, one will find the particle !).

Is the reduction or collapse of the $|$ state $>$ a real (= physical) operation or does it represent only our knowledge (= epistemic) ?

<u>If</u> physical $\longrightarrow$ A non local form of causality exists (= action at a distance).

<u>If</u> epistemic $\longrightarrow$ quantum mechanics "incomplete" : there exists other variables than the quantum state that describe the system.

These variables would tell in which half-box the particle **IS** *before one opens either of them.*

Einstein certainly thought that this argument **proves** the incompleteness of quantum mechanics, since, for him (and probably for everybody else at the time), actions at a distance were unthinkable.

But let us put aside for now the issue of completeness and **prove** non locality directly.

WHAT IS NON LOCALITY ?

Non local causality (causality NOT mere correlation)

Properties

1. Instantaneous
2. a. Extends arbitrarily far

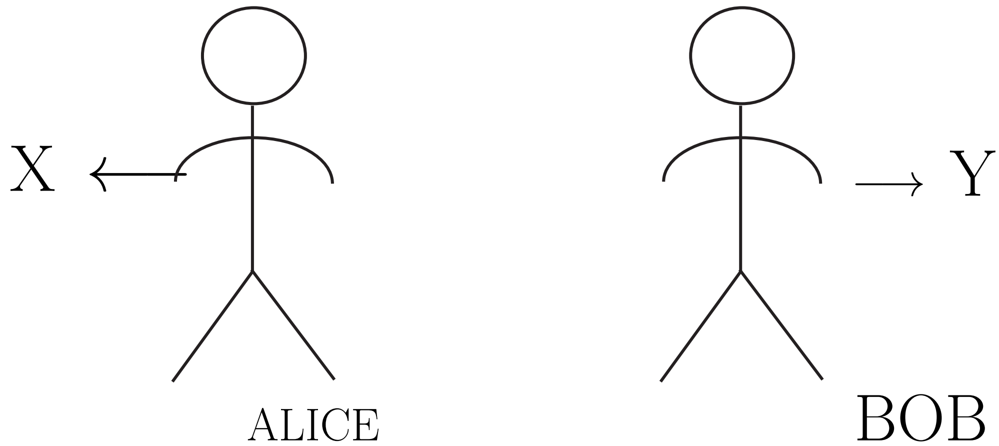
   b. The effect does not decrease with the distance
3. Individuated
4. Can be used to transmit messages

Newton's gravity : 1, 2a and 4

Post-Newtonian physics (e.g. field theories) : 2a and 4

Is there a phenomenon with properties : 1-3 ?
(Not 4 $\rightarrow$ pseudoscience).

PROOF OF NONLOCALITY THROUGH ANALOGY



3 questions 1,2,3

2 answers yes/no

Questions and answers vary. But when the same question is asked at X and Y, Alice and Bob always give the same answer.

Only two possibilities: either the answers are predetermined *or* there exists a form of causality at a distance *after* one asks the questions.

This is the Einstein Podolsky and Rosen (EPR-1935) argument (in Bohm's formulation). Let us call that the **EPR DILEMMA**.

One horn of the dilemma means nonlocality.

The other horn means that the answers are predetermined.

This dilemma concerns what happens in every single experiment, not just in the statistics of their results.

<u>BUT</u>

That second assumption

(alone)

leads to a contradiction with observations made when the questions are different.

Bell (1964)

PROOF

There are 3 Questions 1 2 3

and 2 possible Answers Yes/No

If the answers are given in advance, there exists $2^3 = 8$ possibilities :

1	2	3
Y	Y	Y
Y	Y	N
Y	N	Y
Y	N	N
N	Y	Y
N	Y	N
N	N	Y
N	N	N

In *each case* there are at least *two questions* with the same answer.

Therefore,

$$
\begin{aligned}
&\text{Frequency (answer to } 1 = \text{answer to } 2) \\
+ &\text{Frequency (answer to } 2 = \text{answer to } 3) \\
+ &\text{Frequency (answer to } 3 = \text{answer to } 1) \geq 1
\end{aligned}
$$

BUT,

in some experiments,

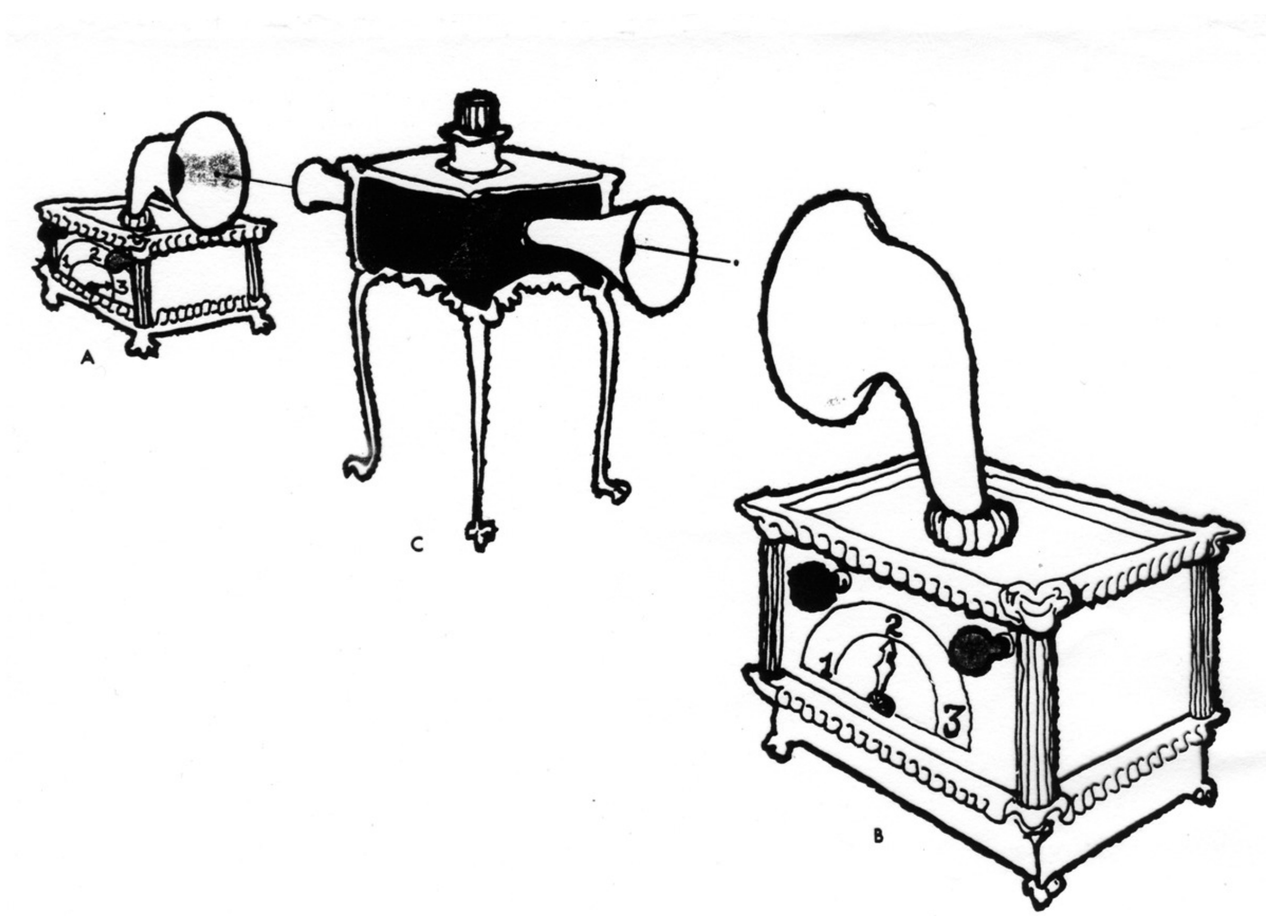
$$
\begin{aligned}
&\text{Frequency (answer to } 1 = \text{answer to } 2) \\
= &\text{Frequency (answer to } 2 = \text{answer to } 3) \\
= &\text{Frequency (answer to } 3 = \text{answer to } 1) \\
= &\frac{1}{4}
\end{aligned}
$$

$$\Rightarrow \frac{3}{4} \geq 1$$

FALSE !

$\Rightarrow$ CONTRADICTION

EXPERIMENTS



EXAMPLE OF “DATA”

1Y1Y	$\underline{\textit{1Y3Y}}$	$1Y2N$
$1N3Y$	$2N3Y$	**2N2N**
$1N2Y$	$3Y2N$	$1Y2N$
$1Y3N$	**3Y3Y**	**1N1N**
2Y2Y	$\underline{\textit{1N2N}}$	$1N2Y$
$3N1Y$	$1Y2N$	$1N3Y$
2N2N	**3N3N**	$\underline{\textit{1Y3Y}}$
1N1N	$3Y2N$	$\underline{\textit{3N2N}}$
$1Y3N$	$\underline{\textit{2Y3Y}}$	**1Y1Y**
$2N1Y$	$3Y2N$	$1N3Y$
2N2N	$\underline{\textit{3N1N}}$	**1Y1Y**
$\underline{\textit{2Y1Y}}$	**1N1N**	$1N3Y$
$2N3Y$	$3Y2N$	$1N2Y$
2Y2Y	$3N1Y$	**3Y3Y**
$1Y3N$	$2N1Y$	$\underline{\textit{3Y2Y}}$
1N1N	$1N2Y$	$3Y2N$
$\underline{\textit{2N1N}}$	**2N2N**	**1Y1Y**
3N3N	$3N2Y$	$1N3Y$

PROOF OF NONLOCALITY USING QUANTUM MECHANICS

A and B are replaced by particles

At X and Y there are are Stern-Gerlach apparatuses that "measure the spin" along some direction.

Below we will let 1, 2, 3 = 3 possible directions for that "measurement".

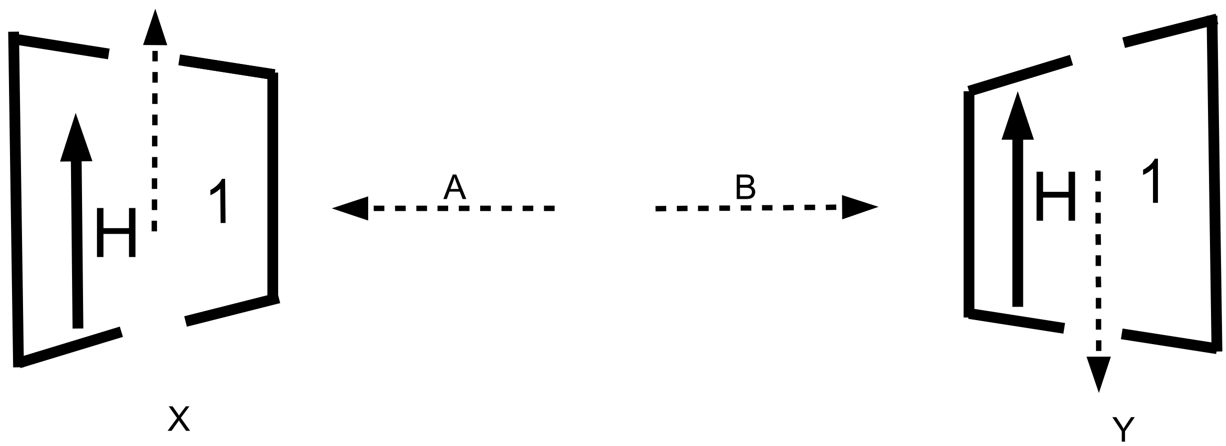
Yes/No = Up/Down.

But let us consider first a

$|$ state of the two particles $>$

$= \frac{1}{\sqrt{2}}(|A\ 1{\uparrow}>\ |B\ 1{\downarrow}> - |A\ 1{\downarrow}>\ |B\ 1{\uparrow}>)$

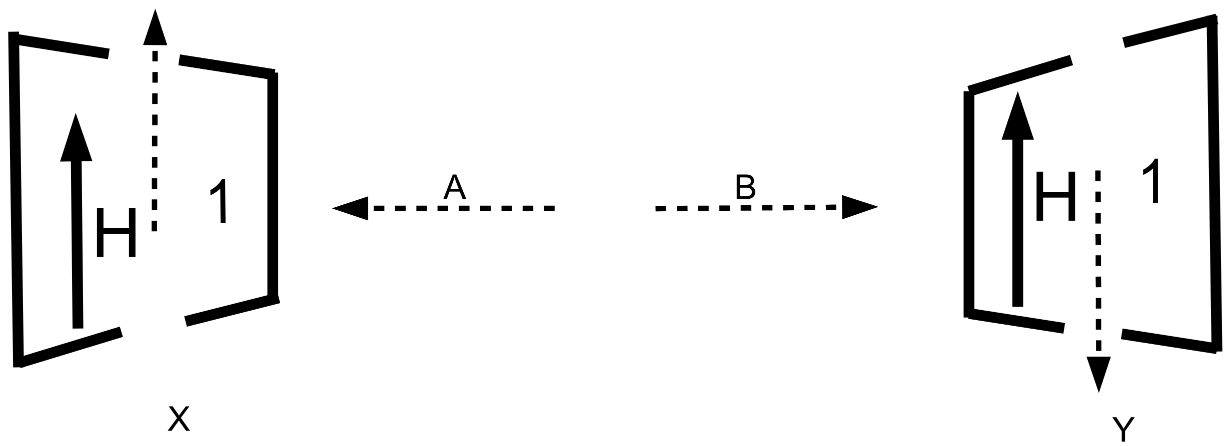
This is called an "ENTANGLED STATE".



Meaning of the state:

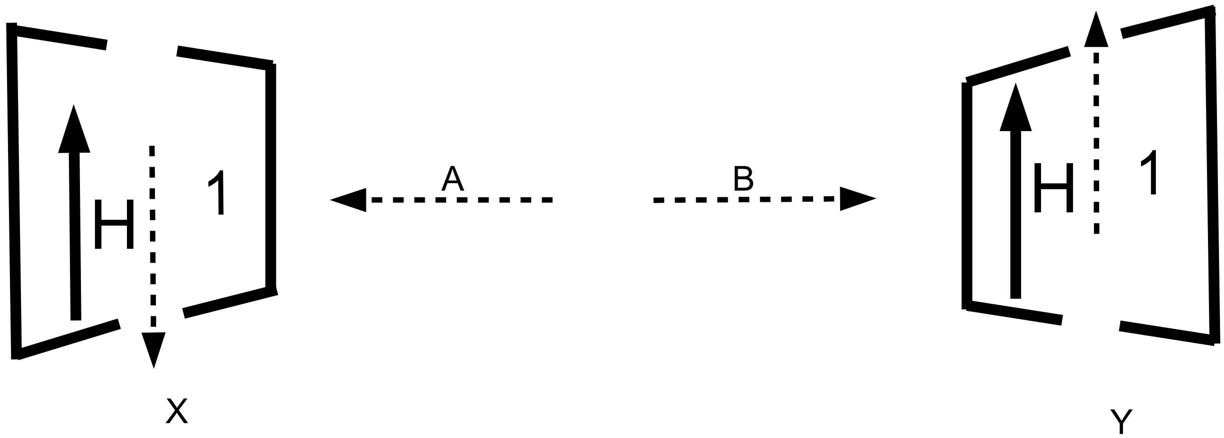
$$\frac{1}{\sqrt{2}}(|A\ 1{\uparrow}> |B\ 1{\downarrow}> -|A\ 1{\downarrow}> |B\ 1{\uparrow}>)$$

One sends two particles A and B, towards boxes located at X et Y, that are perpendicular to the plane of the picture. In each box there is magnetic field H oriented in the vertical direction, denoted 1.



$$\frac{1}{\sqrt{2}}(|A\ 1\!\uparrow> |B\ 1\!\downarrow> -|A\ 1\!\downarrow> |B\ 1\!\uparrow>)$$

One possibility is that particle A goes upwards, meaning in the direction of the gradient of the field and particle B goes downwards, meaning in the direction opposite to the one of the gradient the field.



Another possibility is that particle A goes downwards, meaning in the direction opposite to the one of the field and particle B goes upwards, meaning in the direction of the gradient of the field.

One **never** sees both particles going in the direction of the gradient of the field or in the opposite direction.

Now assume that there is no action at a distance of any sort, namely no influence of the measurement on one side on the result on the other side.

Then, in order to account for those perfect anti-correlations, we are obliged to assume that the results on both sides are predetermined by "instructions" (whether to up or down in a given direction) carried by the particles.

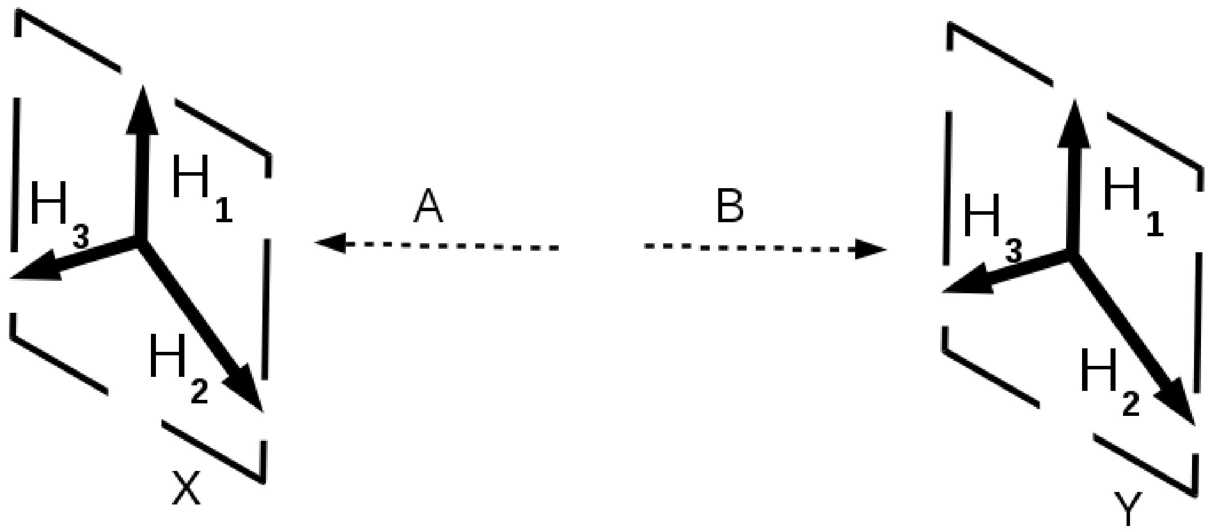
So, let introduce "random variables" $A(1) = \pm 1$, $B(1) = \pm 1$, where $A(1) = +1$ means that the A particle will go in the direction of the gradient of the field, and $A(1) = -1$ means that the A particle will go in the direction opposite to the one of the field, and similarly for $B(1) = \pm 1$.

These are "random variables" in the sense that those values vary from one run of the experiment to the next.

The "random variables" $A(1) = \pm 1$, $B(1) = \pm 1$ are "hidden variables" in the sense that they are not included or determined by the quantum state:

$$= \frac{1}{\sqrt{2}}(|A\ 1{\uparrow}> |B\ 1{\downarrow}> -|A\ 1{\downarrow}> |B\ 1{\uparrow}>)$$

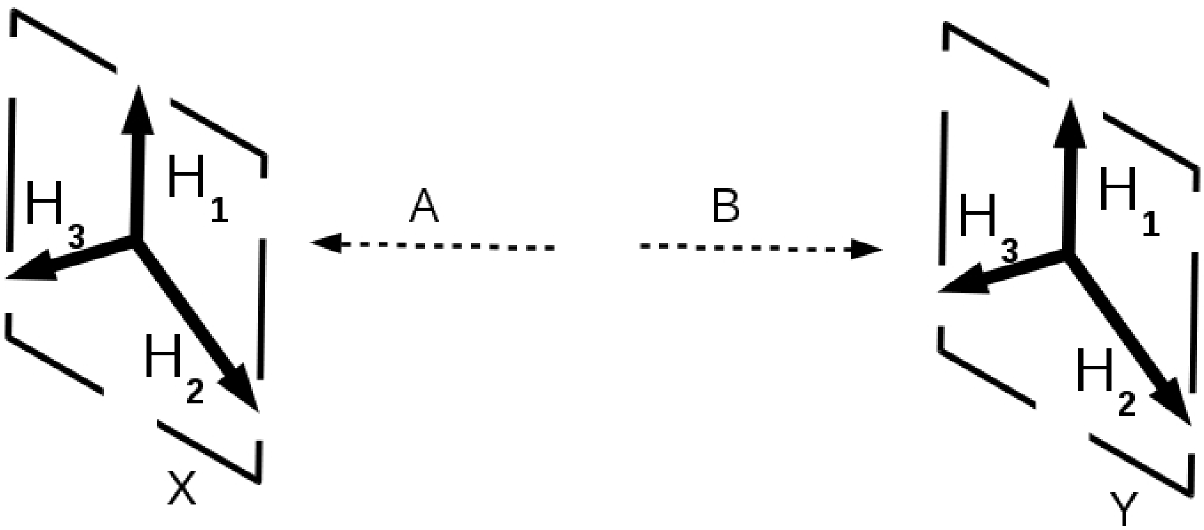
They are analogous to the index of the half-box in Einstein's boxes experiment.



Consider now three possible orientations for the gradient of the magnetic field, denoted H_1, H_2, H_3, in a plane perpendicular to the motion of the particles.

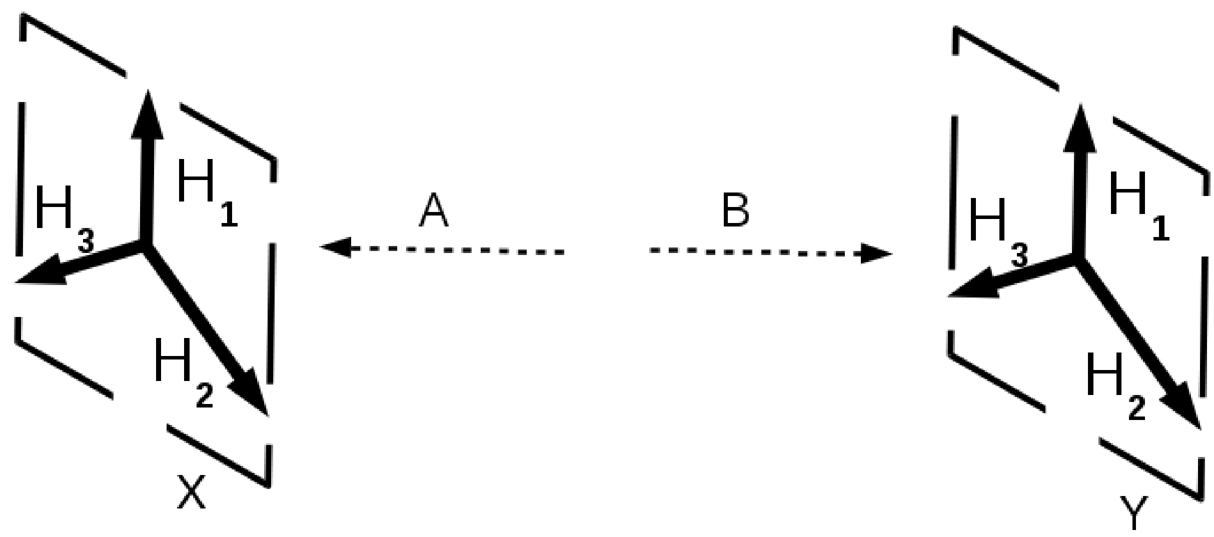
One repeats many times the experiment, by choosing "at random" the orientation of the gradient of the field on both sides.

When the orientations are the same on both sides, the two particles always go in opposite directions.

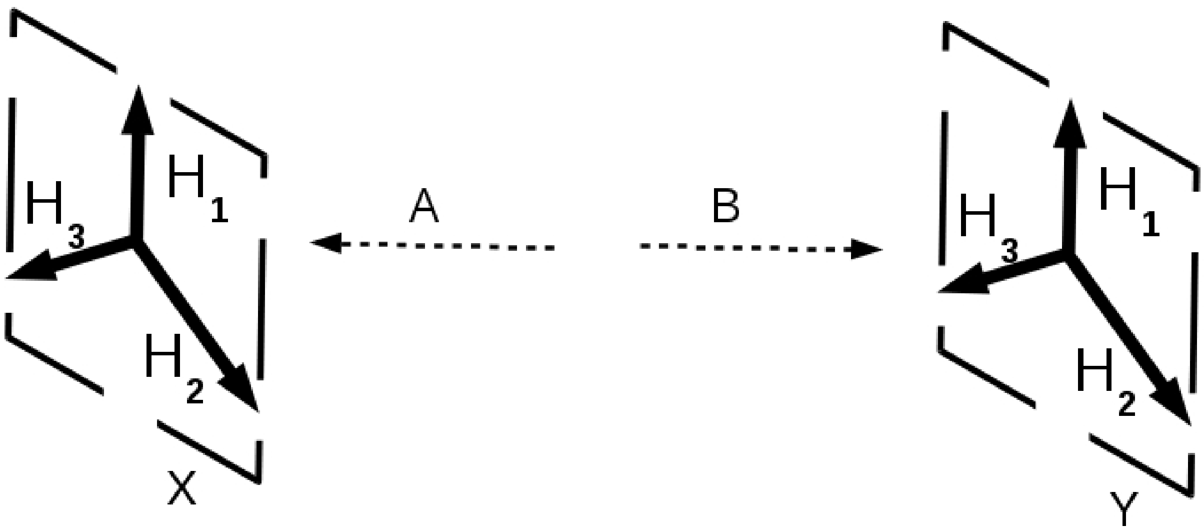


Indeed, the state considered here has the same form in all directions:

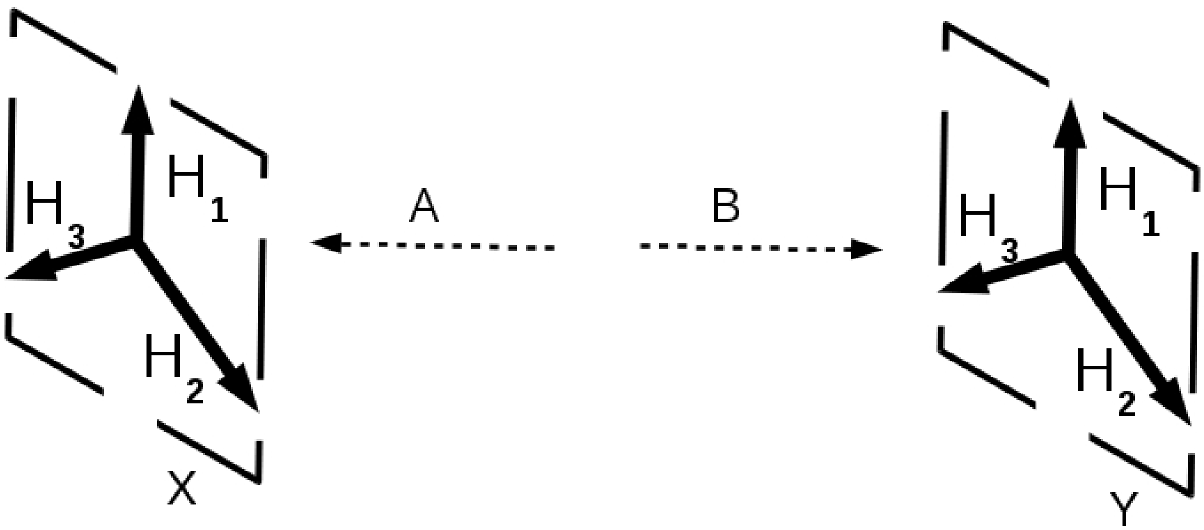
$$
\begin{aligned}
&|\ \text{state of the two particles} >\\
&= \frac{1}{\sqrt{2}}(|A\ 1\uparrow>\ |B\ 1\downarrow> -|A\ 1\downarrow>\ |B\ 1\uparrow>)\\
&= \frac{1}{\sqrt{2}}(|A\ 2\uparrow>\ |B\ 2\downarrow> -|A\ 2\downarrow>\ |B\ 2\uparrow>)\\
&= \frac{1}{\sqrt{2}}(|A\ 3\uparrow>\ |B\ 3\downarrow> -|A\ 3\downarrow>\ |B\ 3\uparrow>)
\end{aligned}
$$



The reasoning made above (as a consequence of simply assuming no action at a distance) implies that we are obliged to assume that the results on both sides are predetermined by "instructions" (whether to up or down in a given direction) carried by the particles, in all three directions.



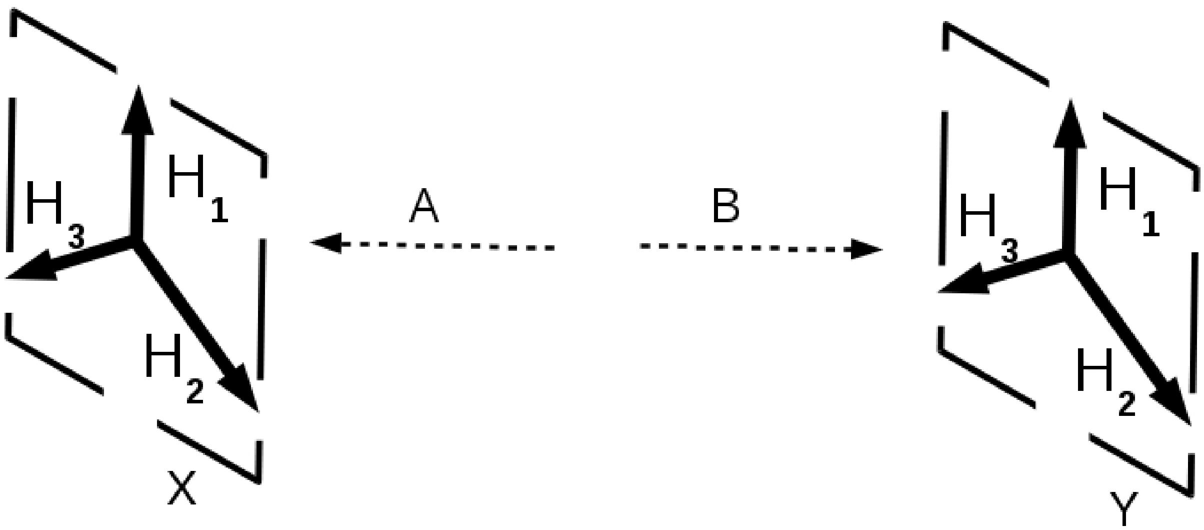
So, let introduce "random variables" $A(\alpha) = \pm 1$, $B(\alpha) = \pm 1$, for $\alpha = 1, 2, 3$ labelling the direction, and where $A(\alpha) = +1$ means that the A particle will go in the direction of the gradient of the field when the latter is oriented in direction α, and $A(\alpha) = -1$ means that the A particle will go in the direction opposite to the one of the field, and similarly for $B(\alpha) = \pm 1$.



But, in order to account for the perfect anti-correlations, we must always have:

$$A(\alpha) = -B(\alpha)$$

$\forall \alpha = 1, 2, 3.$



Now, since $A(\alpha)$ takes only two values and since there are three choices of directions (1, 2, 3), whatever the values of the random variables $A(\alpha)$, we must, for each set of values, have either

$$A(1) = A(2)$$

$$A(1) = A(3)$$

$$A(2) = A(3)$$

(or all three could be equal).

So, by simply assuming that those values exist, we must have:

Frequency (A(1)= A(2))

+ Frequency (A(1)= A(3))

+ Frequency (A(2)= A(3)) $\geq$ 1.

But, since we have

$$A(\alpha) = -B(\alpha)$$

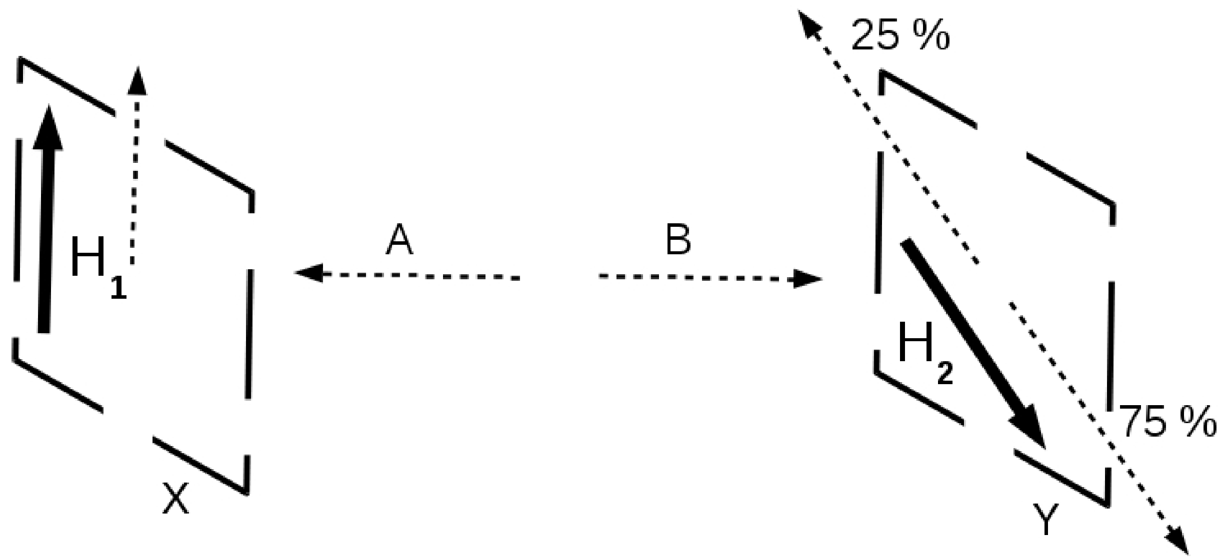
$\forall \alpha = 1, 2, 3.$

we must have

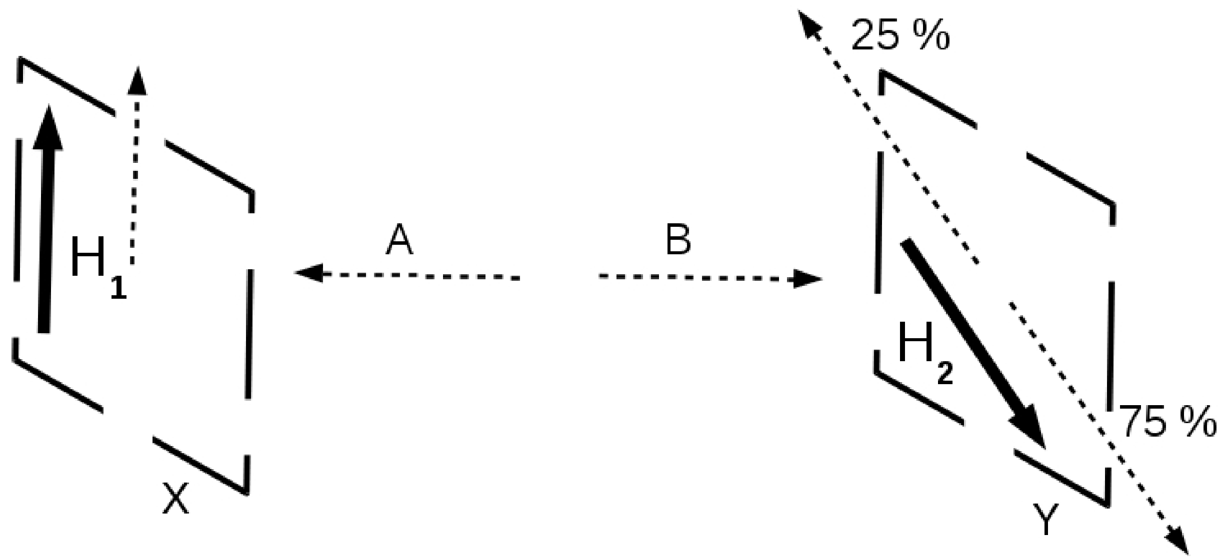
Frequency (A(1)= -B(2))

+ Frequency (A(1)= -B(3))

+ Frequency (A(2)= -B(3)) $\geq$ 1.

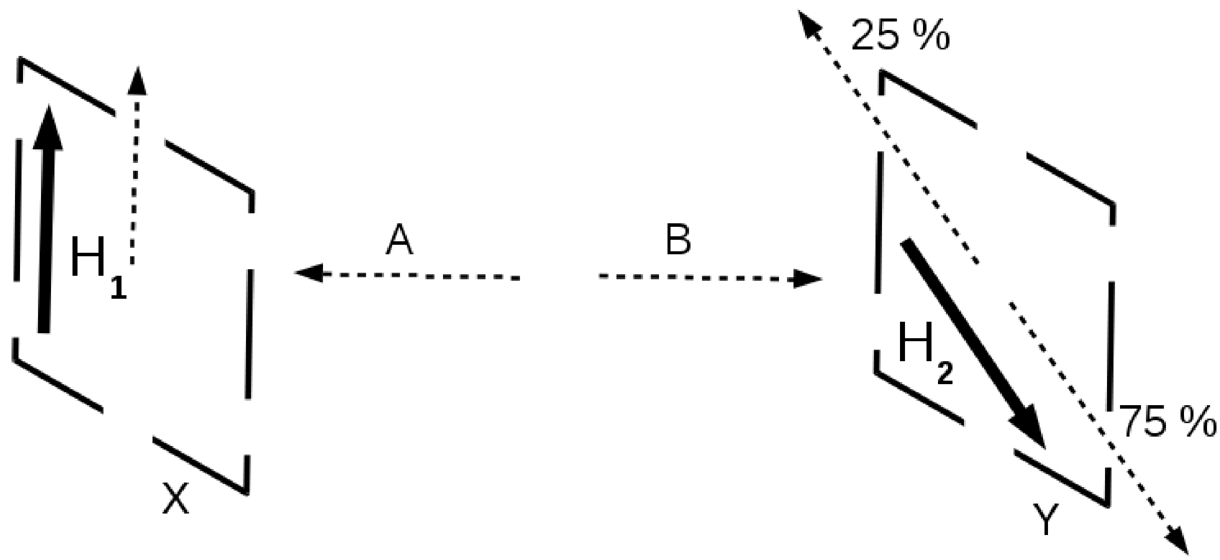


Let us choose, for example, direction 1 at X and direction 2 at Y.



If particle A goes in the direction of the gradient of the field (meaning $A(1) = +1$), as in the picture, then particle B will go in the direction of the gradient of the field (meaning $B(2) = +1$) 75% of the time and in the opposite direction (meaning $B(2) = -1$) 25% of the time (and vice-versa).

One obtains the same results with the 5 other choices of pairs of different orientations of the gradient of the field at X and Y.



But that means that $A(1) = -B(2)$ only a quarter of the time, i.e.

Frequency (A(1)= -B(2))

= Frequency (A(1)= -B(3))

= Frequency (A(2)= -B(3)) $=\frac{1}{4}$.

But then:

Frequency (A(1)= -B(2))

+ Frequency (A(1)= -B(3))

+ Frequency (A(2)= -B(3))

$=\frac{3}{4} < 1$

This contradicts

Frequency (A(1)= -B(2))

+ Frequency (A(1)= -B(3))

+ Frequency (A(2)= -B(3))

≥ 1,

which followed from only assuming that those values $A(\alpha)$, $B(\alpha)$ exist.

And that assumption followed from the one of locality, i.e. no action at a distance of any sort, namely no influence of the measurement on one side on the result on the other side.

Therefore, that latter assumption is false.

Ergo: the world in non-local.

The number $\frac{1}{4}$ mentioned above, for the anti-correlations with an appropriate choice of the directions $1, 2, 3$, is derived in the Appendix.

Let us see how this experiment is described in the quantum formalism:

$$
\begin{aligned}
&|\text{state of both particles} > \\
&= \frac{1}{\sqrt{2}}(|A\ 1{\uparrow}>\ |B\ 1{\downarrow}> - |A\ 1{\downarrow}>\ |B\ 1{\uparrow}>) \\
&= \frac{1}{\sqrt{2}}(|A\ 2{\uparrow}>\ |B\ 2{\downarrow}> - |A\ 2{\downarrow}>\ |B\ 2{\uparrow}>) \\
&= \frac{1}{\sqrt{2}}(|A\ 3{\uparrow}>\ |B\ 3{\downarrow}> - |A\ 3{\downarrow}>\ |B\ 3{\uparrow}>)
\end{aligned}
$$

If one "measures" the spin in direction 1 for the A particle and if one sees $\uparrow$, the state becomes $\Rightarrow |A\ 1{\uparrow}>\ |B\ 1{\downarrow}>$.

If one sees $\downarrow$, the state becomes $\Rightarrow |A\ 1{\downarrow}>\ |B\ 1{\uparrow}>$.

The same holds if one measures the spin in directions 2 or 3; collapse of the quantum state!

But then, the state has changed also *non locally* for the B particle.

Same dilemma as for Einstein's boxes :

reduction of the $|$ state $>$ = physical or epistemic ?

<u>If</u> physical $\longrightarrow$ non locality

<u>If</u> epistemic $\longrightarrow$ "answers" are given in advance, i.e. the particle at B *is* $1\uparrow$ or $1\downarrow$, $2\uparrow$ or $2\downarrow$, $3\uparrow$ or $3\downarrow$, *before* any measurement at A.

The only way to maintain that this collapse is not physical is to assume what we just said:

That there exist "random variables" $A(\alpha) = \pm 1$, $B(\alpha) = \pm 1$, on top of the quantum state that determine which way the particle will go if one measures its spin ($A(\alpha) = +1$ means that the A particle will go in the direction of the gradient of the field, and $A(\alpha) = -1$ means that the A particle will go in the direction opposite to the one of the gradient of the field, and similarly for $B(\alpha) = \pm 1$).

Then, it would make sense to say that the quantum state is only about “information”, and that the collapse of that state occurs only because we “learn” something about the system.

BUT, what Bell shows it that the mere supposition that those variables exist leads to a contradiction!

BELL WAS QUITE EXPLICIT ABOUT WHAT THIS MEANS

"Let me summarize once again the logic that leads to the impasse. The EPRB correlations are such that the result of the experiment on one side immediately foretells that on the other, whenever the analyzers happen to be parallel."

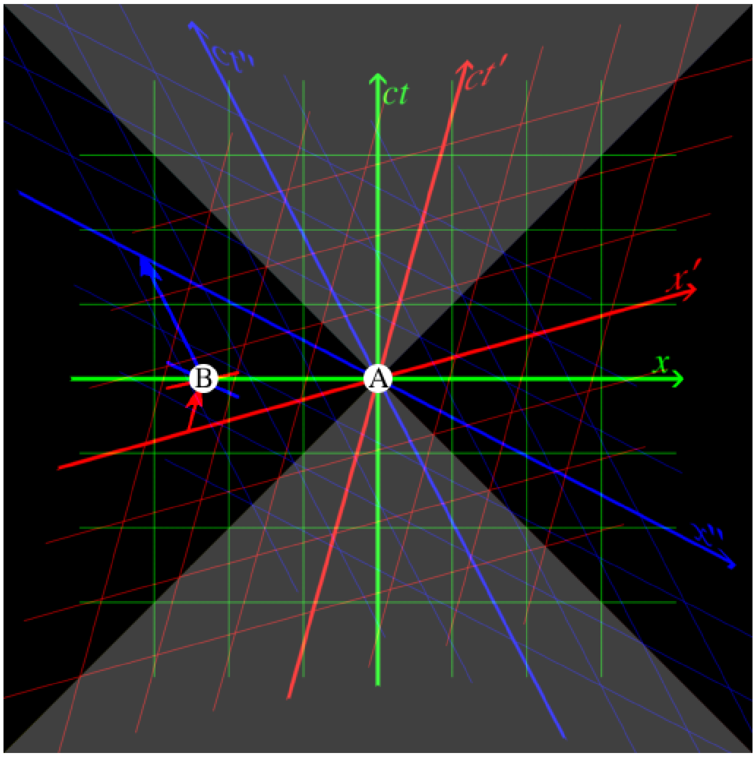
(In EPRB, B refers to Bohm who reformulated the EPR argument in terms in spin, which we use here. EPR spoke of position and momentum.)

"If we do not accept the intervention on one side as a causal influence on the other, we seem obliged to admit that the results on both sides are determined in advance anyway, independently of the intervention on the other side, by signals from the source and by the local magnet setting. But this has implications for non-parallel settings which conflict with those of quantum mechanics. So we *cannot* dismiss intervention on one side as a causal influence on the other."

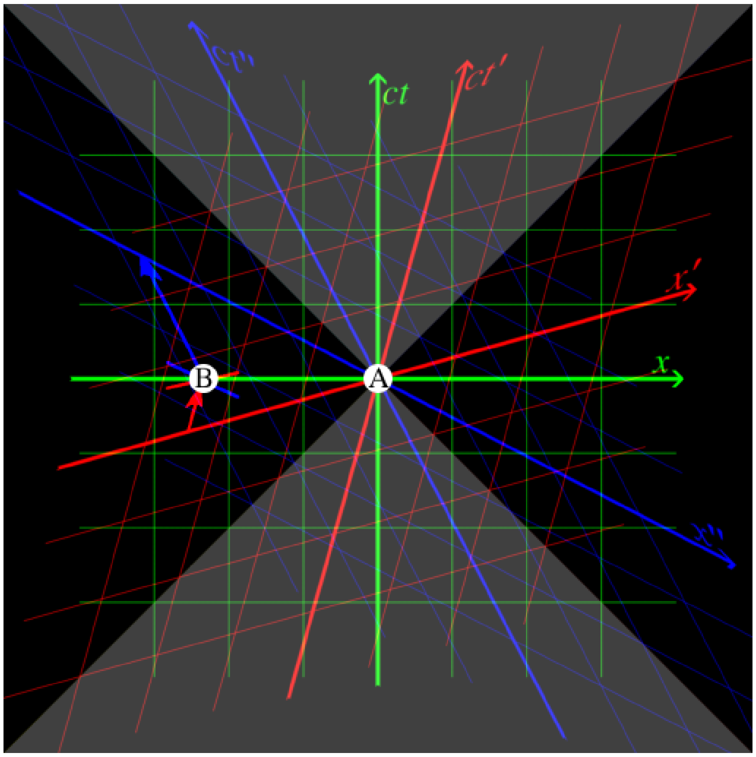
J. BELL

THE TROUBLE WITH RELATIVITY

COMING FROM THE RELATIVITY OF SIMULTANEITY



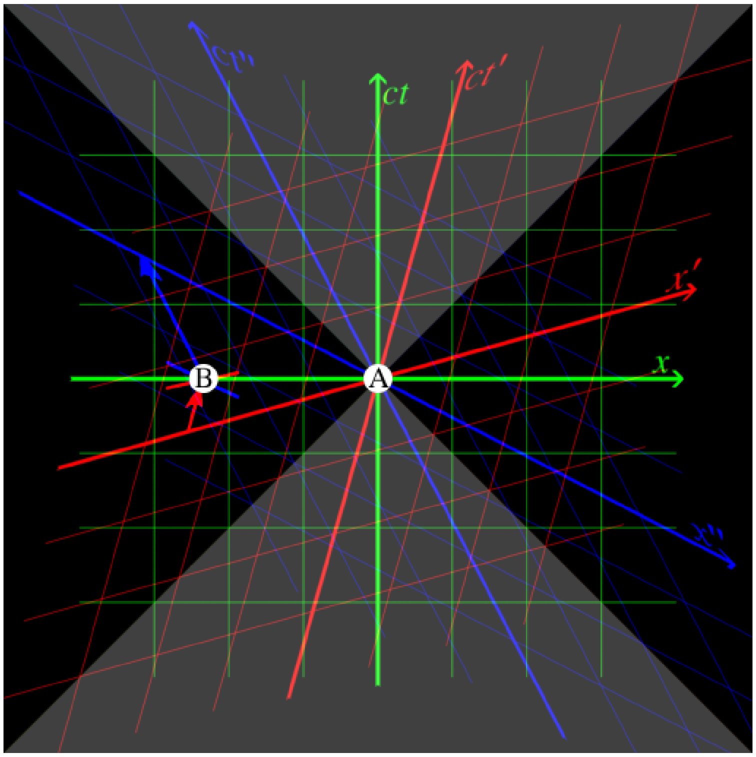
Consider three frames of reference: the green, bue and red lines indicate events that are simultaneous with respect to each of these reference frames



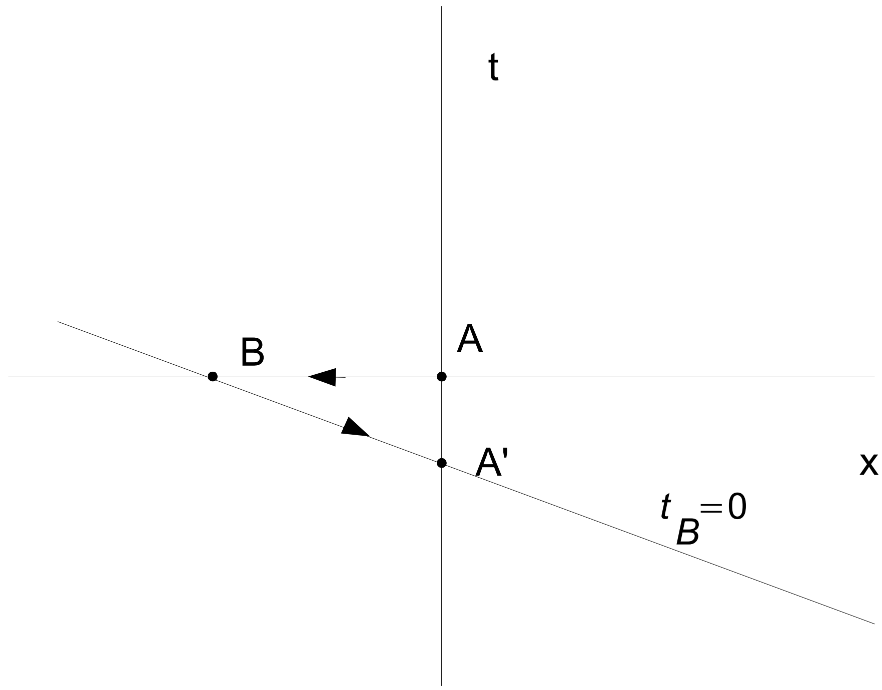
The x axis corresponds to all the events simultaneous with A relative to the green reference frame.

The x' axis corresponds to all the events simultaneous with A relative to the red reference frame.

The x" axis corresponds to all the events simultaneous with A relative to the blue reference frame.

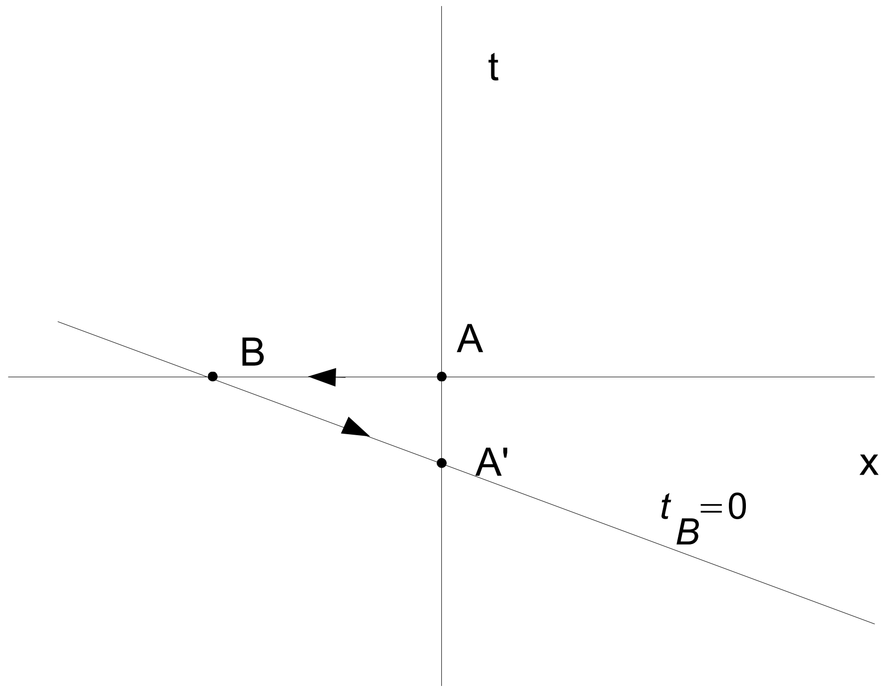


Event B is simultaneous with A relative to the green reference frame but occurs **before** A relative to the blue reference frame and occurs **after** A relative to the red reference frame



The x axis represents the $t = 0$ axis in a reference frame where A is at rest. Suppose that one can send a message instantaneously from A to B (B is in the present of A).

But if in B one considers a reference frame in motion with respect to the one where A is at rest, then the present in that reference frame could be represented by the line $t_B = 0$.



If one can send a message instantaneously from A to B, then B can send a message instantaneously to A', which is the past of A.

That would of course create "causal loops".

What happens in the quantum formalism:

$|\text{state of both particles} >$
$= \frac{1}{\sqrt{2}}(|A\ 1{\uparrow}>\ |B\ 1{\downarrow}> - |A\ 1{\downarrow}>\ |B\ 1{\uparrow}>)$

If one measures the spin in direction 1 at X, before measuring it at Y, and if one sees $\uparrow$, the state becomes $\Rightarrow |A\ 1{\uparrow}>\ |B\ 1{\downarrow}>$.

If one sees $\downarrow$, the state becomes $\Rightarrow |A\ 1{\downarrow}>\ |B\ 1{\uparrow}>$.

One then changes *instantaneously* the state of B.

But if one measured the spin in direction 1 at Y, before measuring it at X, one would change *instantaneously* the state of A.

But who measures first depends on the reference frame !!!

The only solution would be to have an "epistemic" view of the quantum state so that there will be no real action at a distance and the measurements would simply reveal pre-existing values of the spin.

However, Bell showed that this "solution" implies a contradiction ($\frac{3}{4} \geq 1$).

But if there are instantaneous actions, then relativity implies the existence of actions on the past in certain reference frames.

All our intuitive notion of causality collapses, because this notion is based on the idea that causes precede effects in an absolute sense that does not depend on the reference frame.

Unless one introduces a privileged reference frame in which "true" causality holds.

The least one can say is that this contradicts the spirit of relativity!!

What about QFT or relativistic quantum mechanics ?

In standard textbooks, the reduction or collapse of the quantum state is never discussed in relativistic terms $\longrightarrow$ the question raised by EPR and Bell is not even raised.

Luckily, one cannot use EPR-Bell to send messages

If one could, then, as we just saw, relativity implies that one could send messages into one's own past.

BUT:

— Each side sees a perfectly random sequence of YES/NO.

— There is no way to control, by acting on one side, which answer will be received.

— So, one cannot use this mechanism to send messages.

— BUT if each person tells the other which "measurements" have been made (1, 2 or 3), then, they both know which result has been obtained on the other side when the same measurement is made on both sides.

$\Rightarrow$ Then, they both share a common sequence of YES/NO , which is form of "information". Since that information cannot possibly come from the source (because of Bell), some sort of nonlocal transmission of information has taken place.

— That is the basis of quantum cryptography and quantum information.

But the problem of causality remains.

It cannot be solved by just saying "one cannot send messages".

Messages are far too anthropocentric.

If one chooses a privileged reference frame in which true causality holds, then, the argument showing that one cannot send messages also implies that this reference frame is unobservable.

CHOOSE YOUR POISON!

NONLOCALITY IN THE DE BROGLIE–BOHM THEORY

In the de Broglie-Bohm's theory, the state of system is a pair (X, Ψ), where $X = (X_1, \ldots, X_N)$ denotes the actual positions of all the particles in the system under consideration, and

$$\Psi = \Psi(x_1, \ldots, x_N)$$

is the usual quantum state, $(x_1, \ldots, x_N)$ denoting the arguments of the function Ψ. X are the hidden variables in this theory; this is obviously a misnomer, since particle positions are the only things that we ever directly observe (think of the double-slit experiment for example).

So, from the point of view of the de Broglie–Bohm theory, quantum mechanics **is incomplete** : the complete state includes other variables, namely the positions of the particles.

Of course, those variables specify in which half-box the particle **is** before one opens either of them in Einstein's boxes experiment. So, there is no paradox with the boxes from the point of view of the de Broglie–Bohm theory!

In the dynamics of the de Broglie-Bohm's theory, both objects, Ψ and X, evolve in time:

1. SCHRÖDINGER'S EQUATION : for the quantum state, at all times, and whether one measures something or not

$$\Psi_0 \to \Psi_t = U(t)\Psi_0$$

$$i\hbar\partial_t\Psi(x_1,\ldots,x_N,t) = (H\Psi)(x_1,\ldots,x_N)$$

where H is the Hamiltonian: $H = -\frac{1}{2}\Delta + V$, and V is the potential.

THE QUANTUM STATE NEVER COLLAPSES.

2. GUIDING EQUATION The evolution of the positions is guided by the quantum state: writing $\Psi = Re^{iS}$

$$\frac{d}{dt}X_k(t) = \frac{\hbar}{m_k}\nabla_k S(X_1(t), \ldots, X_N(t))$$

for $k = 1, \ldots, N$, where $X_1, \ldots, X_N$ are the actual positions of the particles.

Double slit experiment : numerical solution in the de Broglie-Bohm theory.

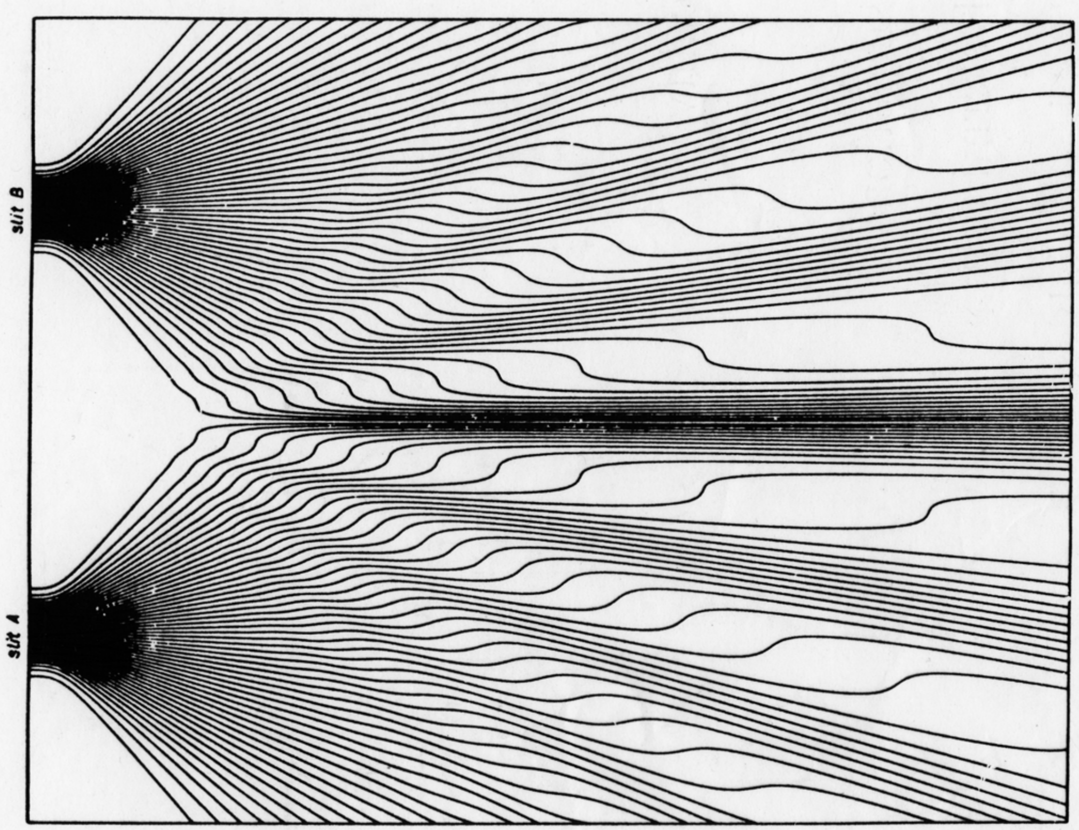


Figure 9.3 Trajectories for two Gaussian slits with a Gaussian distribution of initial positions at the slits.

Motion *in vacuum* highly *non classical*!! Note that one can determine a posteriori through which hole that particle went !

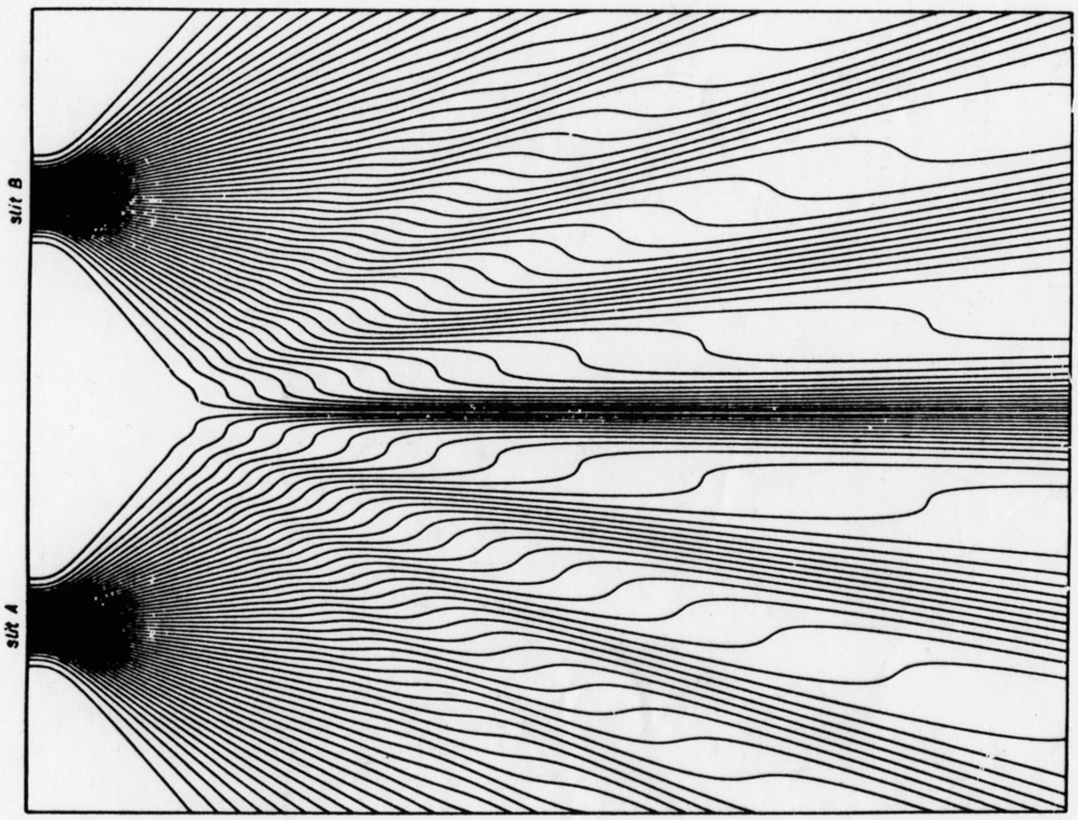


Figure 9.3 Trajectories for two Gaussian slits with a Gaussian distribution of initial positions at the slits.

Note also the presence of a nodal line: by symmetry of Ψ, the velocity is tangent to the middle line; thus, particles cannot cross it.

It is clear that [the results of the double-slit experiment] can in no way be reconciled with the idea that electrons move in paths. [. . .] In quantum mechanics there is no such concept as the path of a particle.

LANDAU AND LIFSHITZ

JOHN BELL:

It is not clear from the smallness of the scintillation on the screen that we have to do with a particle? And is it not clear, from the diffraction and interference patterns, that the motion of the particle is directed by a wave? De Broglie showed in detail how the motion of a particle, passing through just one of two holes in the screen, could be influenced by waves propagating through both holes.

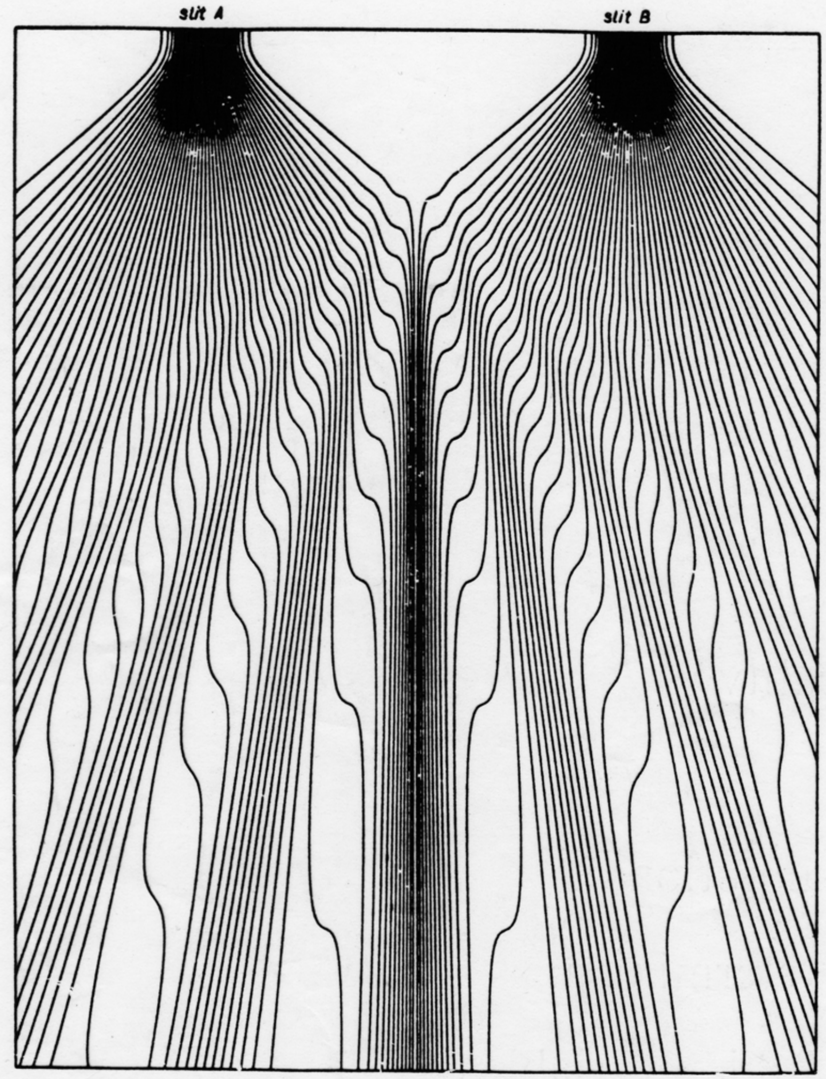


Figure 9.3 Trajectories for two Gaussian slits with a Gaussian distribution of initial positions at the slits.

And so influenced that the particle does not go where the waves cancel out, but is attracted to where they cooperate. This idea seems to me so natural and simple, to resolve the wave-particle dilemma in such a clear and ordinary way, that it is a great mystery to me that it was so generally ignored.

J. BELL

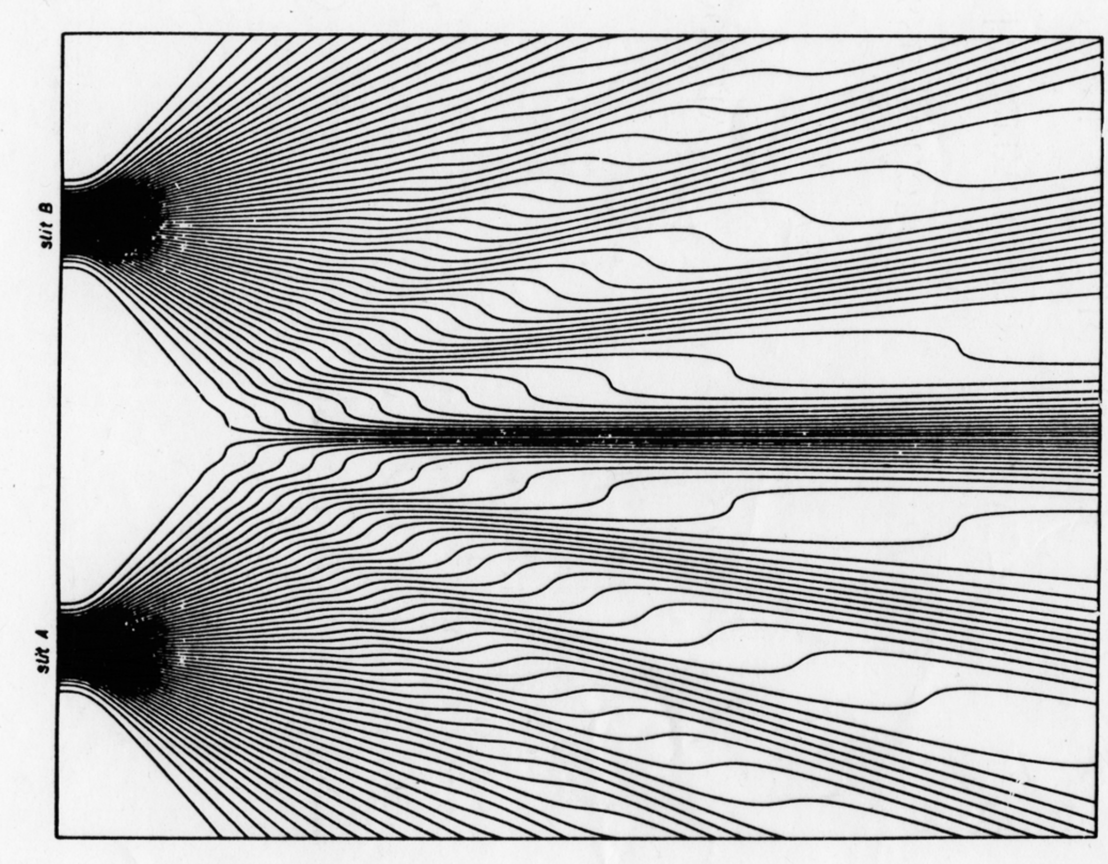
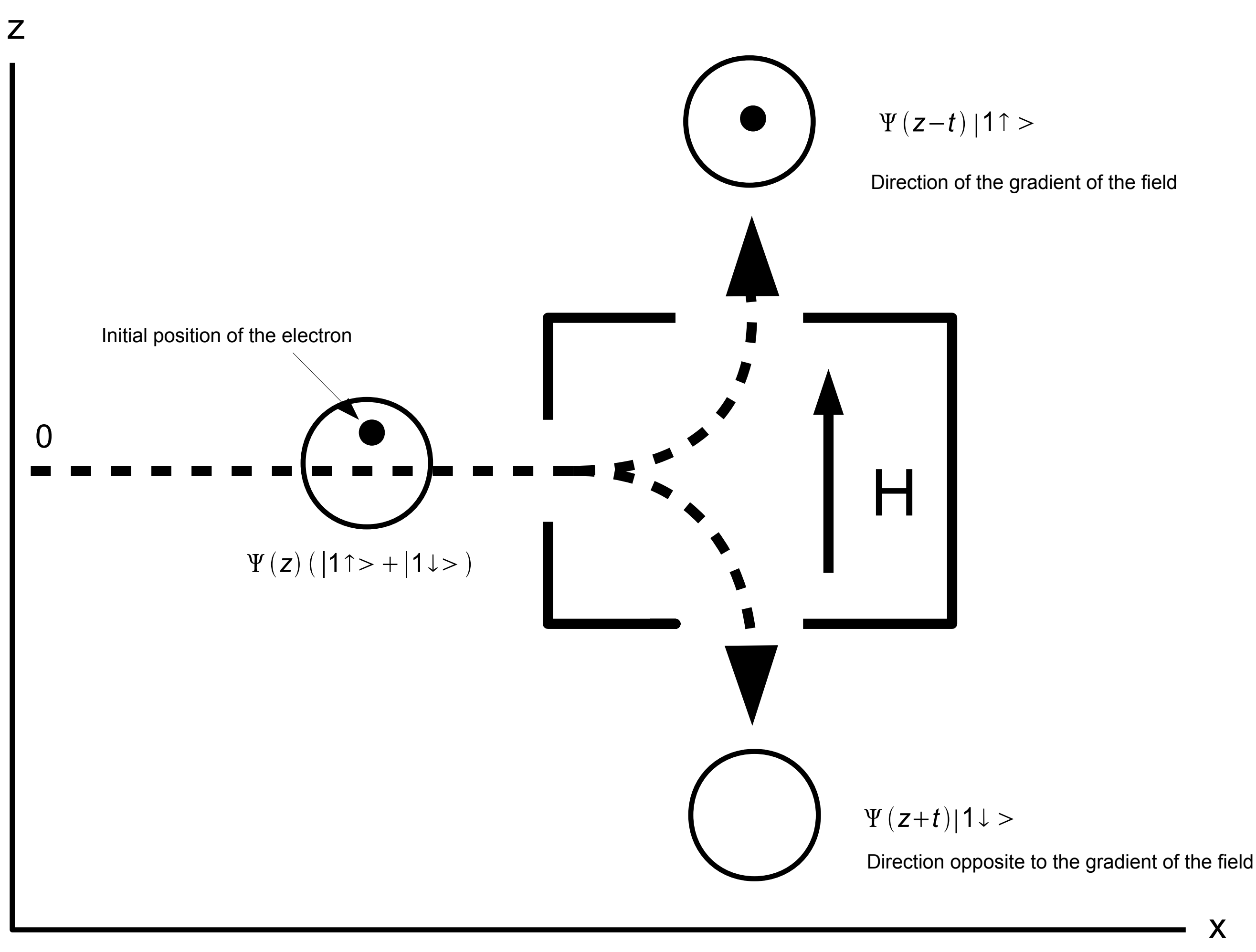


Figure 9.3 Trajectories for two Gaussian slits with a Gaussian distribution of initial positions at the slits.

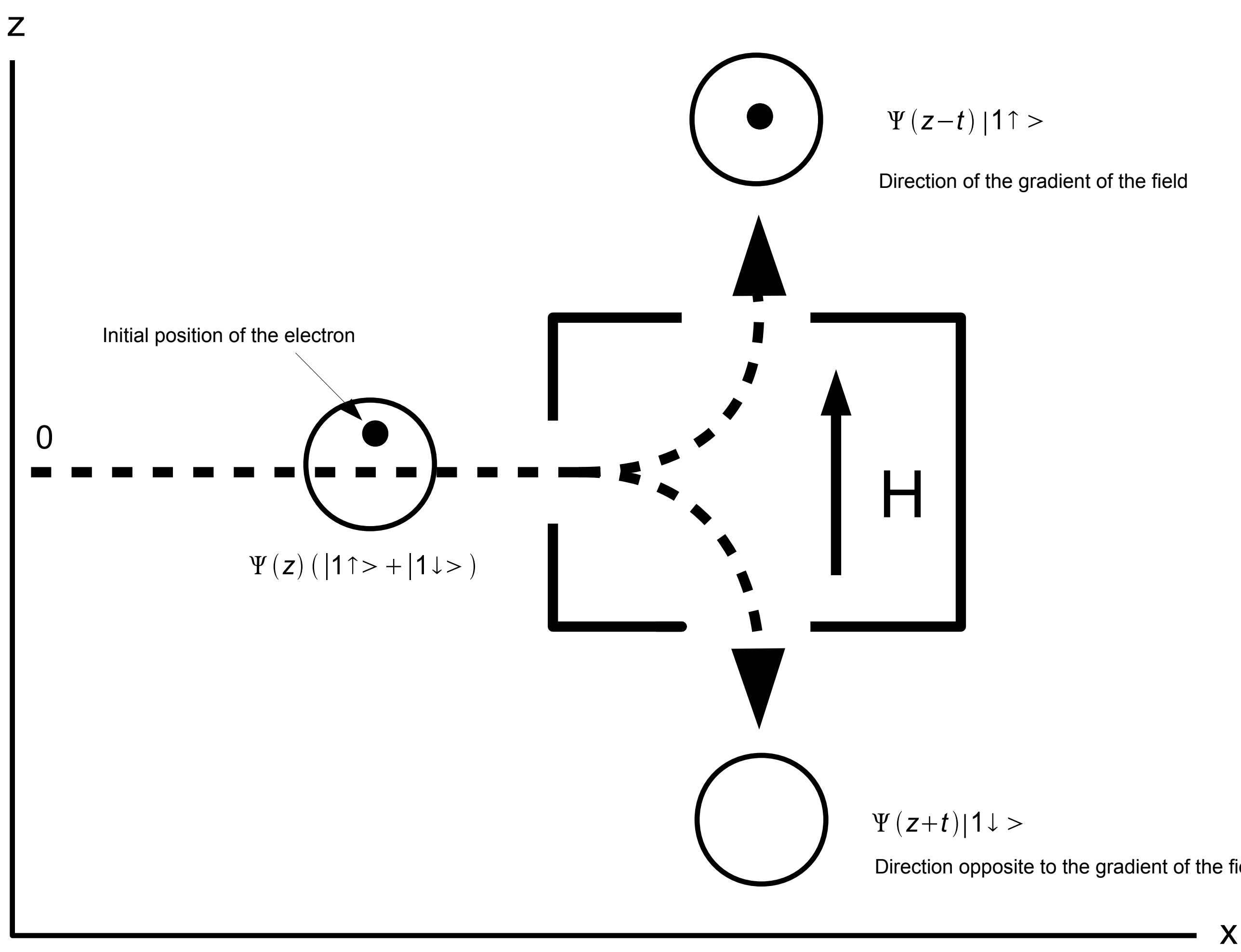
In the de Broglie–Bohm theory, both objects, the quantum state and the particles' positions, evolve according to deterministic laws, the quantum state guiding the motion of the particles.

Thus, since the de Broglie–Bohm theory is deterministic, the result of any quantum measurement will be determined beforehand by the quantum state AND by the configuration of the "measuring device".

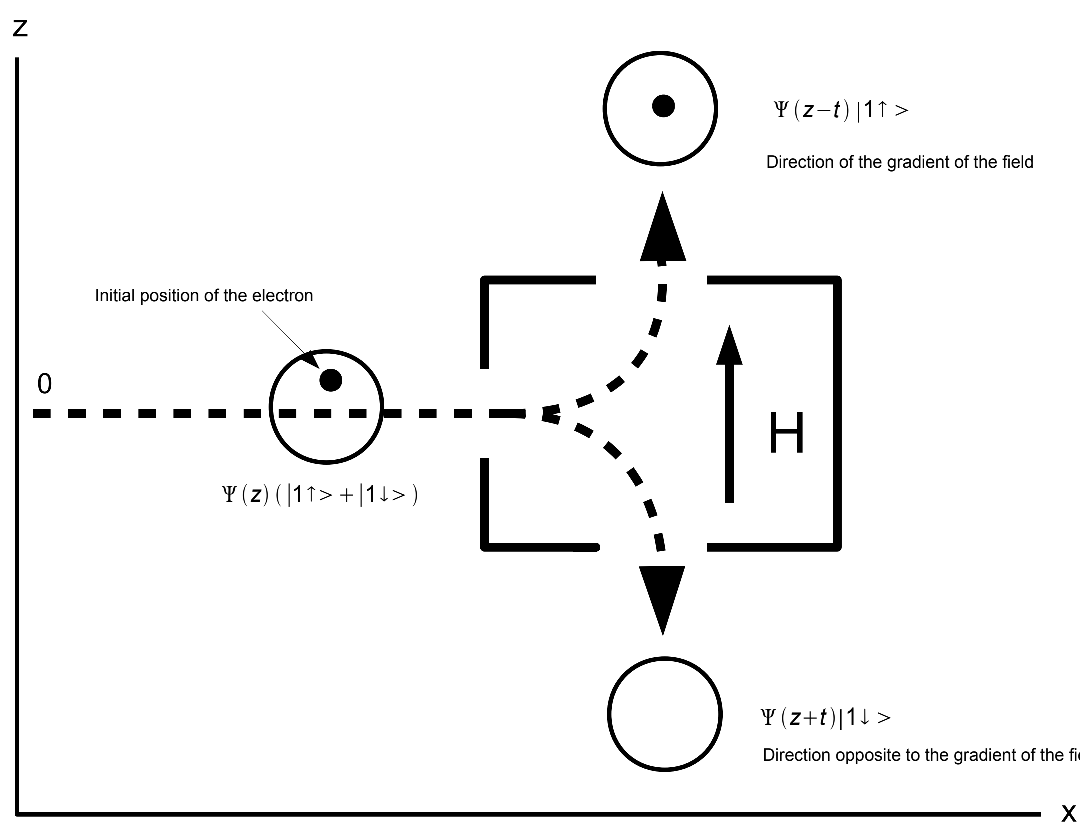
Consider a Stern-Gerlach apparatus "measuring" the spin. Let H be the magnetic field. The arrow indicates the direction of the gradient of the field.



The $|1\uparrow>$ part of the state always goes in the direction of the gradient of the field, and the $|1\downarrow>$ part always goes in the opposite direction.



But if the particle is initially in the upper part of the support of the wave function (for a symmetric wave function), it will always go upwards. That is because there is a nodal line in the middle of the figure that the particles cannot cross.



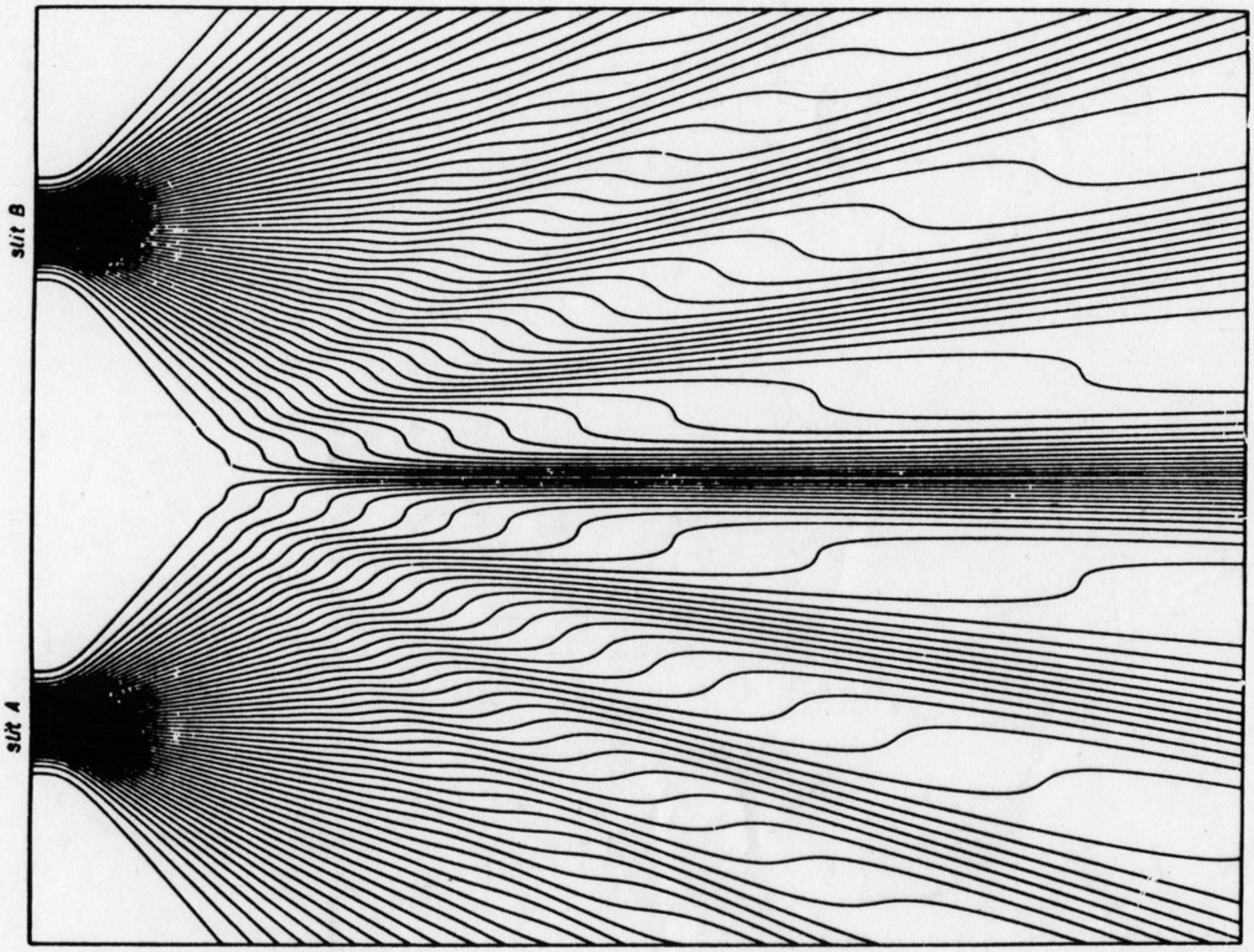
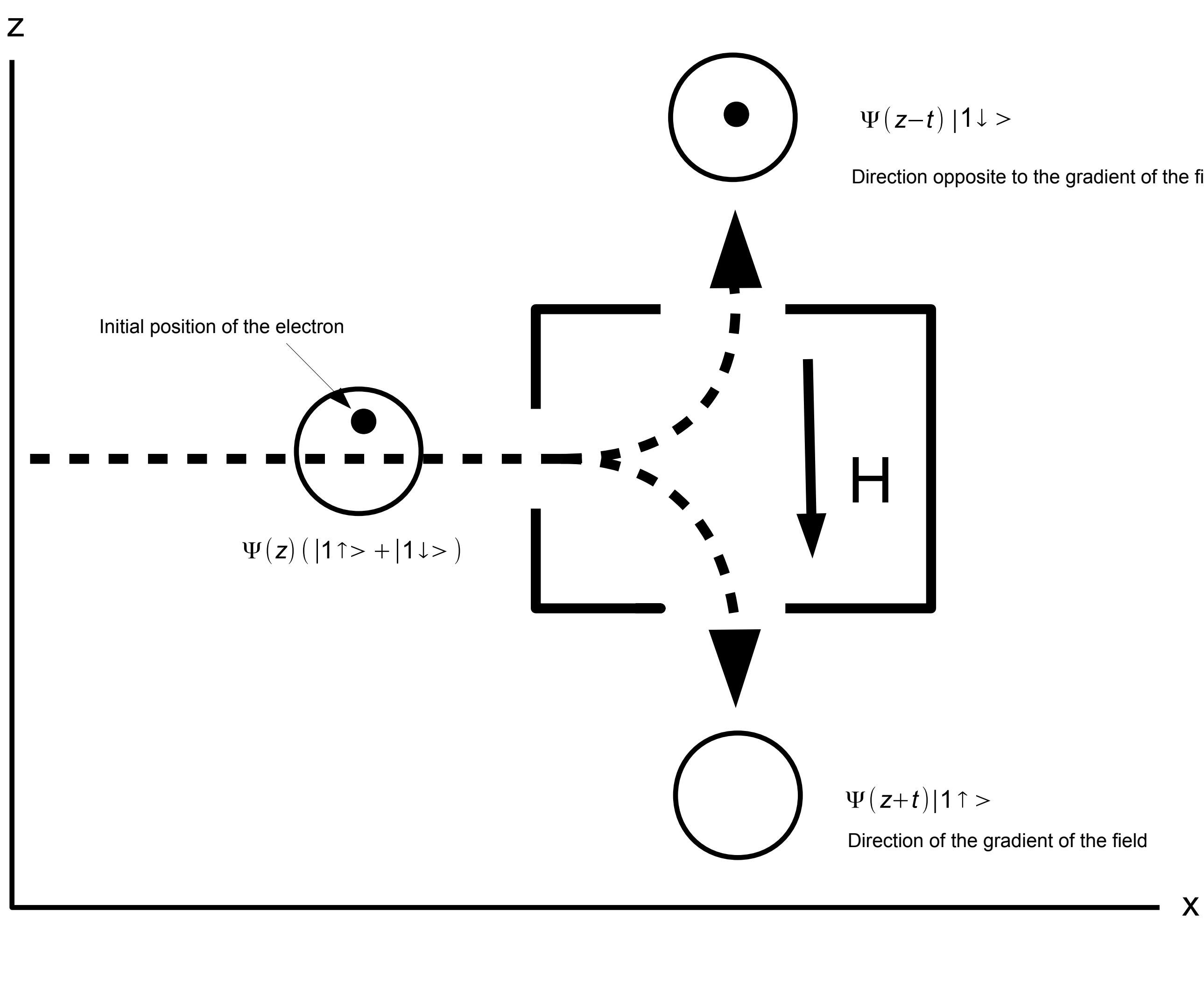


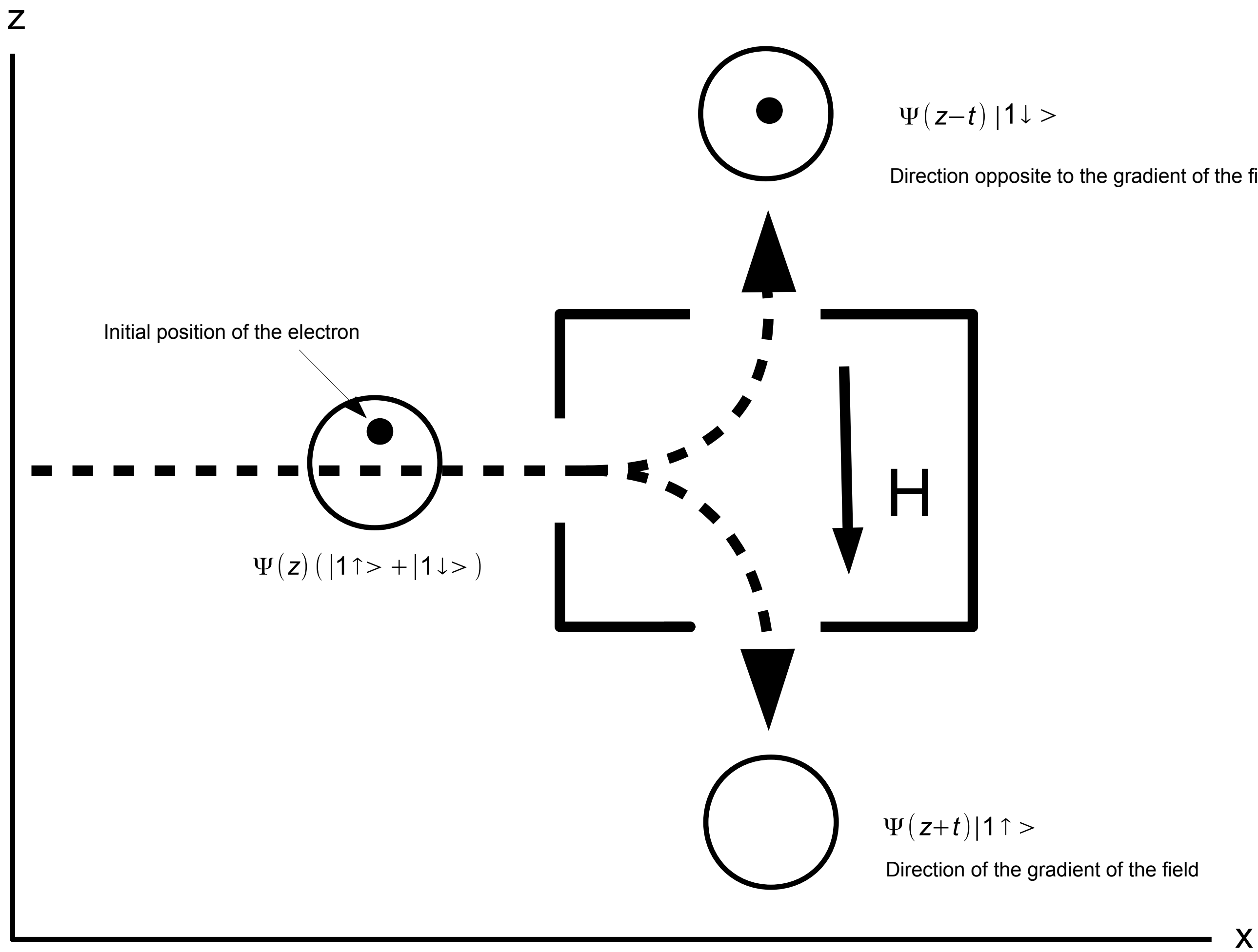
Figure 9.3 Trajectories for two Gaussian slits with a Gaussian distribution of initial positions at the slits.

as here

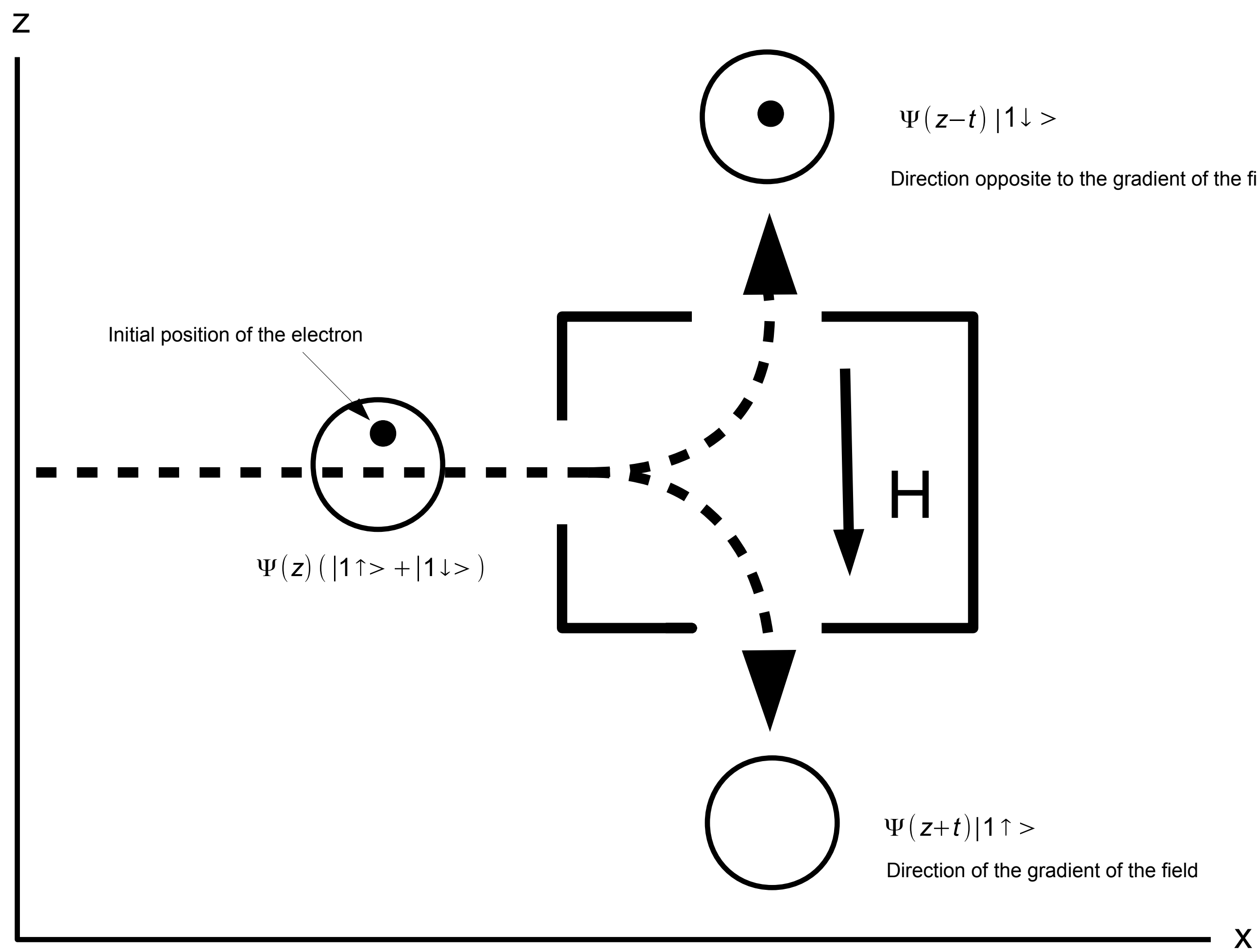
Now, repeat the same experiment, but with the direction of the gradient of the field reversed, and let us assume that the particle starts with *exactly the same wave function and the same position as before.*



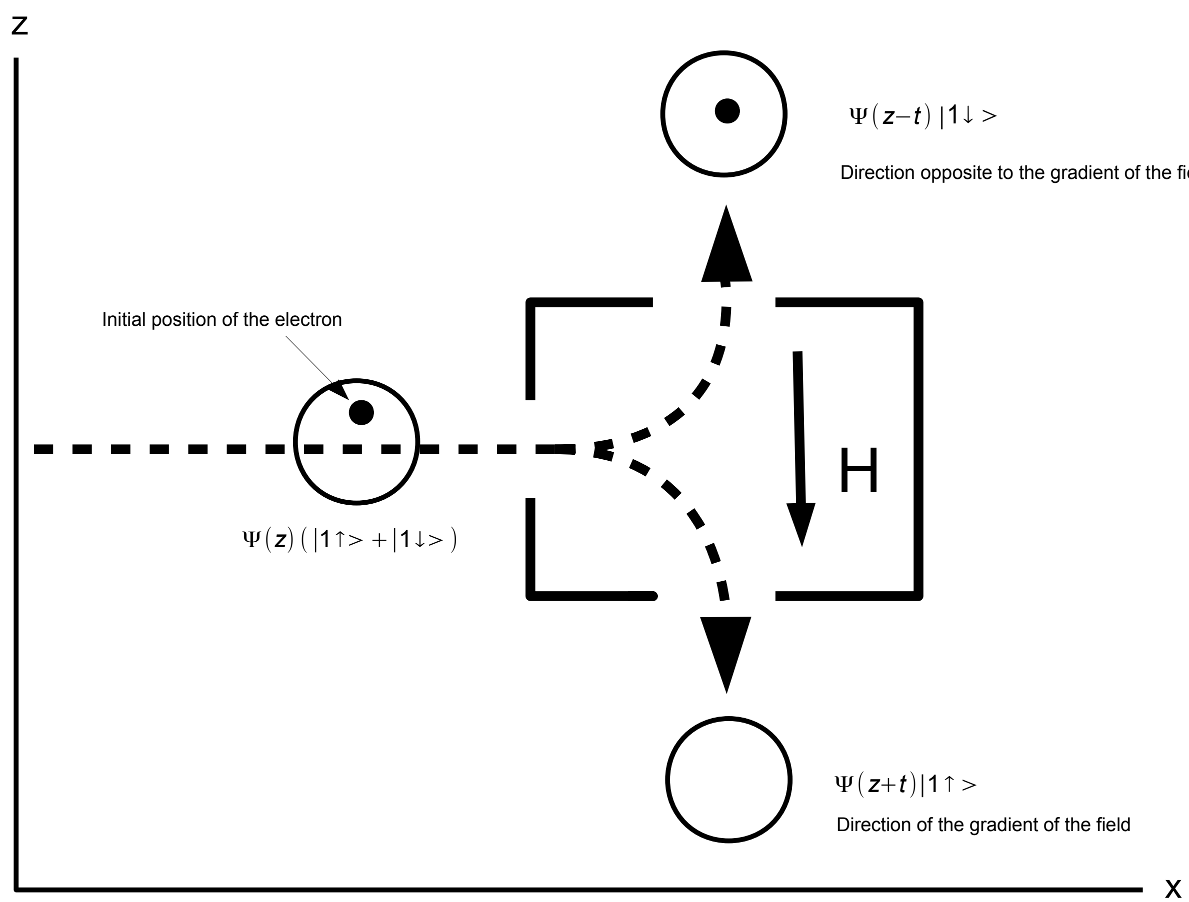
The particle is initially in the upper part of the support of the wave function, and, thus, it will still go upwards, because of the nodal line.



But going upwards means now going in the direction *opposite* to the one of the gradient of the field (since the latter is reversed).



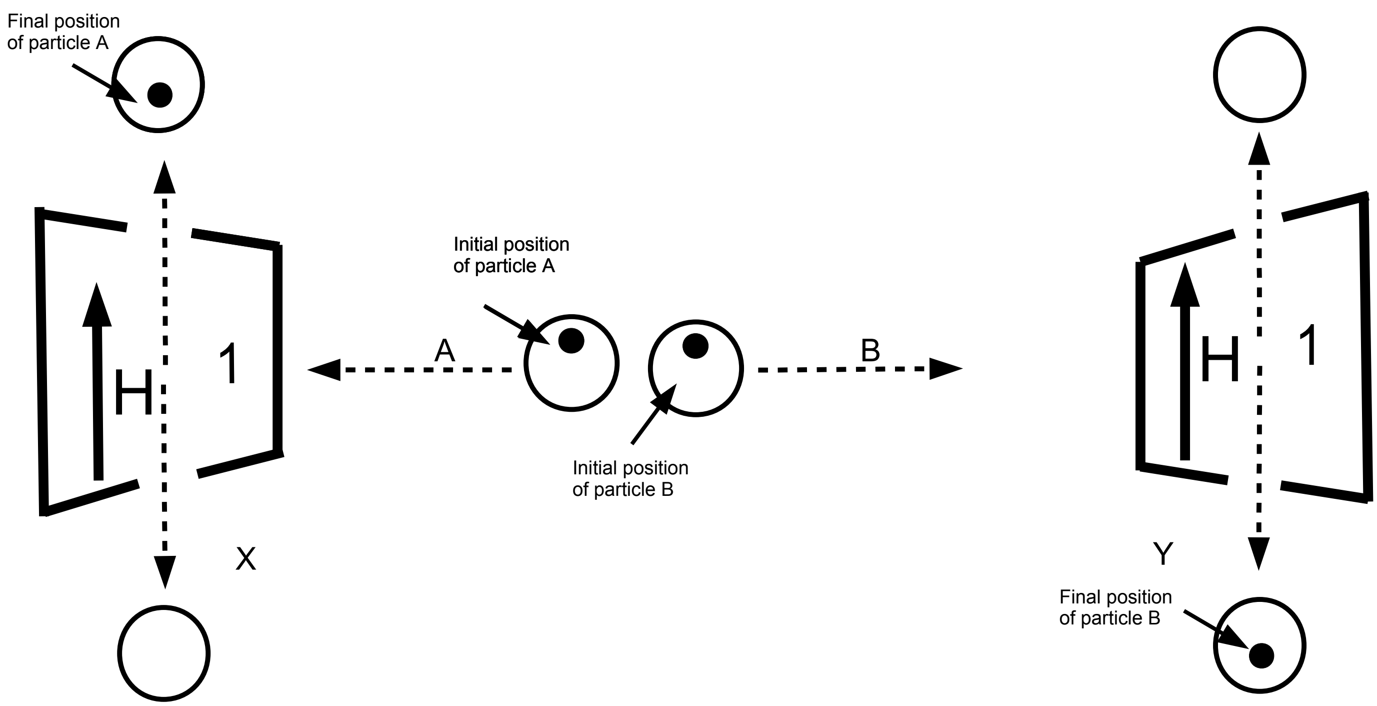
So, the particle whose spin was “up”, will “have” its spin “down”, although one “measures” exactly the same observable (the spin in the vertical direction), with *exactly the same initial conditions (for both the wave function and the position of the particle)*.



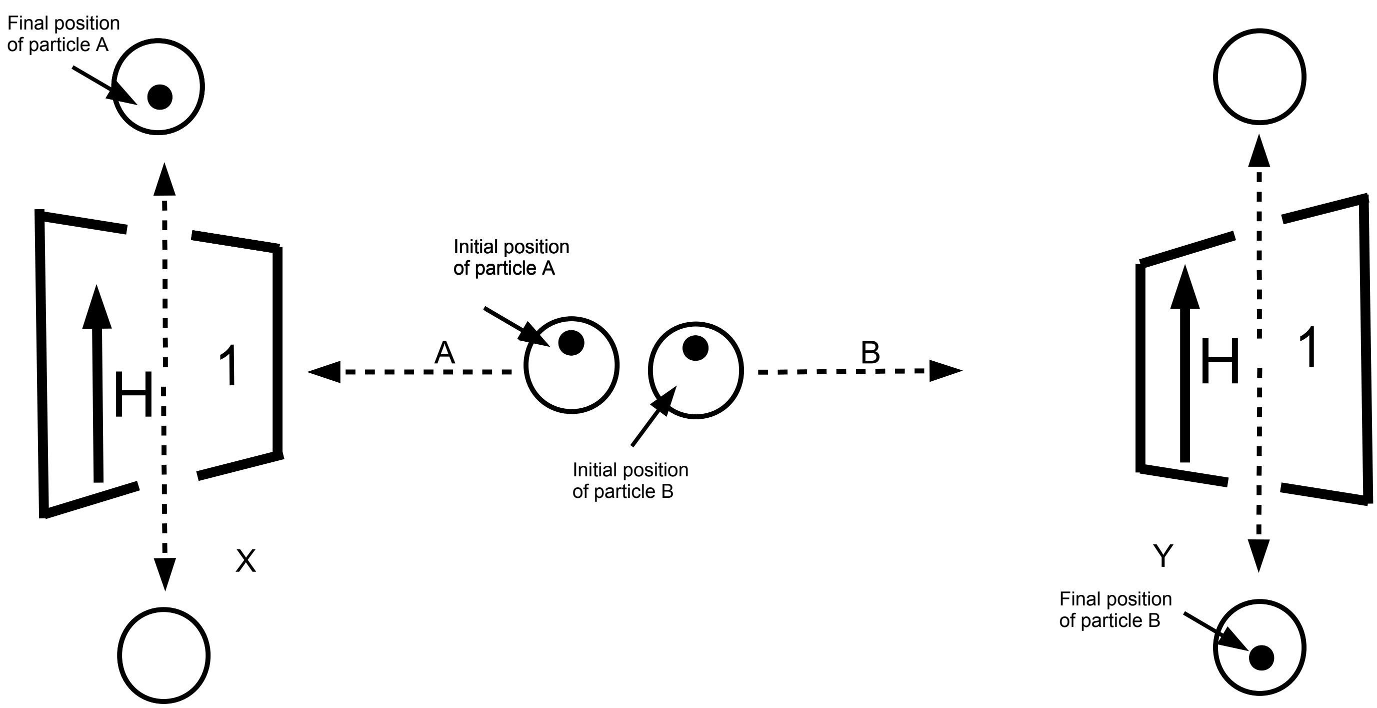
So, with two different arrangements of the apparatus measuring the same spin operator, we get different results, for the same initial conditions of the particle.

This is related to (and explains) the nonlocal character of the de Broglie–Bohm theory.

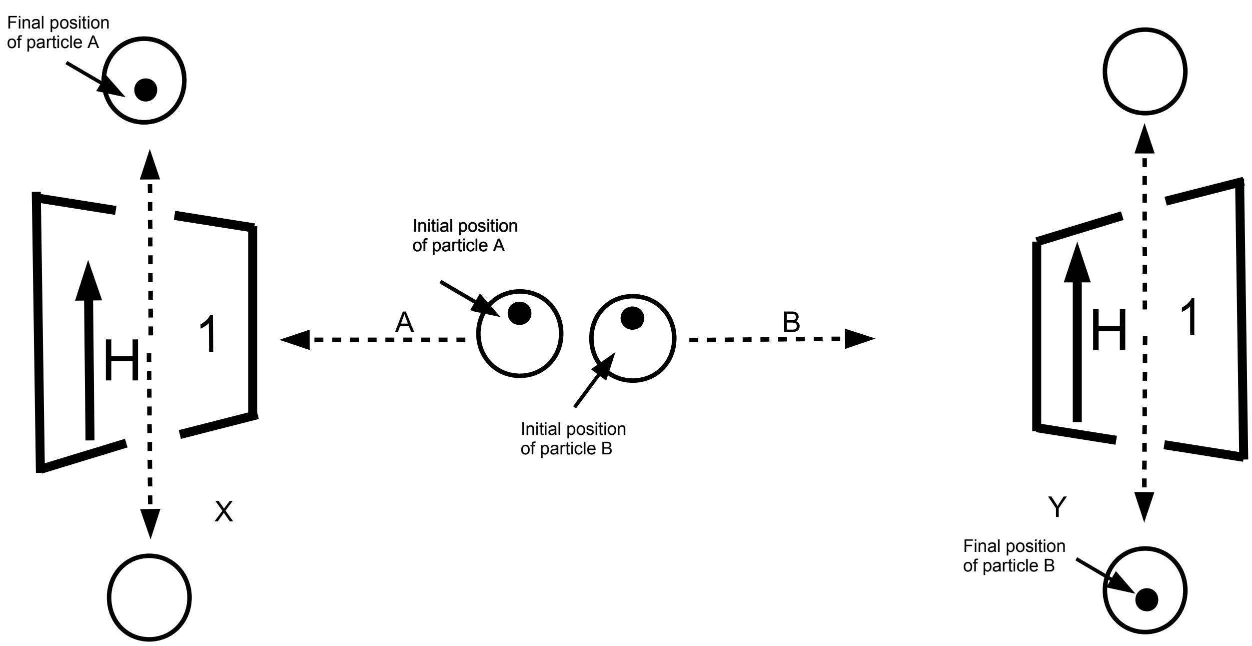
Two particles, A and B are sent towards boxes, located at X and Y, that are perpendicular to the plane of the figure, and in which there is a magnetic field H whose gradient is oriented upwards along the vertical axis, denoted 1. The wave function associated to the particles are represented by disks.



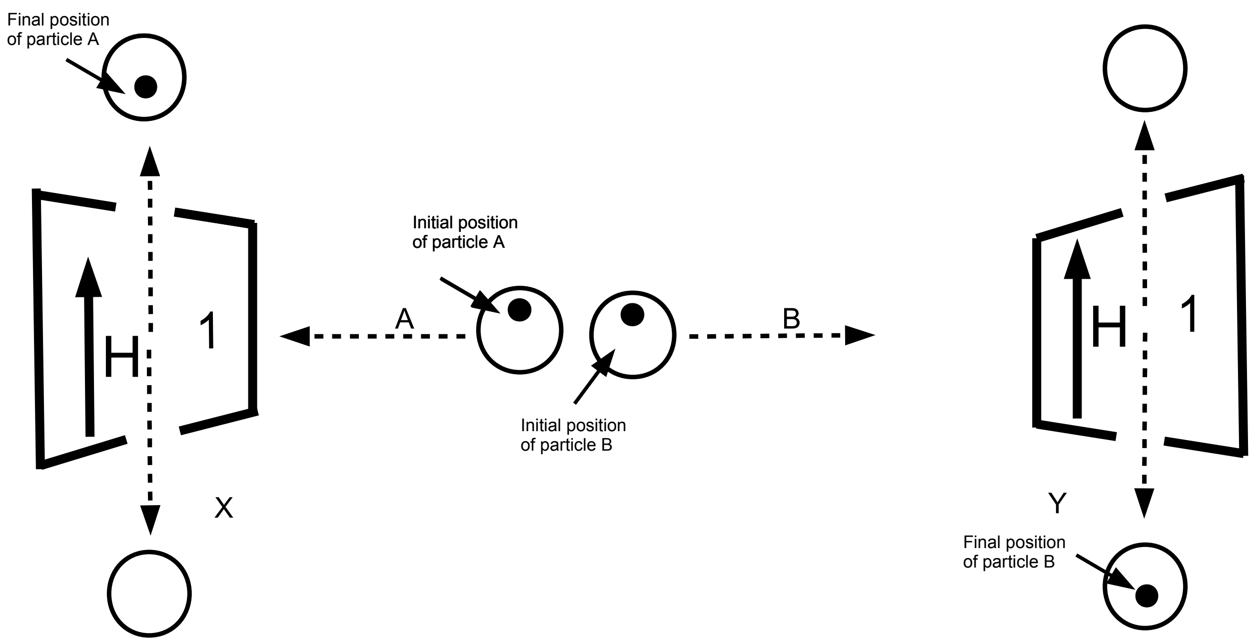
In the boxes, the wave function split into two parts, one going upward in the direction of the gradient of the field, the other going downward, in the direction opposite to the one of the field. The particle positions are indicated by dark dots.



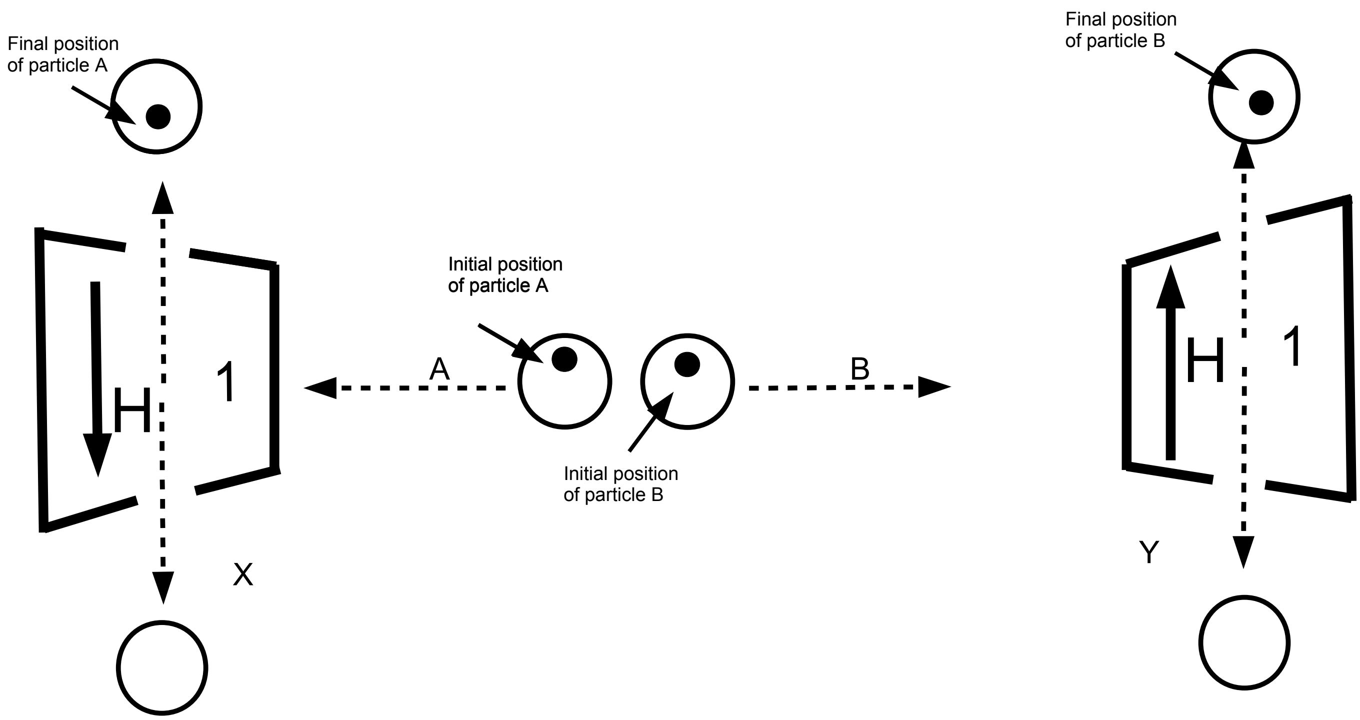
Suppose that we measure the spin of the A particle first. In the de Broglie–Bohm theory, if the A particle starts initially above the horizontal line in the middle of the figure (at the level of the two arrows), it will always go in the upward direction, namely in the direction of the gradient of the field.



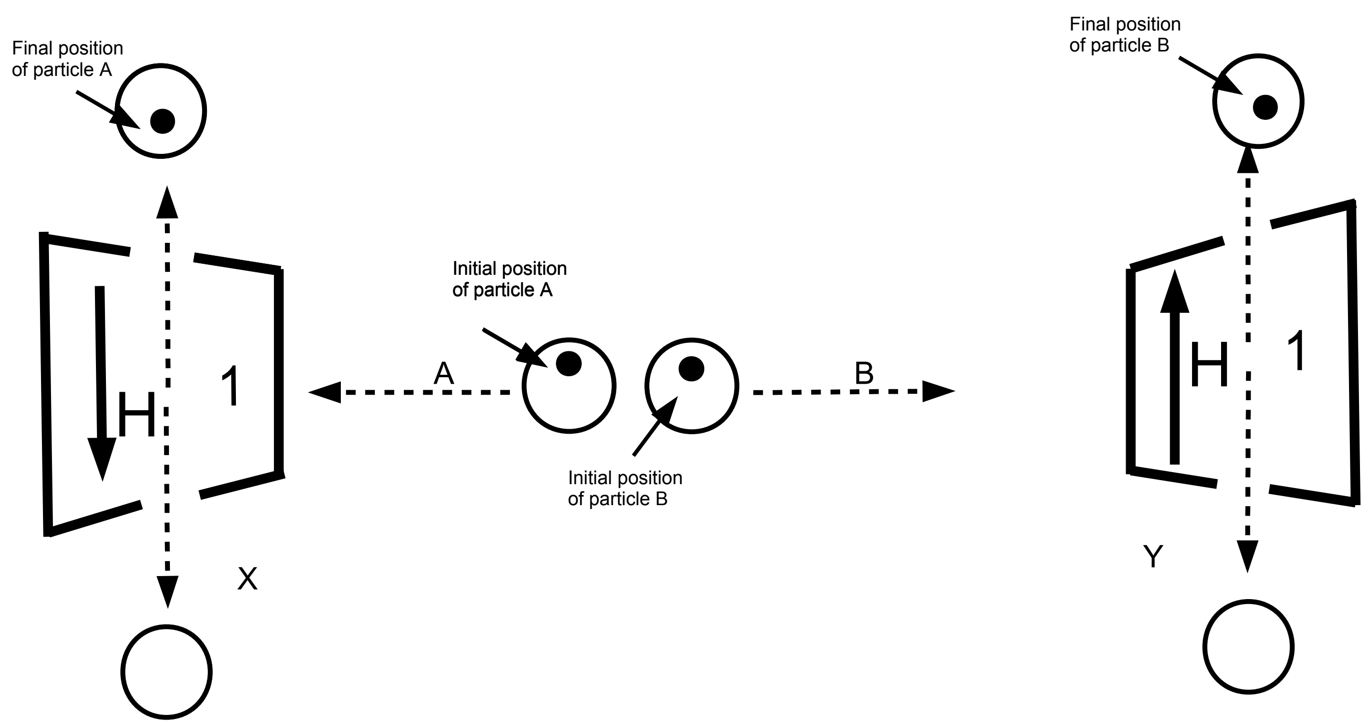
But then, since the wave function of the two particles are such that they are (anti)-correlated, the B particle will have to go in the direction opposite to the one of the field namely downwards.



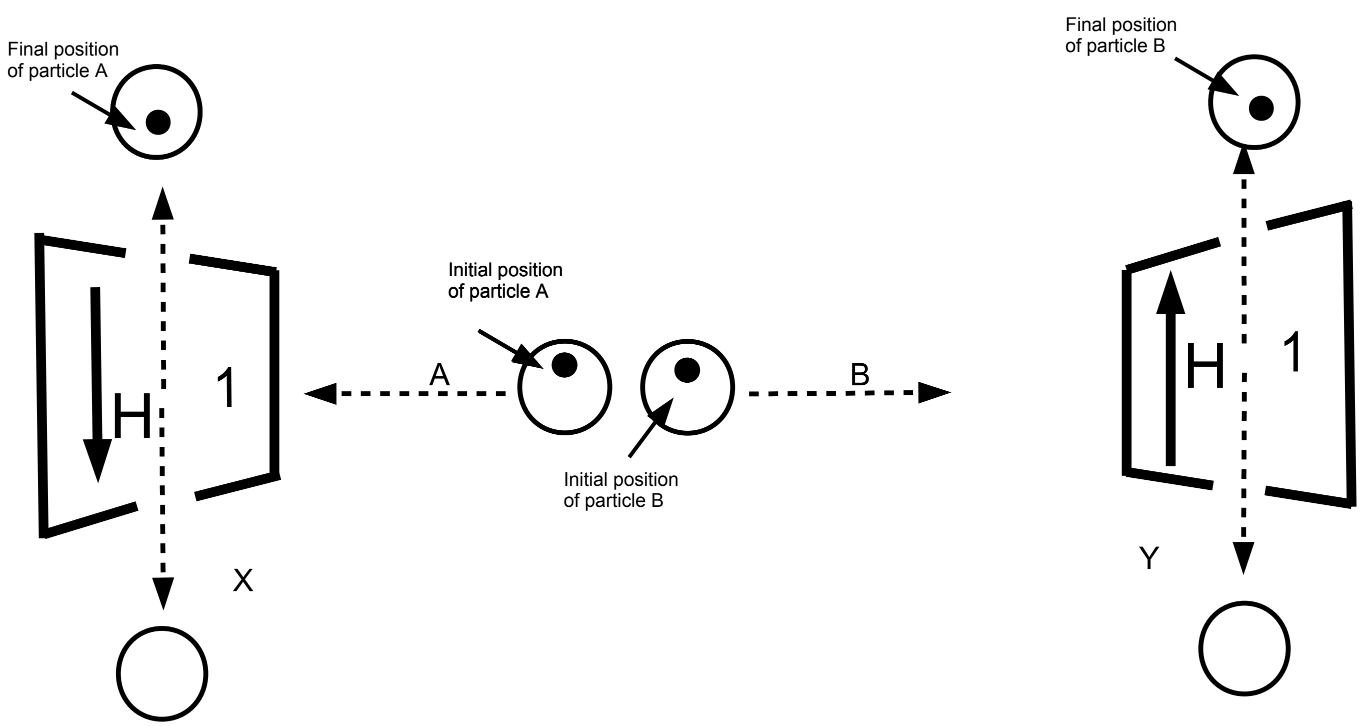
Now, suppose that we reverse the orientation of the gradient of the field on the left relative to the one of the previous figure, but do not change anything on the right and again measure of the spin on the left first.



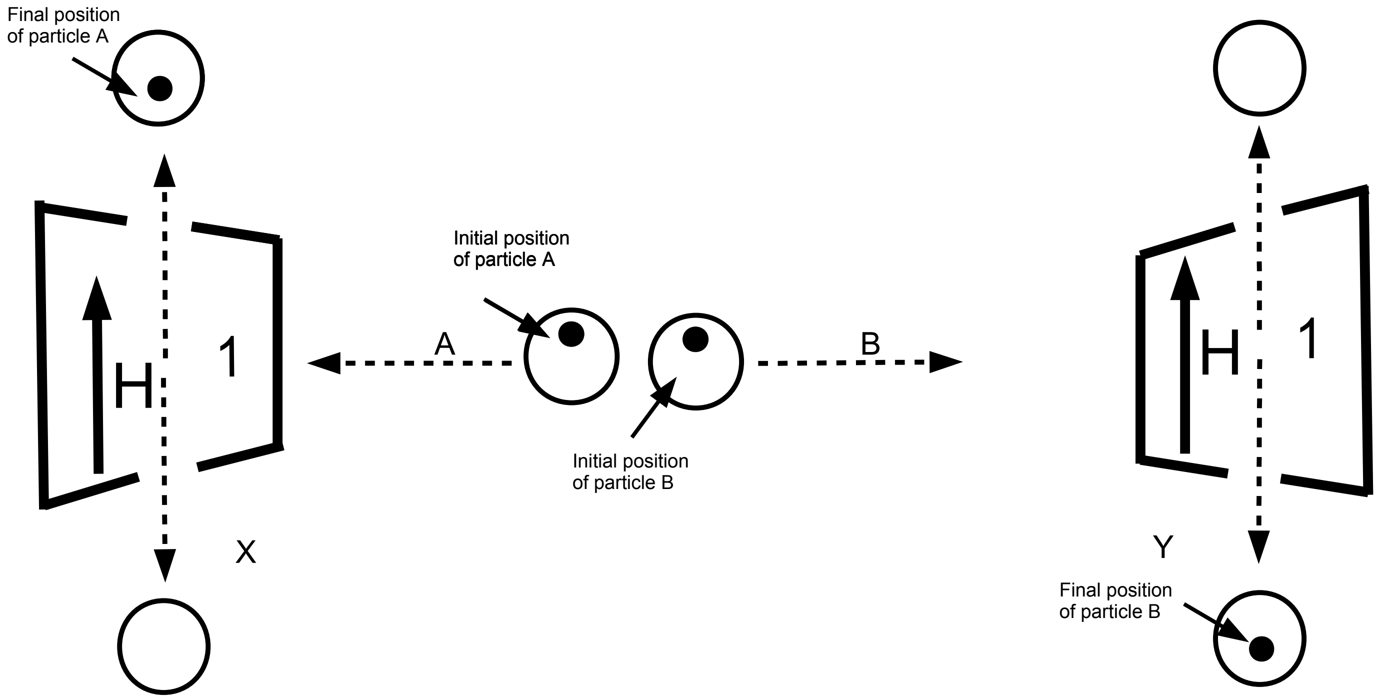
Measure the spin of the A particle first. In the de Broglie–Bohm theory, if the A particle starts initially above the horizontal line in the middle of the figure (at the level of the two arrows), it will always go in the upward direction, namely in the direction opposite to the one of the gradient of the field.

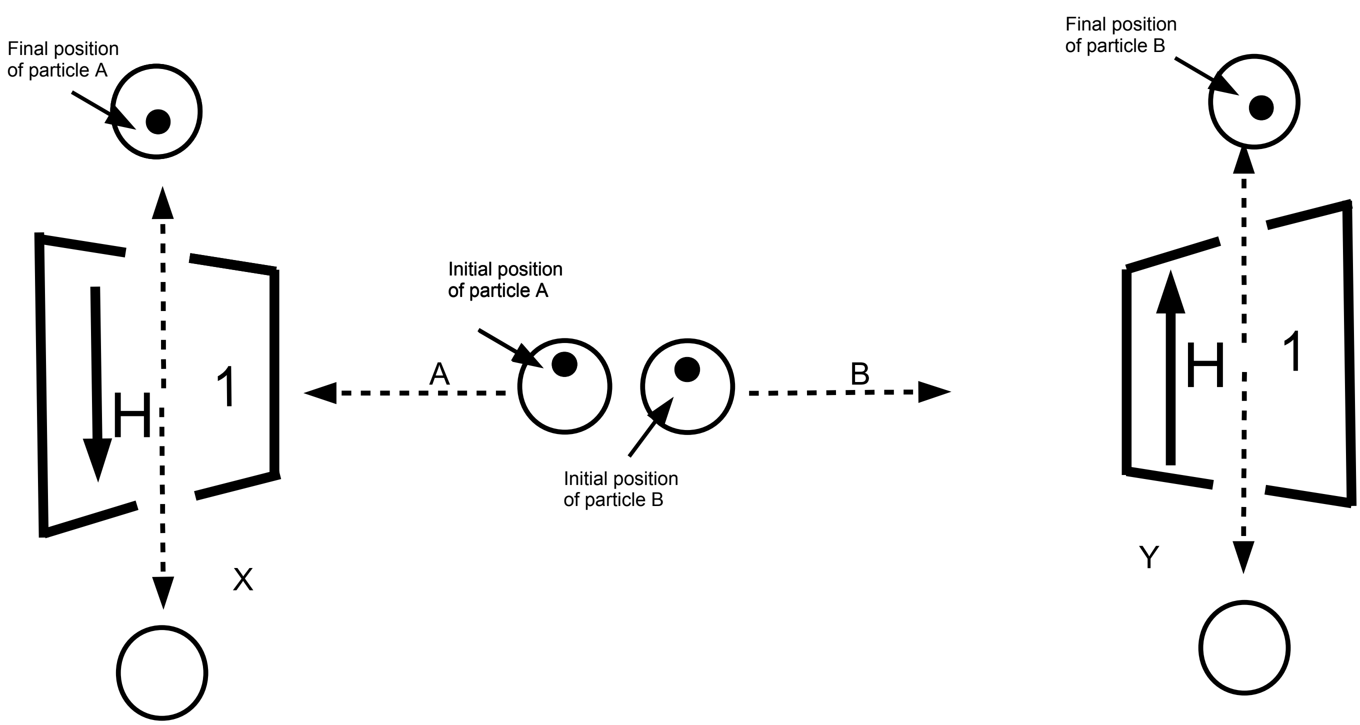


But then, since the wave function of the two particles are such that they are (anti)-correlated, the B particle will have to go in the direction of the gradient of the field, namely upwards.

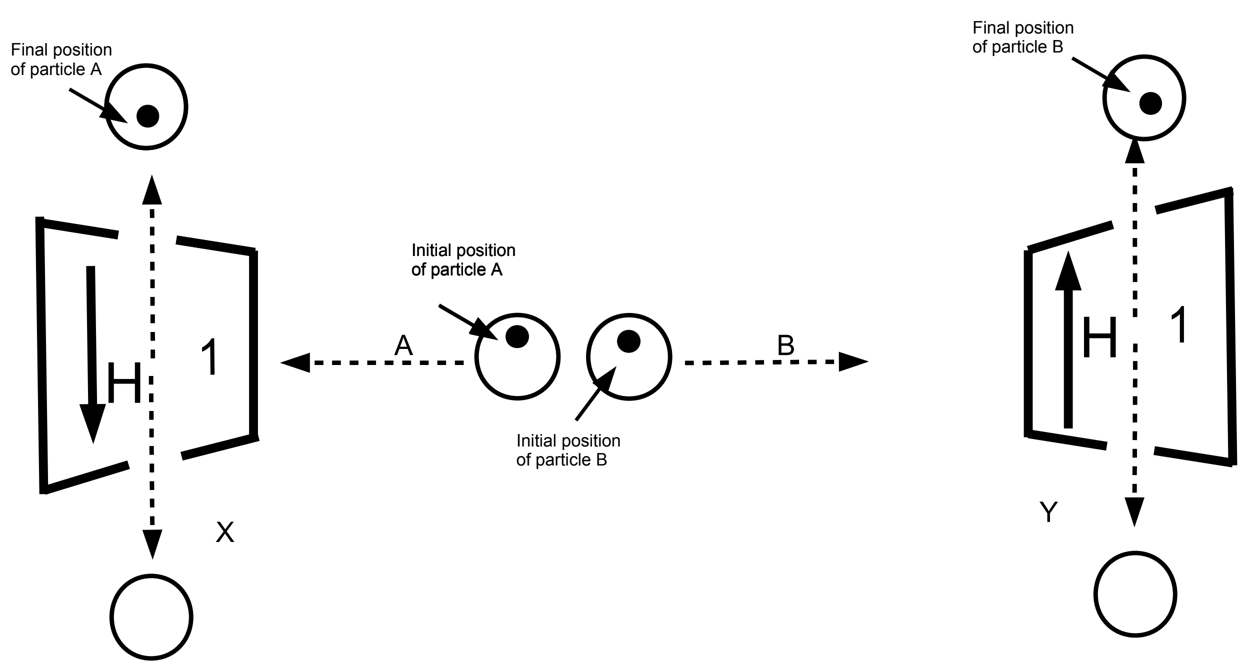


Compare the two figures:

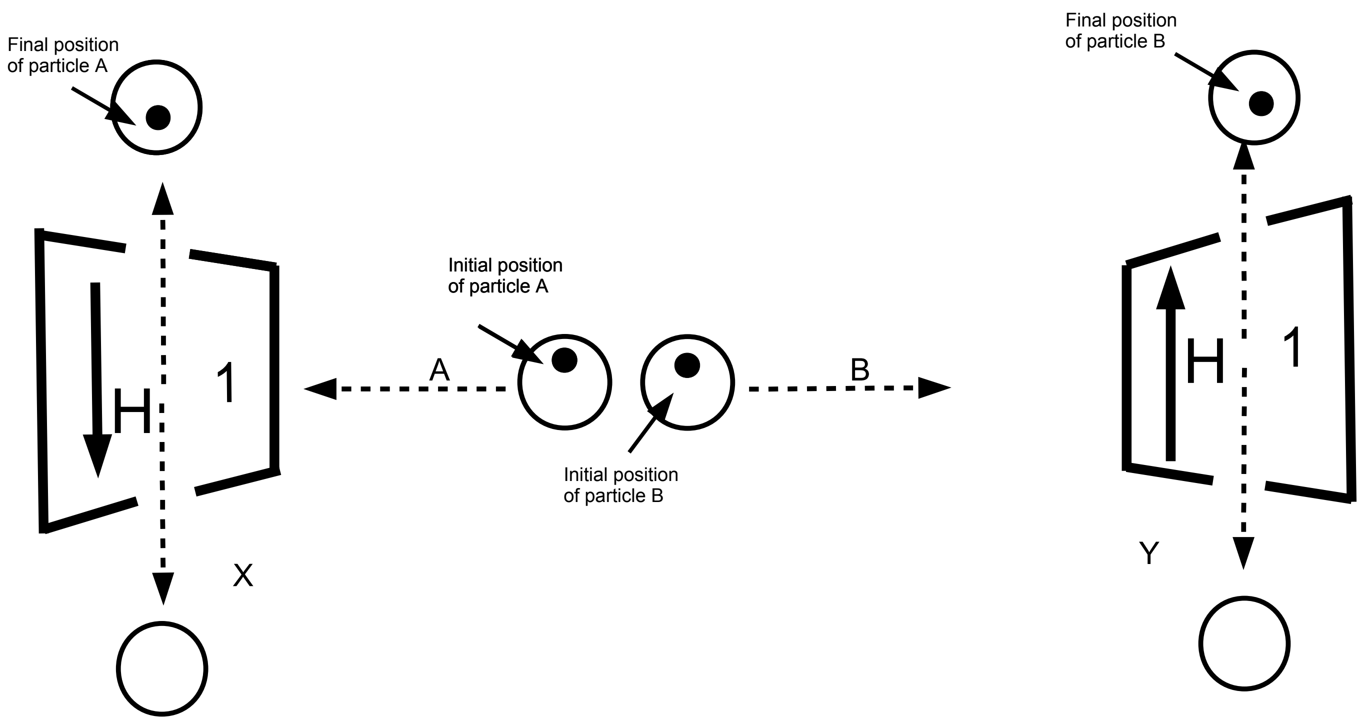




So by changing the orientation of the gradient of the field on the left of the previous figure, while doing nothing whatsoever on the right of that figure, we affect the trajectory of particle B (in one situation, it goes down, in the other one it goes up) which may be arbitrarily far away from the A particle.



This is one of the ways that the action at a distance manifests itself in the de Broglie–Bohm theory.



There is a genuine action at a distance here, since acting on the A particle (by choosing how to "measure its spin") instantly affects the behavior of particle B.

The fact that the de Broglie–Bohm theory is nonlocal is a quality rather than a defect, since we just showed that any theory accounting for the quantum phenomena must be nonlocal.

BELL WAS WIDELY MISUNDERSTOOD

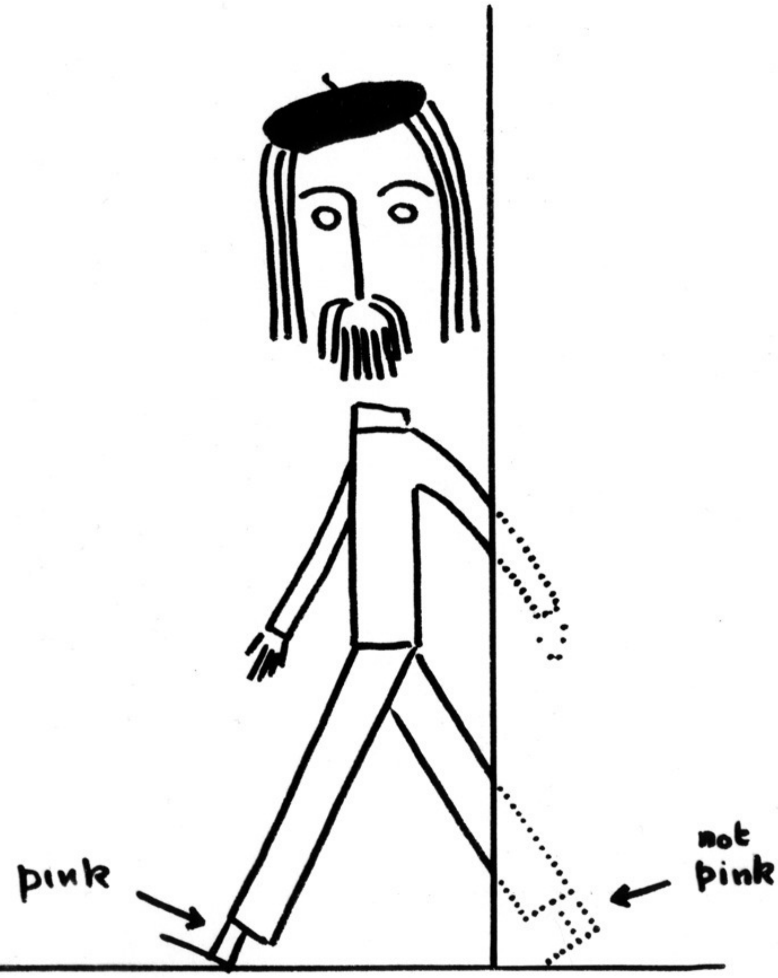
Some theoretical work of John Bell revealed that the EPRB experimental setup could be used to distinguish quantum mechanics from hypothetical hidden variable theories... After the publication of Bell's work, various teams of experimental physicists carried out the EPRB experiment. The result was eagerly awaited, although virtually all physicists were betting on the correctness of quantum mechanics, which was, in fact, vindicated by the outcome.

M. GELL-MANN

Fig 1

Les chaussettes
de M. Bertlmann
et la nature
de la réalité

Fondation Hugot
juin 17 1980



The situation is like that of Bertlmann's socks, described by John Bell in one of his papers. Bertlmann is a mathematician who always wears one pink and one green sock. If you see just one of his feet and spot a green sock, you know immediately that his other foot sports a pink sock. Yet no signal is propagated from one foot to the other. Likewise no signal passes from one photon to the other in the experiment that confirms quantum mechanics. No action at a distance takes place.

M. GELL-MANN (9)

Einstein's view was what would now be called a hidden variables theory. Hidden variables theories might seem to be the most obvious way to incorporate the Uncertainty Principle into physics. They form the basis of the mental picture of the universe, held by many scientists, and almost all philosophers of science. But these hidden variable theories are wrong. The British physicist, John Bell, who died recently, devised an experimental test that would distinguish hidden variable theories. When the experiment was carried out carefully, the results were inconsistent with hidden variables.

S. HAWKING (12)

In the February 2001 issue of *Scientific American*, Max Tegmark and John Archibald Wheeler published an article entitled *100 Years of the Quantum*, where one reads:

> Could the apparent quantum randomness be replaced by some kind of unknown quantity carried out inside particles, so-called 'hidden variables'? CERN theorist John Bell showed that in this case, quantities that could be measured in certain difficult experiments would inevitably disagree with standard quantum predictions. After many years, technology allowed researchers to conduct these experiments and eliminate hidden variables as a possibility.
>
> Tegmark and Wheeler

More recently, the Nobel Prize winner Frank Wilczek wrote:

> In 1964, the physicist John Bell identified certain constraints that must apply to any physical theory that is both local – meaning that physical influences don't travel faster than light – and realistic, meaning that the physical properties of a system exist prior to measurement. But decades of experimental tests, [. . .] show that the world we live in evades those constraints.
>
> Frank Wilczek

All the quotes above commit the same mistake: they ignore the fact that Bell's result, combined with the EPR argument, refutes locality not merely the existence of "hidden variables".

BUT NOT EVERYBODY GOT IT WRONG. After giving an argument similar to the one of Bell, Feynman wrote:

> That's all. That's the difficulty. That's why quantum mechanics can't seem to be imitable by a local classical computer.
>
> I've entertained myself always by squeezing the difficulty of quantum mechanics into a smaller and smaller place, so as to get more and more worried about this particular item. It seems to be almost ridiculous that you can squeeze it to a numerical question that one thing is bigger than another.

R. FEYNMAN (8)

A nice summary:

Contemporary physicists come in two varieties. Type 1 physicists are bothered by EPR and Bell's theorem. Type 2 (the majority) are not, but one has to distinguish two subvarieties. Type 2a physicists explain why they are not bothered. Their explanations tend either to miss the point entirely ... or to contain physical assertions that can be shown to be false. Type 2b are not bothered and refuse to explain why. Their position is unassailable. (There is a variant of type 2b who say that Bohr straightened out the whole business, but refuse to explain how.)

D. MERMIN

CONCLUSION

I know that most men, including those at ease with problems of the highest complexity, can seldom accept even the simplest and most obvious truth if it be such as would oblige them to admit the falsity of conclusions which they have delighted in explaining to colleagues, which they have proudly taught to others, and which they have woven, thread by thread, into the fabric of their lives.

TOLSTOY

APPENDIX 1: PROOF ON NONLOCALITY WITHOUT INEQUALITIES, AND WITHOUT EXPERIMENTS

Introduce Maximally Entangled States

Consider a finite dimensional (complex) Hilbert space $\mathcal{H}$, of dimension N.

A unit vector Ψ in $\mathcal{H} \otimes \mathcal{H}$ is *maximally entangled* if it is of the form:

$$\Psi = \frac{1}{\sqrt{N}} \sum_{n=1}^{N} \psi_n \otimes \phi_n.$$

$$\Psi = \frac{1}{\sqrt{N}} \sum_{n=1}^{N} \psi_n \otimes \phi_n.$$

Since we are interested in quantum mechanics, we will refer to those vectors as *maximally entangled states* and we will associate, by convention, each space in the tensor product to a "physical system," namely we will consider the set $\{\phi_n\}_{n=1}^N$ as a basis of states for physical system 1 (associated to Bob when measurements are made on that system) and the set $\{\psi_n\}_{n=1}^N$ as a basis of states for physical system 2 (associated to Alice when measurements are made on that system).

Now, given a maximally entangled state, one can associate to each operator of the form $1\!\!1 \otimes O$ (meaning that it acts non-trivially only on particle 1) an operator of the form $\tilde{O} \otimes 1\!\!1$ (meaning that it acts non-trivially only on particle 2).

First, one can define an operator U mapping $\mathcal{H}$ to $\mathcal{H}$ by setting

$$U\phi_n = \psi_n,$$

$\forall n = 1, \ldots, N$, and extending U to an anti-unitary operator on all of $\mathcal{H}$.

Using the operator U, the state
$\Psi = \frac{1}{\sqrt{N}} \sum_{n=1}^{N} \psi_n \otimes \phi_n$
can be written as:

$$\Psi = \frac{1}{\sqrt{N}} \sum_{n=1}^{N} U\phi_n \otimes \phi_n.$$

One can check that this formula is the same for any basis.

Then, associate to every operator of the form $1\!\!|\otimes O$ an operator of the form $\tilde{O}\otimes 1\!\!|$ by setting

$$\tilde{O} = UOU^{-1}.$$

Then, if ϕ_n are eigenstates of O, with eigenvalues λ_n,

$$O\phi_n = \lambda_n\phi_n,$$

the states $\psi_n = U\phi_n$ are eigenstates of $\tilde{O}$, also with eigenvalues λ_n:

$$\tilde{O}\psi_n = \lambda_n\psi_n.$$

Let us now generalize the EPR argument by applying this result to spatially separated physical systems.

Suppose that we have a pair of physical systems, whose states belong to the same finite dimensional Hilbert space $\mathcal{H}$. And suppose that the quantum state Ψ of the pair is maximally entangled:

$$\Psi = \frac{1}{\sqrt{N}} \sum_{n=1}^{N} \psi_n \otimes \phi_n.$$

Any “observable” acting on system 1 is represented by a self-adjoint operator O, which has therefore a basis of eigenvectors. Since the representation

$$\Psi = \frac{1}{\sqrt{N}} \sum_{n=1}^{N} U\phi_n \otimes \phi_n$$

of the state Ψ is valid in any basis, we may choose, without loss of generality, as the set $\{\phi_n\}_{n=1}^{N}$ the eigenstates of O. Let λ_n be the corresponding eigenvalues.

$$\Psi = \frac{1}{\sqrt{N}} \sum_{n=1}^{N} U\phi_n \otimes \phi_n$$

Remember that

If ϕ_n are eigenstates of O, with eigenvalues λ_n,

$$O\phi_n = \lambda_n \phi_n.$$

Then, the states $\psi_n = U\phi_n$ are eigenstates of $\tilde{O}$, also with eigenvalues λ_n:

$$\tilde{O}\psi_n = \lambda_n \psi_n.$$

$$\Psi = \frac{1}{\sqrt{N}} \sum_{n=1}^{N} U\phi_n \otimes \phi_n$$

Thus, if one measures that observable O, the result will be one of the eigenvalues λ_n, each having equal probability $\frac{1}{N}$. If the result is λ_k, the (collapsed) state of the system after the measurement, will be $\psi_k \otimes \phi_k$. Then, the measurement of observable $\tilde{O}$, defined by

$$\tilde{O} = UOU^{-1}$$

on system 2, will necessarily yield the value λ_k.

Reciprocally, if one measures an observable $\tilde{O}$ on system 2 and the result is λ_l, the (collapsed) state of the system after the measurement, will be $\psi_l \otimes \psi_l$, and the measurement of observable O on system 1 will necessarily yield the value λ_l.

Let us summarize what we just said:

Principle of perfect correlations.

In any maximally entangled quantum state, there is, for each operator O acting on system 1, an operator $\tilde{O}$ acting on system 2, such that, if one measures the physical quantity represented by operator $\tilde{O}$ on system 2 and the result is the eigenvalue λ_l of $\tilde{O}$, then, measuring the physical quantity represented by operator O on system 1 will yield with certainty the same eigenvalue λ_l, and vice-versa.

The following property will be crucial in the rest of the argument:

Locality. If systems 1 and 2 are spatially separated from each other, then measuring an observable on system 1 has no instantaneous effect whatsoever on system 2 and measuring an observable on system 2 has no instantaneous effect whatsoever on system 1.

Finally, we must also define:

Non-contextual value-maps. Let $\mathcal{H}$ be a finite dimensional Hilbert space and let $\mathcal{A}$ be the set of self-adjoint operators on $\mathcal{H}$. Suppose $\mathcal{H}$ is the quantum state space for a physical system and $\mathcal{A}$ is the set of quantum observables. Suppose there are situations in which there are observables A for which the result of measuring A is determined already, before the measurement.

We would then have a *non-contextual value-map*, namely a map $v : \mathcal{A} \to \mathbb{R}$ that assigns the value $v(A)$ to any experiment associated with what is called in quantum mechanics a "measurement of an observable A." There can be different ways to measure the same observable. The value-map is called non-contextual because all such experiments, associated with the same quantum observable A, are assigned the same value.

We shall need only the following obvious purely mathematical consequence of non-contextuality:

Suppose that, if A_i, $i = 1, \ldots, n$, are mutually commuting self-adjoint operators on $\mathcal{H}$, $[A_i, A_j] = 0$, $\forall i, j = 1, \ldots, n$, and that f is a function of n variables. Then if $B = f(A_1, \ldots, A_n)$, we also have that

$$v(B) = f(v(A_1), \ldots, v(A_n)).$$

Now, the perfect correlations and locality imply the existence of a non-contextual value-map v, for a maximally entangled quantum state.

By the principle of perfect correlations, for any operator O on system 1, there is an operator $\tilde{O}$ on system 2, with which it is perfectly correlated to O.

Thus, if we were to measure $\tilde{O}$, obtaining λ_l, we would know that

$$v(O) = \lambda_l.$$

concerning the result of then measuring O. Therefore $v(O)$ would pre-exist the measurement of O.

But, by the assumption of locality, the measurement of $\tilde{O}$, associated to the second system, could not have had any effect on the first system, and thus, this value $v(O)$ would preexist also the measurement of $\tilde{O}$ and this would not depend upon whether $\tilde{O}$ had been measured. Therefore the map $O \to v(O)$ where O ranges over all operators on system 1, is a non-contextual value-map.

The problem posed by the non-contextual value-map v whose existence is implied by the perfect correlations and locality is that such maps simply do not exist (and that is a purely mathematical result). Indeed, one has the:

Theorem Let $\mathcal{H}$ be a Hilbert space of dimension at least three, and let $\mathcal{A}$ be the set of self-adjoint operators on $\mathcal{H}$. There does not exist a map $v : \mathcal{A} \to \mathbb{R}$ such that:

1) $\forall O \in \mathcal{A}$,

$$v(O) \text{ is an eigenvalue of } O$$

2) $\forall O, O' \in \mathcal{A}$ with $[O, O'] = OO' - O'O = 0$,

$$v(O + O') = v(O) + v(O').$$

Here, we use the implication that, if $B = f(A_1, \ldots, A_n)$, for commuting operators A_i's, we also have

$$v(B) = f(v(A_1), \ldots, v(A_n)),$$

only for $n = 2$ and $f(x, y) = x + y$.

The proof of the existence of a non-contextual value-map v and the theorem on the non-existence of non-contextual value-maps plainly contradict each other. So, the assumptions of at least one of them must be false. Moreover, the theorem on the non-existence of non-contextual value-maps is a purely mathematical result. To derive the existence of a non-contextual value-map v, we assumed only the perfect correlations and locality. The perfect correlations are an immediate consequence of quantum mechanics. The only remaining assumption is locality. Hence we can deduce:

Nonlocality Theorem. The locality assumption is false.

APPENDIX 2

Let us derive the number $\frac{1}{4}$ mentioned above, for the anti-correlations and an appropriate choice of the directions $1, 2, 3$.

Compute first $\mathbf{E}_{\alpha,\beta} \equiv \langle\Psi|\sigma_\alpha^A \otimes \sigma_\beta^B|\Psi\rangle$, where α, β are unit vectors in the directions (1, 2, or 3, specified below) in which the spin is measured at X or Y, and $\sigma_\alpha^A \otimes \sigma_\beta^B$ is a tensor product of matrices, each one acting on the A or B part of the quantum state, with $\sigma_\alpha^A = \alpha_1\sigma_1 + \alpha_2\sigma_2 + \alpha_3\sigma_3$, where, for $i = 1, 2, 3$, α_i are the components of α and σ_i the usual Pauli matrices.

The matrix σ_α^A is the spin operator that is "measured" when one "measures the spin" in direction α for the A particle and similarly for σ_β^B.

So, $\mathbf{E}_{\alpha,\beta} = \langle\Psi|\sigma_\alpha^A \otimes \sigma_\beta^B|\Psi\rangle$ is the expectation value of the measurement of the spin in direction α at X and in direction β at Y, when the quantum state is Ψ.

The quantity $\mathbf{E}_{\alpha,\beta} = \langle\Psi|\sigma^A_\alpha \otimes \sigma^B_\beta|\Psi\rangle$is bilinear in α, β and rotation invariant, so it must be of the form $\lambda\alpha \cdot \beta$, for some $\lambda \in \mathbf{R}$.

For $\alpha = \beta$, the result must be -1, because of the anti-correlations (if the spin is up at A, it must be down at B and vice versa). So $\lambda = -1$, and thus $\mathbf{E}_{\alpha,\beta} = -\cos\theta$, where θ is the angle between the directions α and β.

If we introduce the “hidden variables” $A(\alpha), B(\beta) = \pm 1$, and consider $\mathbf{E}_{\alpha,\beta}$ as an expectation value over those variables, we have:

$$\begin{aligned}\mathbf{E}_{\alpha,\beta} &= P\big(A(\alpha) = B(\beta)\big) - P\big(A(\alpha) = -B(\beta)\big) \\ &= 1 - 2P\big(A(\alpha) = -B(\beta)\big)\end{aligned}$$

and thus

$$P\big(A(\alpha) = -B(\beta)\big) = \frac{1 - \mathbf{E}_{\alpha,\beta}}{2} = \frac{1 + \cos\theta}{2} \,.$$

since $\mathbf{E}_{\alpha,\beta} = -\cos\theta$.

One then chooses the directions

$$1 \longleftrightarrow 0 \text{ degree}\,,$$

$$2 \longleftrightarrow 120 \text{ degree}\,,$$

$$3 \longleftrightarrow 240 \text{ degree}\,.$$

Since $\cos 120 = \cos 240 = -\frac{1}{2}$, we get

$$P\big(A(\alpha) = -B(\beta)\big) = \frac{1 + \cos\theta}{2} = \frac{1}{4}$$

Thus we have perfect anticorrelations only $\frac{1}{4}$ of the time when α and β are different. With our convention, this means that one gets the same answer when one asks different questions on both sides only $\frac{1}{4}$ of the time.

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